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HOW TO MEASURE THE DIMENSION OF THE PARTON FIELD

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Abstract: We show that the anomalous dimension of the fundamental field is connected to the anomalous dimensions of the high spin bilinear operators. The dimensions of these operators can be determined by examining the violations of the Bjorken scaling law in deep-inelastic electron-proton scattering. The structure function at field ω near to one has a power dependence of q^2, the exponent being the anomalous dimension of the "parton" field.

1. Introduction

In recent years many speculations have been made on the possibility that the short-distance behaviour of an interacting theory is different from that of a free theory [1]: the dimensions of nearly all the operators are different from the canonical one; the Green functions of renormalized fields are more singular at short distances that in free field theory; canonical commutation relations are therefore destroyed.

This very interesting conjecture must be tested by examining the off-mass-shell quantities, and this can be done only using currents coupled to leptons. One can measure only the dimensions of those operators which contribute to the leading term of the Wilson expansion of two currents near the light cone; the field is not one of these operators; it therefore seems impossible to measure the dimension d_ϕ of the "parton" field.

The aim of this paper is to show that the situation is not sad: d_ϕ cannot be measured directly, but it can be connected to the observable dimensions of other operators.

In deep-inelastic electron-proton scattering we define in the standard way the function

$$F_2(\omega, q^2) = \nu W_2(\nu, q^2)\,, \qquad \omega = -\frac{2\nu}{q^2}\,.$$

If anomalous dimensions are present, Bjorken scaling is no more valid, but each of the following integrals has asymptotically a simple power behaviour [3]:

$$\int_1^\infty d\omega\, \omega^{-N} F_2(\omega, q^2) \xrightarrow[q^2 \to -\infty]{} C_N \left(\frac{M^2}{-q^2}\right)^{\frac{1}{2}A_N} \tag{1}$$

A_N is the anomalous dimension of the spin N operator and can be determined from a careful analysis of the experimental data.

We prove that in a $\lambda\phi^4$ theory in four dimensions

$$A_\infty \equiv \lim_{N\to\infty} A_N = 2(d_\phi - 1) \equiv 2A_\phi \,, \tag{2}$$

$$A_\infty - A_N = GN^{-H}, \qquad G > 0,\ H > 0\,. \tag{3}$$

We note that the dimension d_N of the operator

$$O_{\mu_1, \mu_N} = \phi^+ \overleftrightarrow{D}_{\mu_1} \ldots \overleftrightarrow{D}_{\mu_N} \phi - \text{traces} \tag{4}$$

is equal to $2 + N + A_N$; the physical interpretation of formula (2) is that the dimension becomes additive in the case of high spin operators.

Additional interest in this fact comes from the observation that for high N the integrals (1) are dominated by the region at $\omega \sim 1$. This implies that [4]

$$F_2\,(\omega, q^2) \to f(\omega) \left(\frac{M^2}{-q^2}\right)^{A_\phi} \tag{5}$$

for fixed ω such that $1 \gg (\omega - 1) \neq 0$.

The anomalous dimension of the fundamental field is thus simply connected to the breakdown of Bjorken scaling at ω near to one.

The difficulties to extend the proof to more realistic situations like the σ model are due to the more complicated structure of the Dyson equations in the presence of more than one fundamental field.

In sect. 2 we discuss the scaling properties of the zero-mass limit and the Dyson equation for the vertex of the O_{μ_1, μ_N} operators with two fields ϕ.

In sect. 3 we study the properties of the Bethe–Salpeter kernel and we present a detailed proof of formulae (2), (3).

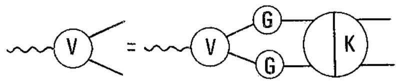
In the last section we discuss our results and some possible extensions.

2. The Dyson equation

The best way to investigate the dimension spectrum of operators is to study the solution of the Dyson equation for the vertex of these operators. To simplify the computations we restrict ourselves to a zero-mass scale-invariant theory. This restriction yields no problem because the dimensions of the operators are mass independent [5] and it has been shown by Symanzik [6] that the Gell–Mann–Low limit of a renormalizable quantum field theory exists and it is scale invariant, provided that a certain function $\beta(g)$ has a zero.

