COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-73/37
26 Giugno 1973

L. Bergamasco, C. Castagnoli, B. D'Ettorre Piazzoli, P. Picchi, K. Sitte and R. Visentin: THE MULTIPLICITY DISTRIBUTION OF SHOWER PARTICLES UNDERGROUND AND THE COSMIC RAY PRIMARY SPECTRUM. -

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

LNF-73/37
26 Giugno 1973

L. Bergamasco(x), C. Castagnoli(x), B. D'Ettorre Piazzoli(x), P. Picchi, K. Sitte(o) and R. Visentin: THE MULTIPLICITY DISTRIBUTION OF SHOWER PARTICLES UNDERGROUND AND THE COSMIC RAY PRIMARY SPECTRUM. -

Ever since the discovery of 'muon bundles", groups of penetrating particles with nearly parallel tracks (Braddick and Hensby(1), Greisen et al.(2), Amaldi et al.(3)), it was hoped that their size and radial distribution would provide evidence on details of the processes of secondary production in high energy collisions. In the following we shall attempt to show from an analysis of experimental results recently obtained that neither the composition of the primary cosmic radiation, nor the characteristics of hadronic interactions, undergo drastic changes in the energy range above some $10^{15}$ eV.

Extensive studies carried out especially by the Moscow group(4) but also elsewhere (Shibata et al.(5), Bergamasco et al.(6)) have established by now that the bundles can be explained as statistical fluctuations of a smooth muon lateral distribution, without introducing special assumptions like coherent or preferential production. This involved some modification in the lateral distribution derived by Greisen(7). With regard to the energetic muons recorded underground, the agreement between the results reported by Vernov et al.(4), and by the Torino group(6), may be emphasized. It is also

---

(x) - Laboratorio di Cosmo-Geofisica del C.N.R., Torino.
(o) - Departement of Physics, University of Freiburg, Freiburg i. Br., and Laboratorio di Cosmo Geofisica del C.N.R., Torino.

a precondition for our analysis which is based on a comparison of the absolute rates of multiple coincidence registered in these two stations.

The experimental arrangement used in our work has been described in more detail previously(6). Briefly, it consisted of two spark chamber arrays $M_1$ and $M_2$, each made up of six two-gaps units separated by layers of 5 cm Pb and 1.8 cm Fe. Photographs of 90° stereoscopic views of both chambers were recorded on a single frame.

Counter telescopes of two 0.7 m x 1.4 m scintillators mounted above and below each spark chamber provided the trigger. The triggering condition was either traversal by more than one penetrating particle of the telescopes $M_1$ or $M_2$ (mode $(M_1+M_2)/2$), or alternatively of more than one particle over the areas of both (mode $(M_1 M_2)$). As a further check signals marking the scintillators struck by a particle were recorded on the photographs, together with other auxiliary information.

A computer programme measured all angles of incidence, and selected the events of "parallel penetrating particles" (PPP), that is, of groups whose trajectories did not converge in the rock above. Only those were included in the present analysis. Daily check runs with single-muon triggering safeguarded uniform performance of the apparatus.

The arrangement was operated at the Mt. Cappuccini Laboratory at a mean depth of 40 m.w.e. for a period of 880 hours during which a total of 1,964 PPP events was registered.

The absolute fluxes I(i), in $s^{-1}$, of events with multiplicity i are listed in Table I. They were determined with the accurate geometry factors reported before(6).

TABLE I

Fluxes I(i) of groups of i parallel penetrating particles.

| Multiplicity Mode | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $(M_1 \cdot M_2)$ | $(8.05 \pm .41) \times 10^{-4}$ | $(8.17 \pm .65) \times 10^{-5}$ | $(2.16 \pm .33) \times 10^{-5}$ | $(5.0 \pm 1.7) \times 10^{-6}$ |
| $(M_1+M_2)/2$ | $(1.06 \pm .07) \times 10^{-4}$ | $(1.23 \pm .25) \times 10^{-5}$ | $(1.71 \pm .45) \times 10^{-6}$ | |

For the following discussion it is important to relate these rates to the mean shower sizes and primary energies of the showers. Making use of the relation

$$(1) \qquad \langle N_e \rangle = \int_{N_e} (dI(i)/dN_e)\, N_e\, dN_e / I(i)$$

this can be done if the shower size spectrum $f(N_e)$ is known. If the multiplicity distribution is essentially determined by statistical fluctuations of the particle density $\Delta$, one has

$$\begin{aligned} & I(i) = (2^i-2)\int f(N)\, dN \binom{n}{i} \iiint (\Delta/n)^i \; x \\ (2) \qquad & x \Bigl[1+(\Delta/n)\, 2\cos\theta\, (X-|x-x'|)(Y-|y-y'|)^{n-1} \quad 2\pi R\, dR\; x \\ & x \Bigl\{(1/Z^2)\cos^{\rho+3+i}\theta \bigl[(X-|x-x'|)(Y-|y-y'|)\bigr]^{i-1}\Bigr\}\, dx\,dy\,dx'\,dy' \end{aligned}$$

Here X, Y, Z are the dimensions of the apparatus, $n = 2\pi . \int \Delta_\mu(N,R)\, dR$, $\rho = 2$, and $\cos\theta = Z/\left[(x-x')^2+(y-y')^2+Z^2\right]^{1/2}$.

