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A. Bramon and M. Greco: A FINITE DISPERSION RELATIONS
APPROACH TO $\eta \rightarrow \pi^+ \pi^- \gamma$ DECAY. -

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ABSTRACT. -

Finite Dispersion Relations are used to obtain $\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma) \simeq (41 \pm 16)$ eV. Through the known branching ratio $(\eta \rightarrow \pi^+ \pi^- \gamma) / (\eta \rightarrow \gamma \gamma)$ we deduce $\Gamma(\eta \rightarrow \gamma \gamma) \simeq 310 \pm 120$ eV, in good agreement with the new determination of the $\eta \rightarrow \gamma \gamma$ partial width.

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It is generally accepted that the precise knowledge of the decay width of the η meson is of considerable relevance in particle physics, not only for the theoretical interpretation of the observed properties of the η meson itself, but also for our understanding of the schemes of particle classification.

In a recent experiment Browman et al.⁽¹⁾ have measured the $\eta \rightarrow \gamma\gamma$ partial width, obtaining

$$(1) \quad \Gamma(\eta \rightarrow \gamma\gamma) = 374 \pm 60 \text{ eV},$$

to be contrasted with the only previously available result

$$(2) \quad \Gamma(\eta \rightarrow \gamma\gamma) = 1000 \pm 220 \text{ eV},$$

obtained by Bemporad et al.^(2,3). Both measurements of $\Gamma(\eta \rightarrow \gamma\gamma)$ have been performed through the Primakoff effect. The new result however, having been obtained at higher energies, is much less sensitive to the uncertainties arising in extracting from the experimental yield the contribution of the simultaneous coherent nuclear production^(x), and seems therefore to be preferred.

The purpose of this work is to present a theoretical approach, based on the use of Finite Dispersion Relations (FDR), which allows a rather self-consistent determination of $\Gamma(\eta \rightarrow \gamma\gamma)$ and therefore a discrimination between the results (1) and (2). Our conclusion favours the new determination of $\Gamma(\eta \rightarrow \gamma\gamma)$.

FDR have been introduced by Aviv and Nussinov⁽⁴⁾ in order to treat the three-body decays in a way compatible with duality.

(x) - The authors are grateful to C. Bemporad for an enlightening discussion on this subject.

Here we apply these ideas to obtain the $\eta \rightarrow \pi^+ \pi^- \gamma$ decay rate, from which one can deduce $\Gamma(\eta \rightarrow \gamma \gamma)$ by using the well established branching ratio⁽³⁾

$$(3) \quad R_\eta \equiv \frac{\eta \rightarrow \pi^+ \pi^- \gamma}{\eta \rightarrow \gamma \gamma} = \frac{(5.0 \pm 0.1)\%}{(38 \pm 1)\%} = 0.132 \pm 0.005.$$

A similar analysis was presented by the authors in a previous paper⁽⁵⁾ and, more recently, by Young and Lassila⁽⁶⁾, who have obtained rather different results⁽⁷⁾.

The matrix element for the $\eta \rightarrow \pi^+ \pi^- \gamma$ transition may be written as

$$T(\nu, t) = A(\nu, t) \varepsilon_{\alpha\beta\gamma\delta} \varepsilon^\alpha_{q_1} \varepsilon^\beta_{q_2} k^\delta,$$

where $q_{1,2}$ are the π^\pm momenta, and k, ε are the momentum and the polarization of the photon. We define $\nu \equiv (s-u)/4$, $s = (q_2 + k)^2$, $t = (q_1 + q_2)^2$ and $\Sigma \equiv s + t + u = m_\eta^2 + 2m_\pi^2$.

According to FDR the amplitude $A(\nu, t)$ receives two types of contributions. The first one comes from the nearby s- and u-channel singularities appearing for values of ν smaller than a fixed cut-off parameter N . In our specific case these low-mass contributions are simply given by the $A_{\frac{1}{2}}^+$ poles as explicitly shown in eq. (4). The second contribution to $A(\nu, t)$ accounts for the remaining high-mass resonant states and is dominated by the leading ρ trajectory exchanged in the t channel, for a sufficiently large N . In the spirit of duality these two contributions are not independent and may be related by means of FESR.

As a total result we obtain⁽⁵⁾:

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$$(4) \quad A(v, t) \simeq \frac{2g_{A\eta\pi}g_{A\pi\gamma}}{s-m_A^2} \left\{ 2t+s - \Sigma - \frac{m_\pi^2}{s} (m_\eta^2 - m_\pi^2) \right\} +$$

$$+ (s \rightarrow u) + \frac{2g_{\rho\pi\pi}g_{\rho\eta\gamma}}{t-m_\rho^2} \frac{(2\alpha'N)^{\alpha-1}}{\Gamma(\alpha)},$$

where $\alpha \equiv \alpha_\rho(t) = 1/2 + t/2m_\rho^2$. The coupling constants appearing in the preceding equation have been defined according to references (5) and (8) and all of them, but $g_{\rho\eta\gamma}$, can be deduced with reasonable accuracy from the ρ and A_2 decay widths. Using the results quoted in PDG compilation⁽³⁾ one gets $g_{\rho\pi\pi} \simeq 6.0$, $g_{A\eta\pi} \simeq 2.8 \text{ GeV}^{-1}$ and $g_{A\pi\gamma} = g_{A\pi\rho} e/f_\rho \simeq 0.46 \text{ GeV}^{-2}$. For future convenience we also introduce the ratio

$$K \equiv g_{\rho\pi\pi} g_{\rho\eta\gamma} / g_{A\eta\pi} g_{A\pi\gamma} \simeq 4.66 \text{ GeV}^3 g_{\rho\eta\gamma}.$$

The first-moment FESR is written as:

$$(5) \quad (t+2m_A^2 - \Sigma) \left[2t+m_A^2 - \Sigma - \frac{m_\pi^2}{2m_A} (m_\eta^2 - m_\pi^2) \right] =$$

$$= K \frac{4\alpha'}{\Gamma(\alpha)} \frac{N^2 (2\alpha'N)^{\alpha-1}}{\alpha+1}.$$

The fulfilment of eq. (5), at least for small values of t , gives the relationship between the cut-off N and the ratio K .

It is now possible to obtain from the above equations an approximate upper bound for $\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma)$, without any additional inputs.

There exists infact, in the context of the model, a lowest limit for the cut-off N . This corresponds to choose the lowest N compatible with the inclusion of the A_2^+ poles in the s-and u-channels, namely

$$N_{\min} = \frac{1}{4} (2 \bar{s}_{\min} - \Sigma + t) \simeq 0.9 \text{ GeV}^2 + \frac{1}{4} t$$

with $\bar{s}_{\min} \simeq m_A^2 + 2m_A \Gamma_A$. The FESR (5) is then well satisfied with $K = K_{\max} \simeq 4.7 \text{ GeV}^2$. Under the assumption of the adequacy of the Regge description in the t channel, also for this low value of N , one obtains:

$$\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma) \lesssim 150 \text{ eV}$$

or, using (3) ,

$$\Gamma(\eta \rightarrow \gamma \gamma) \lesssim 1100 \text{ eV},$$

which is quite near the old experimental result (2).

A more definite prediction can be obtained either by giving a prescription for the choice of the cut-off N , or by an additional information on the coupling constant $g_{\rho\eta\gamma}$. In the first case the choice of N is usually performed by taking an average value between the last resonance included, which in our case is the A_2 meson, and its Regge recurrence. In the second case, in absence of experimental information, one has to trust a specific theoretical estimate for $g_{\rho\eta\gamma}$. The procedure we shall adopt in the following meets simultaneously both criteria. This strongly supports our final result.

Consider the two ratios $\alpha \equiv g_{A\eta\pi}/g_{f\pi\pi}$ and $\beta \equiv g_{\rho\eta\rho}/g_{\rho\omega\pi} = g_{\rho\eta\gamma}/g_{\omega\pi\gamma}$, where the last equality follows from VMD. In the context of the simple quark model, where one assumes that the $\lambda\bar{\lambda}$ component of the η decouples from the purely non-strange quark systems ρ, A_2

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and π , the two ratios α and β become equal and give also a measure of the non-strange quark content of the η meson. This leads to

$$\alpha^2 \simeq 0.5^{(9, \text{x})}, \text{ and}$$

$$K = \frac{g_{\rho\pi\pi} g_{\omega\pi\gamma}}{g_{f\pi\pi} g_{A\pi\gamma}} \simeq 2.6 \text{ GeV}^2,$$

where the values of the coupling constants have been extracted from the results quoted in the PDG compilation⁽³⁾. The FESR (5) is now satisfied for $N = 1.32 (\text{GeV})^2 + t/4$, as shown in Fig. 1. This value of the cut-off corresponds to $\bar{s} = \frac{1}{2}(m_{A_2}^2 + m_{A'_2}^2)$, where A'_2 is the Regge recurrence of A_2 , with $m_{A'_2}^2 = 7/3 m_{A_2}^2$.

Introducing the quoted values of N and K into the amplitude of eq. (4), which turns out to be strongly dominated by the Regge term, one gets:

$$(6) \quad \Gamma(\eta \rightarrow \pi^+ \pi^- \gamma) \simeq g_{\rho\eta\gamma}^2 \frac{g_{\rho\pi\pi}^2}{4\pi} (46 \text{ MeV}^3) \simeq 41 \text{ eV}.$$

It has to be noticed, as already stated⁽⁵⁾, that the FDR approach gives an important modification of about a factor of three to the simple pole model $\eta \rightarrow \rho \gamma$.

The theoretical uncertainties associated to the above result are two fold. In addition to the error arising from the experimental inputs, of about 20%, there are also uncertainties associated to the method of FDR itself, concerning mainly the choice of the cut-off N , which we also estimate of about 20%.