We are interested in operators which give the leading contribution on the light cone: they must be symmetric traceless tensors transforming like the $(\frac{1}{2}n, \frac{1}{2}n)$ representation of the Lorentz group: they can be defined as in (4).

They must satisfy the following Dyson equation:



(6)

where G is the propagator of the field ϕ, V is the $\phi\phi O_{\mu_1,\mu_N}$ vertex, and K is the Bethe–Salpther kernel which is the sum of all two-particle irreducible graphs. The inhomogeneous term cannot be present because it transforms under scale transformations in a different way from the other term [7,8].

If we introduce the Fourier transform of the functions, into the Dyson equation, it can be rewritten as

$$V_{\mu_1,\mu_N}(p+K, -p+K) = \int d^4q\, V_{\mu_1,\mu_N}(q+K, -q+K)\, G(q+K)\, G(-q+K)$$

$$\times K(q+K, -q+K; p+K, -p+K) . \tag{7}$$

The scale properties of the functions involved in the Dyson equation are

$$G(\lambda p) = \lambda^{-4+2d_\phi} G(p),$$

$$K(\lambda p_1, \lambda p_2; \lambda p_3, \lambda p_4) = \lambda^{4-4d_\phi} K(p_1, p_2; p_3, p_4) ,$$

$$V_{\mu_1,\mu_N}(\lambda p_1, \lambda p_2) = \lambda^{-2d_\phi + d_N} V_{\mu_1,\mu_N}(p_1, p_2) . \tag{8}$$

This equation must be satisfied at arbitrary values of the external momenta; we study it when K is equal to zero.

The existence of the Fourier transform of the vertex in this particular situation, i.e. the absence of infrared divergences, can be proved when the dimension of O_{μ_1,μ_N} is greater than two using the Araky–Haag–Ruelle [9] theorem on cluster decomposition in zero-mass theories.

From eq. (8)

$$V_{\mu_1,\mu_N}(p, -p) = \alpha (p^2)^{-d_\phi + \frac{1}{2}(d_N - N)} [p_{\mu_1} \ldots p_{\mu_N} - \text{traces}] . \tag{9}$$

The constant of proportionality is of no interest, eq. (8) being homogeneous in the vertex.

We introduce an arbitrary vector t_μ:

$$V_{\mu_1,\mu_N}(p,-p)\,t^{\mu_1}\dots t^{\mu_N} = C_N(pt)\,(p^2)^{-d_\phi+\frac{1}{2}(d_N-N)}\,, \tag{10}$$

where C_N is the Chebyshev polynomial of order N.

The Dyson equation at zero momentum transfer can therefore be rewritten as

$$(p^2)^{-\delta}\,C_N(pt) = \int \mathrm{d}^4q\,\bar{K}(p,-p;q,-q)\,(q^2)^{-\delta}\,C_N(qt)\,, \tag{11}$$

where

$$\delta = d_\phi - \tfrac{1}{2}(d_N-N) \equiv A_\phi - \tfrac{1}{2}A_N\,,$$

$$\bar{K}(p,-p;q,-q) = K(p,-p;q,-q)\,G^2(q)\,,$$

$$\bar{K}(\lambda p,-\lambda p;\lambda q,-\lambda q) = \lambda^{-4}\,\bar{K}(p,-p;q,-q)\,. \tag{12}$$

The possible values for the dimension of a spin-N operator are obtained improving the validity of (12).

In the next section we shall prove that irrespectively of the detailed form of K, eq. (11) admits solutions in which A_N goes to $2A_\phi$ when N goes to infinity.

3. The solutions of the Dyson equation

The key ingredient we need to prove our statements is the fact that $\bar{K}(0,0;q,-q)$ is free of infrared divergences also in a zero-mass field theory. This fact was first recognized by Baker, Johnson and Wiley in their fundamental paper on zero-mass electrodynamics. Their proof can be extended without difficulties also to the case of $\lambda\phi^4$ theory. This result united with the scaling properties of the function $\bar{K}$ tell us that

$$\bar{K}(p,-p;q,-q) \xrightarrow[q^2\to\infty]{} (q^2)^{-2}\,,$$

i.e. the short distance singularity of the kernel is logarithmic in the x-space.

