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# POSSIBLE TESTS OF SCALE INVARIANCE IN VERY HIGH-ENERGY $e^+e^-$ COLLISIONS☆

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The differential cross section for the process $e^+e^- \to e^+e^-X$ is connected to properties of the asymptotic zero mass theory. Assuming Wilson's expansion relations between different measurable quantities are obtained.

Recent theoretical efforts motivated by the discovery of canonical scaling behaviour [1] in deep inelastic electroproduction [2] gave the hint to some generalizations of Current Algebra [3]. In this connection the fundamental step was Wilson's idea [4] of operator product expansion at short-distances. In Wilson language, Bjorken scaling, experimentally observed at SLAC, corresponds to an infinite tower of local tensor operators with increasing spin and with scale dimensions dictated by free-field theory i.e. $l_J = 2 + J$ [5]; these operators give the relevant contribution to the product of two electromagnetic currents at light-like distances [5].

Various models have been proposed which share these properties: a particular realization of this canonical scheme is the ligh-cone Current Algebra proposed by Fritzsch and Gell-Mann [3]. Nevertheless it has been shown that it is very difficult to construct an interacting quantum field theory of fermions without anomalous dimensions [6]. However, relatively small anomalous dimensions are consistent with present experimental evidence [7].

Crucial tests for Bjorken scaling can be performed in inclusive processes involving two high-mass photons: examples of such reactions are $e^-p \to e^-\mu^+\mu^-X$; $e^+e^- \to \mu^+\mu^-X$, and $e^+e^- \to e^+e^-X$, where connected and disconnected matrix elements of four currents are probed [8].

In the present note we shall confine ourselves to processes of the last type where the analysis is less difficult due to the much simpler structure of the vacuum state. Cross sections whose typical orders of magnitude are estimated to be approximately $10^{-37}$–$10^{-38}$ cm$^2$, can hopefully be measured in the second generation of $e^+e^-$ colliding beams machines. These cross sections are related to the imaginary part of the forward elastic scattering amplitude of two highly virtual photons.

The crucial point is that this amplitude, when all the masses are large, can be rigorously proven to be related to the zero mass limit [9].

We stress that this is not a trivial statement because the forward configuration could in principle originate infrared effects which would destroy scale invariance.

A first consequence of the previous statement is that, if no anomalous dimensions occur in nature, it is ver hard to avoid parton model predictions for the amplitude [8].

In particular it is interesting to observe that this result holds at all order in the coupling constant in a $\lambda\phi^4$ theory with negative $\lambda$ [10].

On the other hand, if anomalous dimensions are present, which means failure of Bjorken scaling, parton models predictions are lost and a non-trivial asymptotically scale invariant theory [11] (perhaps with small anomalies) is expected.

It has been recently emphasized [12] that theories which possess a scale invariant limit are also conformal invariant so the constraints which come out from this larger symmetry may be tested in these experiments. This kind of constraints essentially comes from the fact that conformal symmetry fixes the functional form of two and three-points functions of operators in terms of their dimensions [13]; the additional input of Wilson expansion $A(x)B(0) = \Sigma_n c^n_{AB}(x^2)O_n$ implies the relation [14]

$$c_{ABn} = c^n_{AB}c_{nn} , \qquad (1)$$

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where $c_{ABn}$, $c_{nn}$, $c^n_{AB}$ are the normalizations of the two and three point functions $\langle O_n O_n\rangle$, $\langle ABO_n\rangle$ and the coefficient of $O_n$ in the Wilson expansion respectively.

An important application of the ideas sketched above was very recently given by Crewther [15] who found the constraint

$$4S = KR\ ,$$

using asymptotic conformal and U(3)⊗U(3) invariance: Here $S$ is the Adler anomaly [16], $K$ is the contribution of the axial current to the Wilson expansion of two e.m. currents [17] and $R$ is the ratio $\sigma_{e^+e^-\to x}/\sigma_{e^+e^-\to\mu^+\mu^-}$ at asymptotic energy.

The purpose of the present note is to point out that other relations can be obtained in the two photon channel. In fact the cross section for the process $e^+e^-\to e^+e^-X$ is related to the Fourier transform of the vacumm expectation value $\langle 0|[T(J_\alpha(y)J_\beta(x)),\ T(J_\mu(0)J_\nu(z))]|0\rangle$ where all points are light-like to each others [8].

The dynamics of the process is contained in several structure functions $f_i(k_1^2, k_2^2, P^2)$ connected to the following matrix elements

$$M_{\alpha\beta;\mu\nu} = \sum_x (2\pi)^4\delta(P-P_x)M^*_{\alpha\beta}M_{\mu\nu} = \int d^4x d^4y d^4z\ \exp[i(z-x)P-2iyk_2] \times \langle 0|[T(J_\alpha(y)J_\beta(x)),\ T(J_\mu(0)J_\nu(z))]|0\rangle\ , \qquad (2)$$

where $P = k_1 + k_2$ and $k_1$, $k_2$ are the photon momenta.