Our result (6) implies, through eq. (3):

(x) - Additional implications of this result, both theoretical and experimental, are discussed in ref. (9).

$$\Gamma(\eta \rightarrow \gamma\gamma) \simeq (310 \pm 120) \text{ eV},$$

which strongly favours the new determination of the $\eta \rightarrow \gamma\gamma$ partial width.

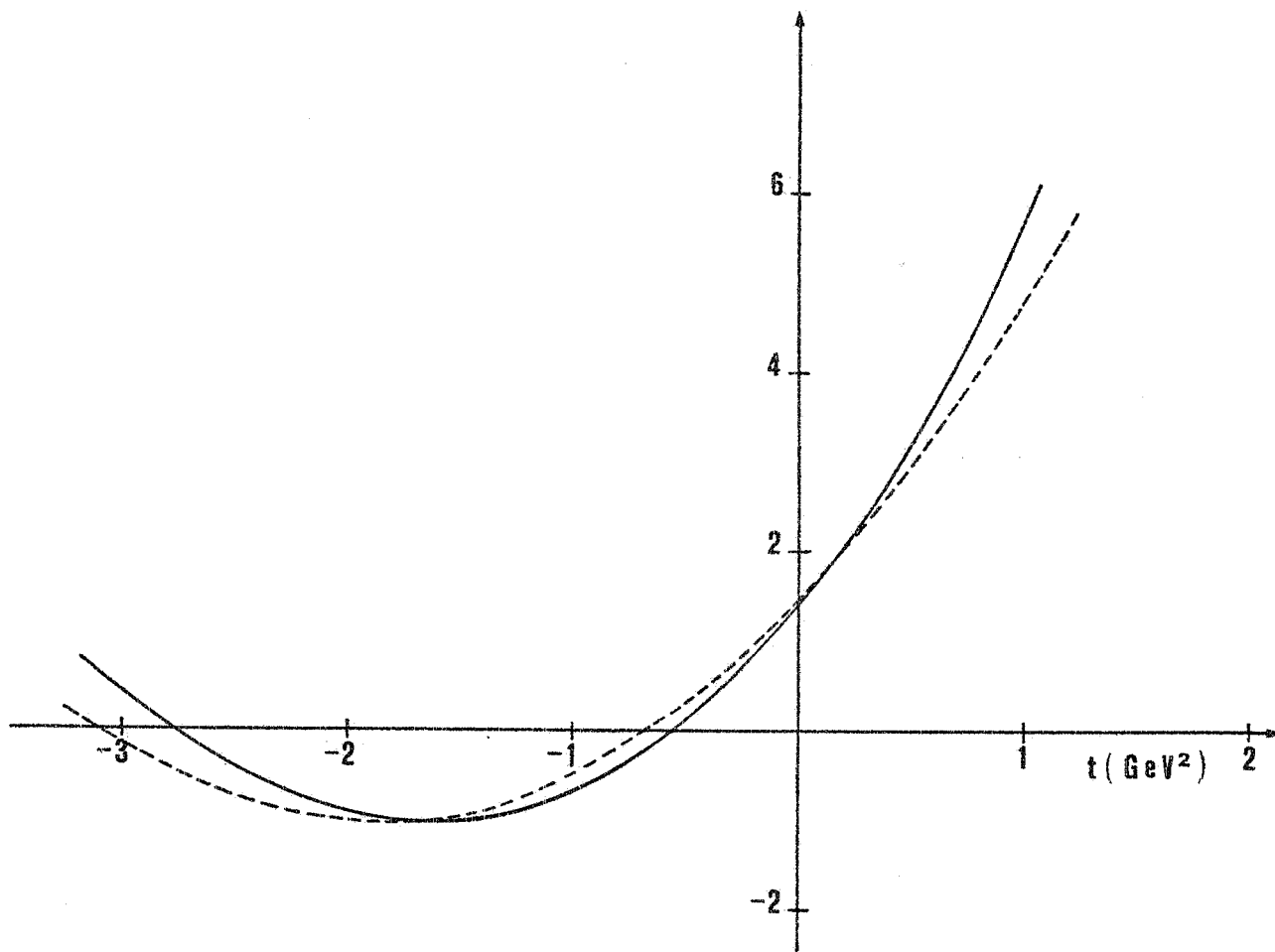


FIG. 1 - Saturation of the sum rule of eq. (5). Full and dashed lines represent Regge and resonance sides respectively. Ordinates are in arbitrary units.

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- (7) - The authors of ref. (6) give an estimate of $\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma)$ which is about a factor of three greater than ours. The discrepancy, in our opinion, is due to the over-estimate of the A_2 contribution to the amplitude $A(\nu, t)$ of eq. (4). They put in fact $s = m_{A_2}^2$, in the numerator of the A_2 contribution (r.h.s. of eq. (4) and eq. (7) of ref. (6), which makes a great difference in the evaluation of the actual decay width, where it holds $s \ll m_{A_2}^2$.
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