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E. Etim[(x)], S. Ferrara[(o)] and A.F. Grillo: POSSIBLE IMPLICATIONS OF THE GRIBOV-LIPATOV RECIPROCITY RELATIONS IN INCLUSIVE PROCESSES. -

To leading logarithmic order in perturbation theory Gribov and Lipatov[(1)] have shown that knowledge of the structure functions in either the scattering or annihilation channel is sufficient to determine the structure functions in the other channel completely[(2)]. The connection between the two sets of functions is

$$F_1(\frac{1}{\omega}, q^2) = \mp \omega \bar{F}_1(\omega, q^2)$$
(1)
$$F_2(\frac{1}{\omega}, q^2) = \mp \omega^3 \bar{F}_2(\omega, q^2)$$

where on the lhs are the scattering and on the rhs the annihilation structure functions and the upper sign goes with spin half particles and the lower with spin zero. Eq. (1), which can be tested experimentally is to be distinguished from analytic continuation, i.e.

$$F_1(\omega, q^2) = \bar{F}_1(\omega, q^2)$$
(2)
$$F_2(\omega, q^2) = \bar{F}_2(\omega, q^2)$$

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which relates one set of functions to the other in their unphysical region. Taken together eqs. (1) and (2) imply for instance

(3) $$F_2\left(\frac{1}{\omega}, q^2\right) = -\omega^3 F_2(\omega, q^2)$$

which is certainly a much stronger restriction on $F_2(\omega, q^2)$ than is ordinarily placed on analytic functions. It can be shown, in fact, that an analytic function regular at $\omega = 1$ which satisfies eq. (3) has necessarily the form[1,3]

(4) $$F_2(\omega, q^2) = (\omega - 1)^a \omega^b \phi\left(\left(\frac{\omega - 1}{\omega + 1}\right)^2; \quad q^2\right)$$

with $(-)^{a+1} = 1$, $a+2b=-3$ and $\phi(z;q^2)$ regular at $z = 0$.

Experimentally the reciprocity relations will not be tested simply be matching two functions but through their consequences. The purpose of this note is to discuss some of these consequences which will be tested soon at Frascati and other e^+e^- storage ring facilities.

The differential cross section for the inclusive production of a spin J hadron in an e^+e^- collision, integrated over the angles is given by[4]

(5) $$\frac{d\sigma}{d\omega} = \mp \left(\frac{2J+1}{4}\right)^2 \sigma_1(s)\, \omega^2 \bar{F}_2(\omega)$$

where $\sigma_1(s) = 4\pi\alpha^2/3S$ is the total cross section for muon pair production at CM energy $\sqrt{s}$ and the signs have the same meaning as in eq. (1).

We shall assume that the theory verifies Bjorken scaling so that the structure functions have no explicit q^2-dependence. The multiplicity of the particle of mass M is then given from eqs. (1) and (5) by

(6) $$\langle n(s) \rangle = \frac{(2J+1)}{2R} \int_1^{\sqrt{s}/2M} \frac{d\omega}{\omega} F_2(\omega)$$

where $R = \sigma_{TOT}(s)/\sigma_1(s)$, with $\sigma_{TOT}(s)$ the total cross section for e^+e^- annihilation into hadrons.

In the parton model the integral in eq. (6) is interpreted as the average number of partons in a hadron. Because an infinite sea of neutral partons and pairs thereof is not excluded this integral could diverge with an arbitrary power of s. One immediate consequence of the reciprocity relations is that this arbitrariness is very strongly contained and cannot exceed the limiting growth of the multiplicity $\langle n(s)\rangle \sim \sqrt{s}$ allowed by kinematics. Actually, making use of the fact that $F_2(\omega)$ defines a total cross section the energy dependence of the integral can be further restricted by invoking the Froissart bound

$$F_2(\omega) \underset{\omega \to \infty}{\lesssim} (\ln \omega)^2 \tag{7}$$

thus limiting $\langle n(s)\rangle$ to grow no faster than $(\ln \sqrt{s})^3$.