In the present process two-possible light-cone limits can be done: $k_1^2, k_2^2, P^2$ large with $k_1^2/P^2$ fixed and $k_2^2/P^2 \to 0$ and $k_1^2, k_2^2, P^2$ large with $k_1^2/k_2^2$ fixed and $P^2/k_2^2 \to 0$.

In the first limit we are probing the region $x^2 \to 0$ while in the second limit $(x-y)^2 \to 0$, $z^2 \to 0$. Insertion of Wilson expansions in these limits corresponds to different expansion of the forward amplitude. In the first case matrix elements of the operators involved in the expansion at zero momentum transfer are involved. In the second case the same vertices are considered but not in the forward direction: these operators carry momentum $P^2$. A comparison of these two different sequences of limits may provide a deep and direct test of asymptotic conformal invariance as the knowledge of the vertex function in the forward direction is sufficient to compute it everywhere. In the list of the operators $O_n$ there are operators whose properties are well known like the axial current and the energy momentum tensor: one can derive sum rules which possibly test the algebraic structure of the underlying theory.

Let us consider in more details the axial current contribution to the four currents amplitude.

Using Wilson expansion one gets [15]

$$J^Q_\beta(x)J^Q_\mu(0) = \frac{-1}{x^4}\frac{K}{3\pi^2}\, d^{QQl}\epsilon^{\delta\sigma}_{\beta\mu}x_\delta A^l_\sigma(0) + \dots\ , \qquad (3)$$

where $Q$, $l$ are internal symmetry indices, $Q$ referring to the e.m. charge. The axial current $A^l_\sigma$ contribution to the four-current amplitude for $x^2 \to 0$ is

$$\langle 0|J^Q_\alpha(y)J^Q_\beta(x)J^Q_\mu(0)J^Q_\nu(z)|0\rangle \sim -\frac{1}{x^4}\frac{K}{3\pi^2}\, d^{QQl}\epsilon^{\delta\sigma}_{\beta\mu}x_\delta\langle 0|J^Q_\alpha(y)A^l_\sigma(0)J^Q_\nu(z)|0\rangle + \dots \qquad (4)$$

Using the fact that the functional form of the axial-vertex is fixed by conformal invariance to be the same as in free field theory [18] (up to normalization), one gets the relation

$$\tfrac{4}{3}SK = \tfrac{16}{3}S^2/R = H\ . \qquad (5)$$

Here $H$ is the measured quantity in $e^+e^- \to e^+e^-X$ which can be expressed as an integral over the structure functions. This number, in parton models, turns out to be proportional to the mean value of the fourth power of the charge of the partons [8].

In the quark parton model the relation (5) is identically satisfied with $S = \frac{1}{6}$, $\langle Q^2\rangle = \frac{2}{3}$, $\langle Q^4\rangle = \frac{2}{9}$. This relation remains unchanged if the "color" [19] of quarks is introduced, in fact: $S = \frac{1}{2}$, $\langle Q^2\rangle = 2$, $\langle Q^4\rangle = \frac{2}{3}$. Note that the above relation is insensitive to the presence of anomalous dimensions since the dimension of the axial current is fixed to be 3 by current algebra.

Finally we mention that a similar relation, which is however, sensitive to the validity of the Bjorken scaling, could be obtained for the stress tensor. In fact, if noncanonical scaling holds, one would expect, according to ref. [20], that the only second rank tensor of dimension exactly 4 is the stress-tensor $\theta_{\mu\nu}$. In this case

$$J_\beta(x)J_\mu(0) \underset{x\to 0}{\sim} (c/x^2)\,t^{\rho\sigma}_{\beta\mu}\,\theta_{\rho\sigma}(0) + \dots\ , \qquad (6)$$

and the related contribution to the four-current amplitude is

$$(c/x^2)\, t_{\mu\nu}^{\rho\sigma} \langle 0 | J_\alpha^Q(y)\, \theta_{\rho\sigma}(0)\, J_\nu^Q(z) | 0 \rangle\,, \qquad (7)$$

where the normalization of this vertex can be expressed, using Ward identities for the conformal group generators, in terms of $R$, i.e. the normalization of the vacuum expectation value $\langle 0 | J_\beta^Q(x) J_\mu^Q(0) | 0 \rangle$ for $x \to 0$, and $c$ is fixed by the Callan-Gross sum rule in ep electroproduction [21]. In this case one has the relation

$$cR = L,$$

where $L$ can be related to another integral over the structure functions measured in virtual $\gamma\gamma$ interactions. However, we remark that the above result is valid only if renormalization of dimensions occurs [22]. In fact in free field theory there are several second rank operators having the same dimension of the stress tensor which give an unknown contribution to the above sum rule.

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