We can use this information to write a Deser–Gilbert–Sudershan (D.G.S.) representation for spacelike p of the form [12,13]

$$\bar{K}(p,-q;q,-q) = \int_0^\infty \mathrm{d}a \int_{-1}^{1} \mathrm{d}b\,\frac{\bar{\sigma}(a,b,p^2)}{[a+K^2+p^2+2bpK]^2}\,. \tag{13}$$

The limits of integration are fixed as usual by the spectrum condition.

We insert this representation in the Dyson equation and we make a Wick rotation on the internal integration to avoid any possible convergence problem.

It is now clear that if $\bar{K}$ would be a regular function, at fixed A_N the r.h.s. would go exponentially to zero, so that the Dyson equation could not ever be satisfied unless we chose values of A_ϕ such that the Q^2 integral becomes singular: this happens just for $A_N = 2A_\phi$. This would imply that in the large N limit A_N goes to A_ϕ in exponentially rapid way. The real situation is different: $\bar{K}$ is not a completely regular function, but its singularity can be controlled using the D.G.S. representation; the final result will be that A_N goes to A_ϕ with a speed that is proportional to some inverse power of N, the power being fixed by the threshold behaviour of the kernel.

To proceed further we exchange the D.G.S. representation with the q integral. The q integral can be performed using the formula

$$\int \frac{\mathrm{d}^4q\,(q^2)^{-\delta}\,C_N(tq)}{[p^2+q^2+2bpq+a]^2} = \pi^2(a+p^2)^{-\delta}\,b^N B(N+2-\delta,\delta)$$

$$\times\,{}_2F_1\left(\delta,N+2-\delta;N;\frac{bp^2}{a+p^2}\right)C_N(pt) \tag{14}$$

(we now use the Euclidean metric).

This formula reduces in the large-N limit to

$$\pi^2\Gamma(\delta)N^{-\delta}b^N\,[p^2(1-b)+a]^{-\delta}\ . \tag{15}$$

The self-consistency conditions deduced from the Dyson equation can be written for large N in the following way

$$\pi^2\Gamma(\delta)N^{-\delta}\int_0^\infty \mathrm{d}a\int_{-1}^{1}\mathrm{d}b\,\sigma(a,b,p^2)\,b^N\,[1-b+a/p^2]^{-\delta} = 1\ . \tag{16}$$

The r.h.s. of (16) must go to zero at fixed δ when N goes to infinity; both the factors $N^{-\delta}$ and b^N go to zero. A $\delta(b-1)$ singularity cannot be present without producing a too singular behaviour of the kernel at threshold.

The conclusion is that for large N and small δ the r.h.s. can be approximated by GN^{-H}/δ where H and G are positive numbers connected to the threshold singularity of the kernel. Solving for δ and recalling definition (12) we find the result

$$2A_\phi - A_N \xrightarrow[N\to\infty]{} GN^{-H}. \tag{17}$$

4. Conclusions

We have just found that the anomalous dimensions of the high-spin operators are connected to the anomalous dimension of the parton field. $A_N = A_\phi - GN^{-H}$.

Unfortunately present SLAC experiment do not pose relevant restrictions on the value of the index A_∞: one should know the behaviour of the structure functions at

ω near to one but outside the resonance region and this implies an enormous value of q^2: future experiments will hopefully yield more stringent limits on A_∞.

A by-product of our analysis is a new proof of two different known results: Bjorken scaling ($A_N = 0\ \forall N$) can be valid only in an asymptotic free theory (A_ϕ=0); if Bjorken scaling is not valid all the integrals in [1] must scale with a different power.

As a final remark we note that our results can be readily extended both to a $\lambda\phi^3$ theory in $6 + \epsilon$ dimensions and to a $\lambda\phi^4$ theory in $4 - \epsilon$ dimensions. They agree with the explicit calculations of Mack [17] and Wilson [18].

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