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INTERMEDIATE BOSON (W) MASS.

L. Bergamasco^(x), B. D'Ettorre Piazzoli^(x), P. Picchi⁽⁺⁾ and R. Visentin:
THE NEUTRINO DIFFERENTIAL CROSS SECTION ($d\sigma/dQ^2$) PREDICTIONS AS FUNCTION OF INTERMEDIATE BOSON (W) MASS. -

In the present study of the weak interactions there is the problem of finding an intermediary of the weak force. The simplest possibility lies in the existence of one or more intermediate bosons W. Activity to discover such a particle is enormous.

The Batavia National Laboratories accelerator will permit experiments with neutrinos of energy up to 500 GeV, and one of the detector will be a big bubble chamber filled with hydrogen, deuterium or neon⁽¹⁾.

In this work we give the differential cross section $d\sigma/dQ^2$ for the interactions of neutrinos⁽²⁾ of energies $E_\nu = 100, 200, 500$ GeV on Ne nuclei calculated for different values of M'_W .

The measurements of $d\sigma/dQ^2$ with bubble chambers are rather simple demanding only the identification of the prompt negative muon and the value of its momentum and scattering angle. These

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data may be very useful if the W decay into hadrons is dominant⁽³⁾.

The reactions which we have considered are the following:

$$(1) \quad \nu + N_e \rightarrow \mu^- + h \quad (\text{virtual } W)$$

$$(2) \quad \nu + N_e \rightarrow \mu^- + W^+ + h \quad (\text{real } W)$$

where W is the usual spin one W-boson and h the hadronic final state. To the lowest order of perturbation theory these processes are represented by the diagrams of Fig. 1, where

P_ν = four-momentum of neutrino,

P_μ = four-momentum of muon

r = four-momentum of the initial hadronic state

r' = four-momentum of the final hadronic state

Q = $P_\nu - P_\mu$ = four-momentum transferred from leptons to hadrons and W.

We define moreover :

s = r - r' = four-momentum transferred to the hadronic vertex (for the process (1) Q = s)

$\nu = E_\nu - E_\mu$ = energy transferred from leptons to hadrons and to W,

$\nu_h = E_{r'} - E_r$ = energy transferred to the hadronic vertex (for process

$$(1) \quad \nu_h = \nu)$$

M_N = the nucleon mass

M_A = the mass of the hadronic initial state (nucleon or nucleus),

M_W = the W boson mass,

ϕ = the azimuthal angle of the final hadronic state momentum around \vec{Q} .

The basical formula (which gives the cross-section ν -nucleon) for reaction (1), Fig. 1a, is

$$(3) \frac{d^2 \sigma^+}{d\nu dQ^2} = \frac{G^2}{4\pi} \frac{1}{E_\nu^2} \left[Q^2 W_1^+ + (2E_\nu^2 - 2E_\nu \nu - \frac{Q^2}{2}) W_2^+ + W_3^+ \frac{2E_\nu - \nu}{2M_N} Q^2 \right] \frac{1}{(1 + \frac{Q^2}{2M_W^2})}$$

where $G = 1.02 M_p^{-2} \cdot 10^{-5}$ is the Fermi coupling constant (M_p = proton mass).

$W_{1,2,3}^+$ are the form factors describing the process (the plus or minus sign refers respectively to neutrino or antineutrino). In the evaluation of eq. (3) we assume⁽⁴⁾ to hold for W_1^+ the scale invariance.

The total ν -nucleon cross section results for $M_W = \infty$:

$$\sigma_{\nu \text{ nucleon}} = \frac{\sigma_{\nu p} + \sigma_{\nu n}}{2} = 0.68 \times 10^{-38} E_\nu \text{ (cm}^2\text{)}$$

are in good agreement with the Cern experimental results $\sigma_{\nu \text{ nucleon}} = (0.69 \pm 0.14) \times 10^{-38} E_\nu \text{ (cm}^2\text{)}$ for $E_\nu < 10 \text{ GeV}$.

Reaction (2) (see Fig. 1b) represents the W direct production by neutrinos (to lowest order the cross-section for $\bar{\nu}$ is identical).