If in the scattering channel Regge and Bjorken limits are interchangable one would have for large ω

$$F_2(\omega) \xrightarrow[\omega \to \infty]{} \sum_i b_i \, \omega_i^{\alpha_i - 1} \tag{8}$$

Substituted in eq. (6) this gives a logarithmically growing multiplicity for the Pomeron contribution and a finite multiplicity for ordinary Regge exchanges. The constant coefficient b of the Pomeron term in eq. (8) is known from fits to the proton structure function namely, $b = 0.22$[5]; the corresponding coefficient for the pion b' follows from the quark model ratio $b'/b = 2/3$. Using eq. (6) and taking $R = 2$ the logarithmic parts of the proton and pion multiplicities become $\langle n_p(s)\rangle = 0.15 \ln(\sqrt{s})$ and $\langle n_\pi(s)\rangle = 0.05 \ln(\sqrt{s})$ respectively, while for the entire multiplet of baryons and pseudoscalar mesons one finds $\langle n_{TOT}(s)\rangle = 2.8 \ln(\sqrt{s})$. The calculation of the finite contributions to the multiplicities is more dependent on the dynamics. For the baryons (roughly about 16 times the mean of the proton and neutron) an estimate from the experimental[6] value of the integral in eq. (6) with $R = 2$ gives a value of about 0.32 comparable to the uncertainty in the same number. The main contribution to the baryon multiplicity therefore comes from the logarithm. For the pseudoscalars the situation is just the very opposite since experimentally[7] the multiplicity of charged secondaries (mostly pions) at Frascati beam energy (1.5 GeV) is about 3. At that energy the logarithmic contribution to the pion multiplicity is far below this number. Compared to what obtains in hadron-hadron scattering things are again working in the reverse order.

For single particle inclusive distribution in e^+e^- annihilation, energy conservation is summarized in the sum rule

$$2\,\sigma_{TOT}(s) = \sum_i \int_{2M/\sqrt{s}}^{1} d\omega\, \omega \frac{d\sigma^{(i)}}{d\omega} \tag{9}$$

where the summation extends over all stable hadrons. In terms of the structure functions and making use of eq. (1) this reads

$$4R = \sum_i (2J_i+1) \int_1^{\sqrt{s}/2M} \frac{d\omega}{\omega^2} F_2^{(i)}(\omega) \quad (10)$$

The integral in eq. (10), like that in eq. (6), is known for the proton and neutron from SLAC[6]. Its interpretation in the parton model is that it measures the average charge per parton in the hadron, whence the variety of estimates which have been given by invoking quark models of all descriptions[8]. The meaning of this integral in eq. (10) is certainly simpler and physically more attractive.

Extending the summation in eq. (10) over stable baryons and pseudoscalar mesons and assuming the contribution from each baryon to be approximately the mean of the proton and neutron contributions one gets an interesting bound

$$R > 1 \quad (11)$$

on the ratio R which seems to be respected by experiments[7]. Simple quark models with $R < 1$ seem thus to be excluded[9].

Experimentally a good fit to the proton and neutron structure functions has been given in terms of three adjustable parameters[10]. Assuming the same can be done for the pion we consider two simple alternatives[11]

$$F_2^{\pi}(\omega) = C_1\left(1-\frac{1}{\omega}\right)^2 + C_2\left(1-\frac{1}{\omega}\right)^3 + C_3\left(1-\frac{1}{\omega}\right)^4 \quad (12.1)$$

$$F_2^{\pi}(\omega) = C_1\left(1-\frac{1}{\omega}\right) + C_2\left(1-\frac{1}{\omega}\right)^2 + C_3\left(1-\frac{1}{\omega}\right)^3 \quad (12.2)$$

corresponding, according to the Drell-Yan threshold theorem[12], to asymptotic elastic form factors falling off as $(1/q^2)^{3/2}$ and $(1/q^2)$ respectively. The coefficients C_i are determined from the constant asymptotic value of $F_2^{\pi}(\omega)$ (2/3 that of the proton), the constant value of the ratio R (the energy sum rule eq. (10)) and the constant value of the pion multiplicity as discussed above. Treating the baryons as done previously one has sufficient equations to determine the coefficients C_i in terms of the annihilation parameters R and $\langle n_\pi \rangle$, values of which must be so assigned as to make $F_2^{\pi}(\omega)$ always positive. In Fig. 1 we have plotted the structure functions so determined for $\langle n_\pi \rangle = 0.8$, $R = 4$ (curve (a)) $\langle n_\pi \rangle = 0.4$, $R = 2$ (c), using eq. (12.1) and for