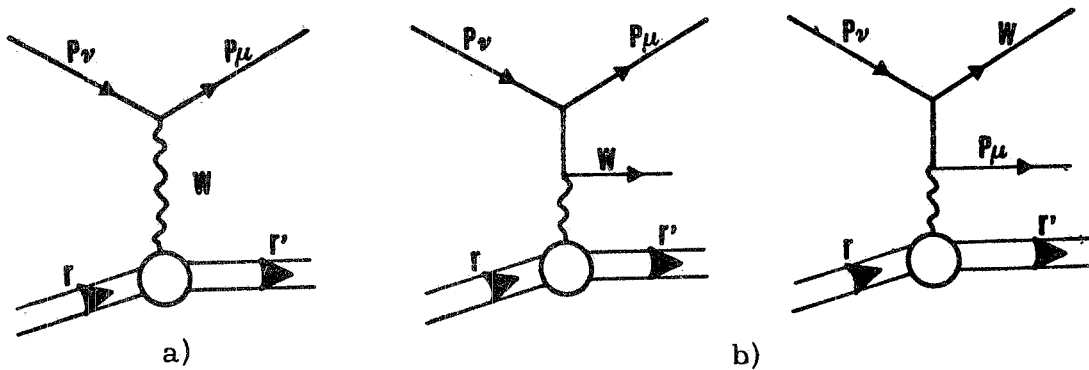


FIG. 1 - a) The diagram of virtual W production by neutrinos; b) The diagram of real W production by neutrinos.

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For a target, of charge Z , at rest we have:

$$(4) \quad \frac{d^2 \sigma_W}{dQ^2 d\nu} = \frac{Z^2 \alpha^2}{(2\pi)^2} g^2 \frac{1}{E_\nu^2 M_A} \frac{1}{|\vec{Q}(Q^2, \nu)|} \int_{s_{\min}^2}^{s_{\max}^2} \frac{ds^2}{s^4} \mathcal{F}(Q^2, \nu, s^2)$$

where α is the fine structure constant $1/137$ and $g^2 = \frac{M_W^2}{\sqrt{2}} G$ is the semi-weak coupling constant.

The function $\mathcal{F}(Q^2, \nu, s^2)$ is given explicitly after the analytical evaluation of the integral:

$$(5) \quad \mathcal{F}(Q^2, \nu, s^2) = \frac{1}{2M_A} \int_0^{2\pi} T_{\mu\nu} W_{\mu\nu} d\phi$$

The tensor $T_{\mu\nu}$ which arises after summation over the lepton spins and W polarizations, represents the semi-weak and electromagnetic vertex. The hadronic vertex is described in the usual way by the tensor $W_{\mu\nu}$:

$$(6) \quad W_{\mu\nu} = -W_1(s^2) \left(\delta_{\mu\nu} - \frac{s_\mu s_\nu}{s^2} \right) + \frac{W_2(s^2)}{M_A^2} \left(r_\mu - \frac{(rs)}{s} s_\mu \right) \left(r_\nu - \frac{(rs)}{s} s_\nu \right)$$

where for incoherent production on nucleon we use:

$$W_1(s^2) = \frac{1}{2} s^2 G_M^2; \quad W_2(s^2) = 2M_A^2 (1+\tau)^{-1} (G_E^2 + \tau G_M^2)$$

$$\tau = \frac{-s^2}{4M_N^2}$$

$$G_E(\text{proton}) = G_M(\text{proton})/2.79 = -G_M(\text{neutron})/1.91 \approx (1. \frac{-s^2}{2})^{-2} .71$$

$$G_E(\text{neutron}) = 0.$$

while for coherent production on nucleus we have:

$$W_1(s^2) = 0, \quad W_2(s^2) = 2M_A^2 |F(s^2)|^2, \quad F(s^2) = e^{-\frac{s^2 a^2}{6}},$$

with

$$a = 5.1 A^{1/3} (\text{GeV}/c)^{-1}.$$

For the inelastic channel (the cross section on nucleon is about 1/2 of the incoherent cross section on proton) the structure functions are quite different and depend on the ν_h .