$\langle n_\pi \rangle = 0.6$, $R = 4$ (b) and $\langle n_\pi \rangle = 0.5$, $R = 2$ (d), using eq. (12.2). Compared to the proton structure function one notes the following distinguishing features:

(i) the overall large values of the structure functions, in particular their large peak values,
(ii) the maxima occur approximately within the same range of ω between 2.5 and 4, as for the proton and neutron,
(iii) the relatively narrow spread,
(iv) the decreasing behaviour (except for curve (d)) of the structure functions at values of ω at which the constant (maximum) value is about reached for the proton(6, 10).

These features are evidently very closely related; taken together they have a direct bearing on the structure of the pion. Clearly the most remarkable aspect is the overall large values; the question thus arises as to how the structure functions of other hadrons would compare with those of the nucleon and the pions.

If the constant asymptotic behaviour of the proton structure function, already evident at modest values of ω, is a real effect, it would appear from Fig. 1 that the same behaviour, for large momentum transfers, will not be manifest in the pion structure function at presently available energies. Thus if the behaviour in Fig. 1 suggests a trend it may be premature to say what the proton structure function is doing for large ω.

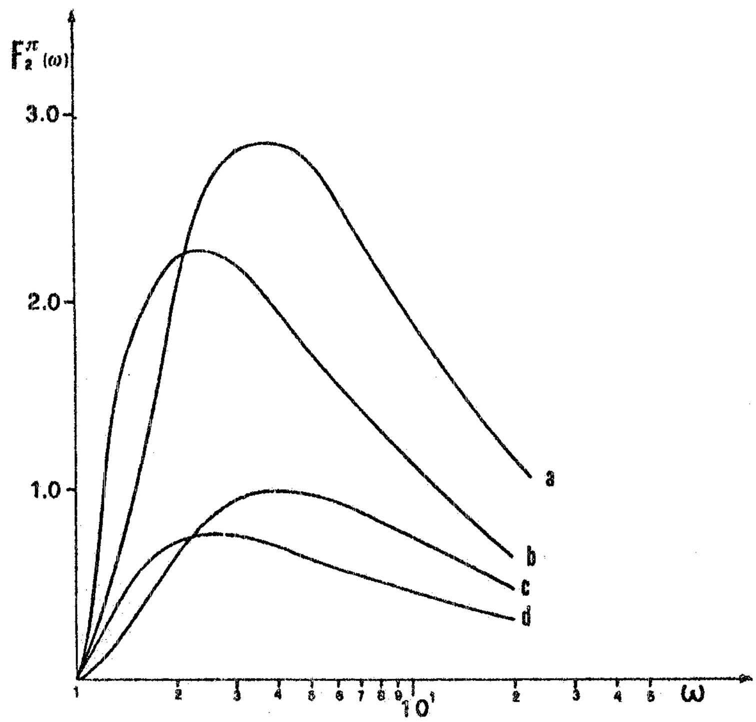

FIG. 1 - Plot of the pion structure functions for various values of the parameters $\langle n_\pi \rangle$ and R.

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(11) - The determination of the pion scattering structure function carried out here is not just an interesting theoretical application of the reciprocity ideas, firstly because data on the pion structure function may be forthcoming from Serpukhov and secondaly because by comparing the computed pion structure function with the proton's important general conclusions may be drawn regarding the shape, asymptotic behaviour and other charactestics which are supposed to reflect the internal structure of hadrons.

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For this theorem to be also applicable to spin zero particles the threshold behaviour must be understood as referred to the differential cross sections.