Assuming the scaling behaviour of $\nu_h W_2(s^2, \nu_h)$ we have⁽⁵⁾:

$$W_1(s^2, \nu_h) = \left(1 - \frac{\nu_h^2}{s^2}\right) \frac{W_2(s^2, \nu_h)}{1+R}$$

$$W_2(s^2, \nu_h) = \frac{M_N}{\nu_h} F(x); \quad F(x) = 4 \frac{(1 - e^{-(x-1)})}{1+x/20}$$

with

$$\left\{ \begin{array}{l} \nu_h = \frac{1}{2M_N} (\pm s^2 + M_B^2 - M_N^2) \\ x = \frac{-2M_N \nu_h}{s^2} \\ R = \sigma_L / \sigma_T \simeq 0 \end{array} \right.$$

$M_B^2 = r'^2$ is the squared mass of the hadronic final state.

After saturation, the explicit form of $T_{\mu\nu} W_{\mu\nu}$ depends on Q^2, ν, s^2 and on rational functions of the powers (up to the second) of $\cos\theta$: it follows that $\mathcal{F}(Q^2, \nu, s^2)$ can be easily given in closed form.

The calculation of $d\sigma/dQ^2$ requires only two numerical integrations over ν (between $\nu_{\min}(Q^2)$ and $\nu_{\max}(Q^2)$) and s^2 .

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For the inelastic channel we must carry out a further integration over the squared mass of the final hadronic state:

$$\frac{d\sigma_W^{\text{in}}}{dQ^2} = \int_{r'^2_{\text{min}}}^{r'^2_{\text{max}}} \frac{d\sigma_W}{dQ^2 dr'^2}(r'^2) dr'^2$$

All integration limits are unambiguously defined analitically. We have checked the validity of the calculations by integrating the differential cross section over Q^2 . The total cross sections (with the anomalous magnetic momentum of the W, $K=0$) agree to within 5% with those of ref. (5).

The differential cross section per proton on a Neon target is

$$\begin{aligned} \frac{d\sigma}{dQ^2} / \text{proton} &= \frac{d\sigma^+}{dQ^2} (\text{proton}) + \frac{A-Z}{Z} \frac{d\sigma^+}{dQ^2} (\text{neutron}) + \\ &+ \frac{d\sigma_W}{dQ^2} (\text{proton}) + \frac{A-Z}{Z} \frac{d\sigma_W}{dQ^2} (\text{neutron}) + \frac{1}{Z} \left(1 - \frac{1}{Z}\right) \frac{d\sigma_W}{dQ^2} (\text{coherent}) \\ &+ \frac{d\sigma_W^{\text{in}}}{dQ^2} (\text{proton}) + \left(\frac{A-Z}{Z}\right) \frac{d\sigma_W^{\text{in}}}{dQ^2} (\text{neutron}). \end{aligned}$$

In our calculations we assume equal the proton and neutron inelastic cross-sections for W production, and we dont include corrections for Fermi motion and the Pauli principle.

In Fig. 2 we give $(d\sigma/dQ^2)/\text{proton}$ for $E_\mu \gg 1.5 \text{ GeV}$ e $M_W = \infty, 10, 5 \text{ GeV}$ ⁽⁶⁾.

For $Q^2 \lesssim 20 \text{ GeV}^2/c^2$ and for masses $M_W \gtrsim 10 \text{ GeV}$ there are no peculiar differences.

In Fig. 3 we shown $(d\sigma/dQ^2)/\text{proton}$ for $1.5 \ll E_\mu \ll 0.15 E_\nu$, GeV and $M_W = \infty, 10, 5 \text{ GeV}$. These curves are sharply different, even for low Q^2 . This difference results ⁽³⁾ from the fact that in process (2) a large energy transfer to real W is accompanied by a low Q^2 .

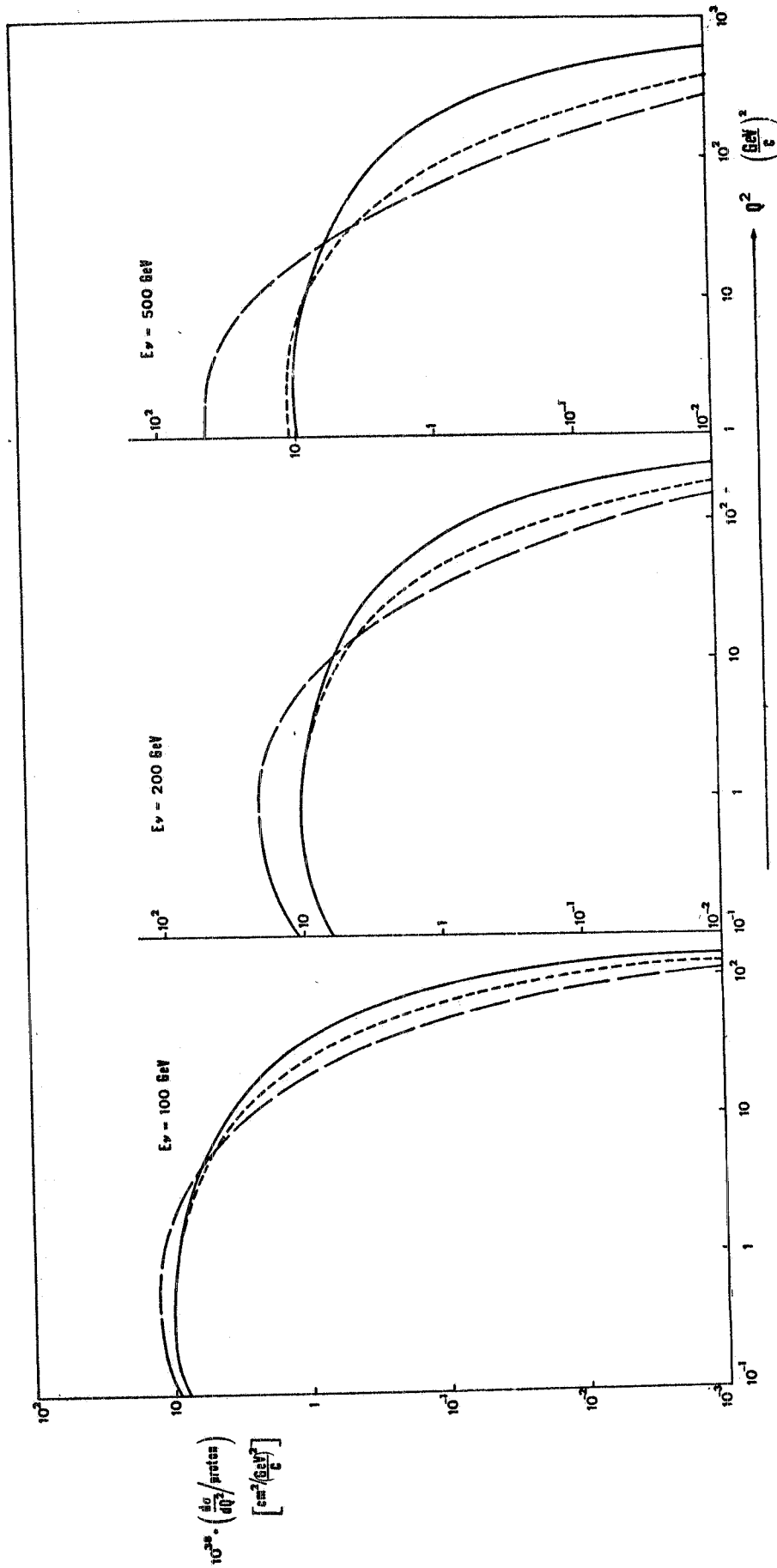


FIG. 2 - Inelastic neutrino ($d\sigma/dQ^2$)/proton for $E_\nu \gg 1.5 \text{ GeV}$, $M_W = \infty$ (full lines), $M_W = 10 \text{ GeV}$ (dotted lines), $M_W = 5 \text{ GeV}$ (dashed lines).

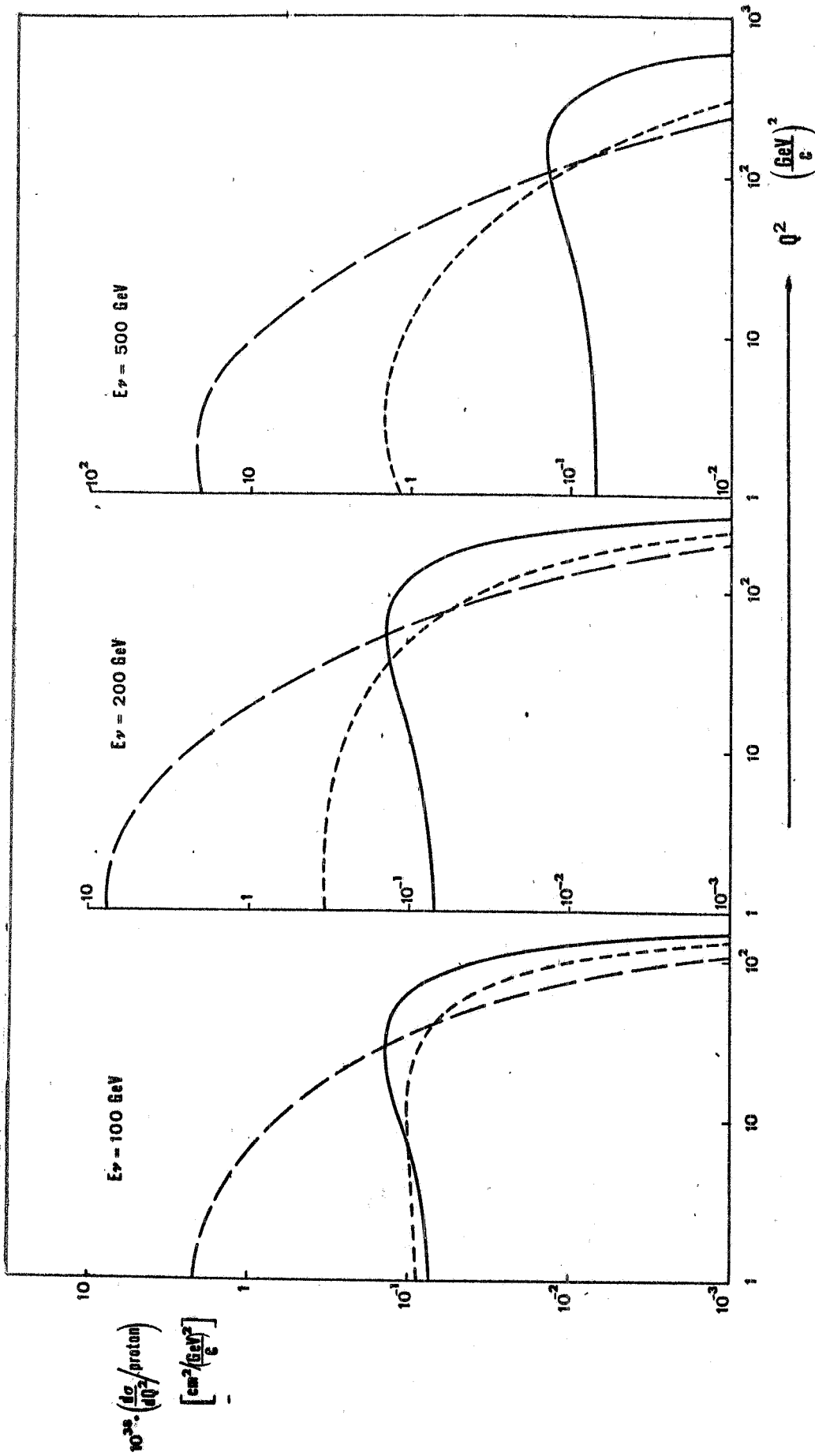


FIG. 3 - Inelastic neutrino ($d\sigma/dQ^2$)/proton for $1.5 \lesssim E_\mu < 1.5 E_\nu$, GeV
 $M_W = \infty$ (full lines), $M_W = 10 \text{ GeV}$ (dotted lines), $M_W = 5 \text{ GeV}$ (dashed lines).

The conclusions which may be drawn are synthesized in Fig. 4 which shows $(d\sigma/dQ^2)/\text{proton}$ for different Q^2 values (1, 10, 50, 100, 200) as a function of M_W^{-1} , namely:

- Ratios between different Q^2 are very sensible to M_W .
- Also for $E_\nu = 100$ GeV it is possible to carry out experimental measurements which can evidence masses M_W of about 15-20 GeV.

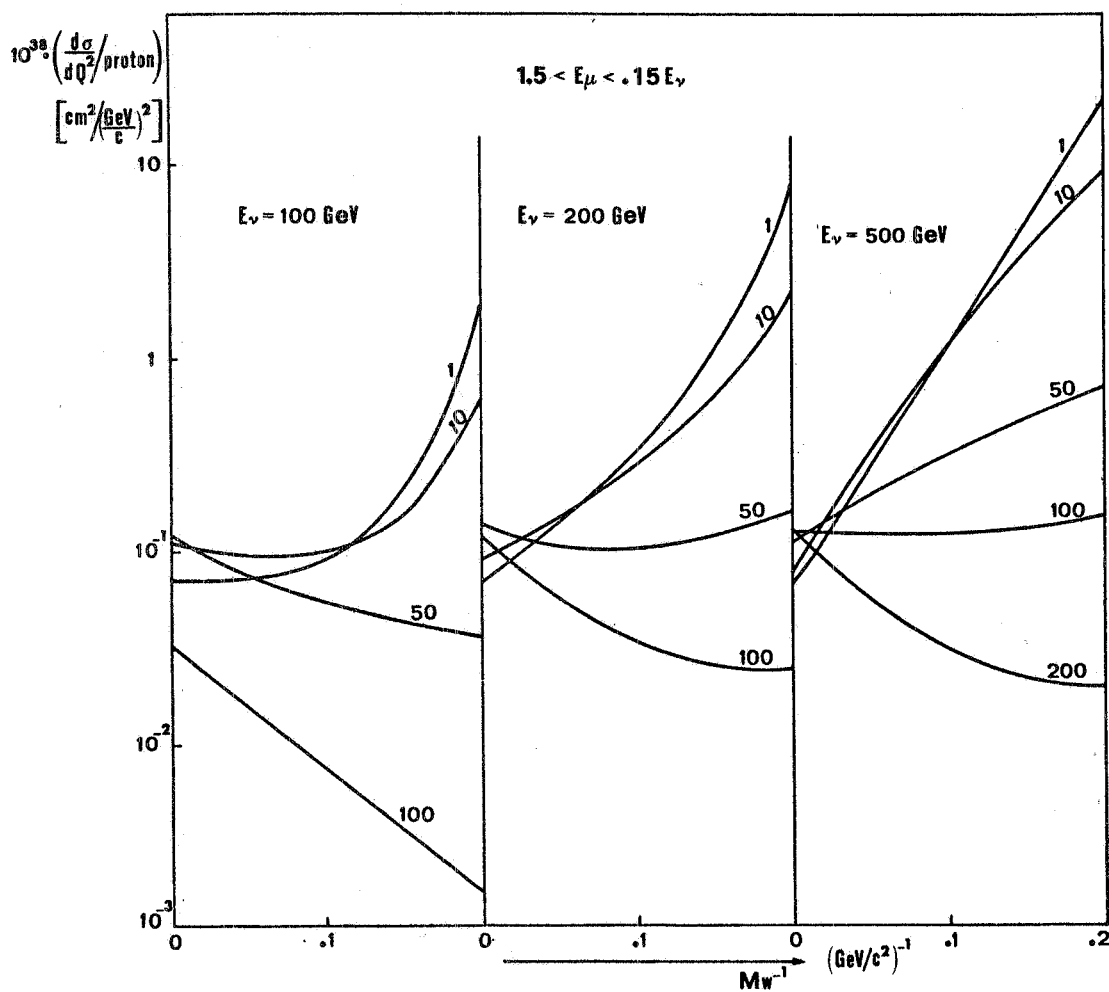


FIG. 4 - Inelastic neutrino $(d\sigma/dQ^2)/\text{proton}$ for different Q^2 values (1, 10, 50, 100, 200) as a function of M_W^{-1} .

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FIGURE CAPTIONS. -

FIG. 1 - a) The diagram of virtual W production by neutrinos;

b) The diagram of real W production by neutrinos,

FIG. 2 - Inelastic neutrino $(d\sigma/dQ^2)/\text{proton}$ for $E_\mu > 1.5 \text{ GeV}$,

$M_W = \infty$ (full lines), $M_W = 10 \text{ GeV}$ (dotted lines), $M_W = 5 \text{ GeV}$ (dashed lines).

FIG. 3 - Inelastic neutrino $(d\sigma/dQ^2)/\text{proton}$ for $1.5 < E_\mu < 1.5 E_\nu \text{ GeV}$

$M_W = \infty$ (full lines), $M_W = 10 \text{ GeV}$ (dotted lines), $M_W = 5 \text{ GeV}$ (dashed lines).

FIG. 4 - Inelastic neutrino $(d\sigma/dQ^2)/\text{proton}$ for different Q^2 values (1, 10, 50, 100, 200) as a function of M_W^{-1} .