From reasons which will be discussed below we have evaluated I(i) with the shower size spectrum and the muon lateral distribution reported by Vernov et al.(4). The average shower sizes thus obtained for various multiplicities i and for both triggering modes are reproduced in Fig. 1. The interpretation of our results rests on three considerations:

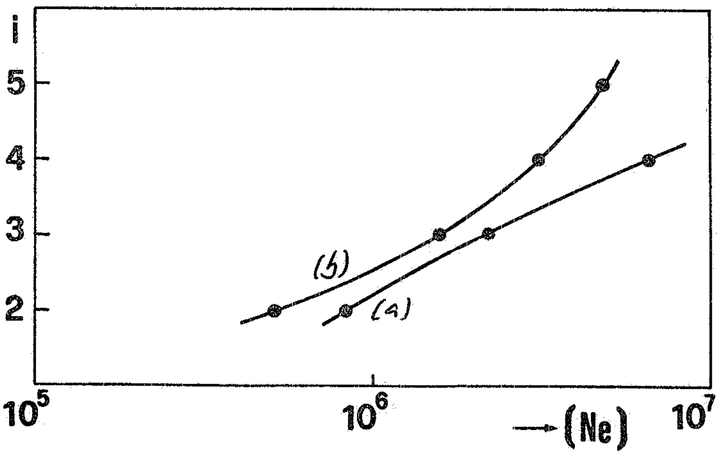

FIG. 1 - Particle multiplicities i as a function of the average shower size $\langle N_e \rangle$ : (b) triggering mode $(M_1 \cdot M_2)$ (a) triggering mode $(M_1+M_2)/2$.

(i) An evaluation of the absolute rates of PPP events according to (2) requires knowledge of two parameters, the muon density $\Delta_\mu$ and the shower size distribution $f(N_e)$.

(ii) The absolute rate is strongly dependent on the geometry factors, and hence on the dimensions of the apparatus which determine the mean distance R from the axis of the recorded groups.

(iii) This in turn implies that arrays of different dimensions even at equal depth do not test the customary assumption of permissible factorization in describing the particle density.

$$\Delta(N_e, R) = N_e^\alpha f(R) \rightarrow N_e^\alpha R^{-\beta} \tag{3}$$

in the same range. Since any change in the interaction characteristics or in the primary composition will results in a variation of the parameters $\alpha$ and $\beta$ of (3), this change will be revealed by a comparison of the results obtained with apparatus of different dimensions. Agreement of observed rates with those calculated for the recording array signifies little. But agreement of the data from several apparatus with calculations using the same shower parameters does prove their constancy. Note, in particular, that in (2) the density $\Delta_\mu(N_e, R)$ enters raised to the ith power.

Thus in principle already our records for $(M_1+M_2)/2$ and $(M_1 \cdot M_2)$ could be used to check whether a unique factorized density relation is valid over the entire range of observations. But a much better test is provided by a comparison of our rates with those of the Moscow group(4). Accordingly we present in Fig. 2(a) and (b) our experimental results on the frequencies I(i), together with the rates calculated with the Moscow data (shown in full lines). Fig. 2(a) refers to $(M_1+M_2)/2$, Fig. 2(b) to $(M_1 \cdot M_2)$.

The general agreement is quite satisfactory, and in our opinion significant. Within the range of shower sizes covered by our experiment, roughly between $10^5$ and $10^7$ particles, the Moscow data as well as those of many other studies indicate a marked change in the slope of the shower size spectrum. For this three possible reasons have been discussed. Firstly it could be due to a change in the composition of the primary radiation, a rising dominance of heavy nuclei. Secondly it could be ascribed to a change in the characteristics of hadron interactions at very high energies. Finally it could, of course, reflect a genuine change in the primary spectrum, perhaps resulting from an energy dependence of the propagation processes.

In order to distinguish between these alternatives, we have recalculated the theoretical frequencies I(i) with three modifications.

In the first case a spectrum with constant slope was used, and the parameters $\alpha$ and $\beta$ were left unchanged. This gave the results shown in the dashed-dotted lines of Fig. 2. Next the Moscow size spectrum was retained but $\alpha$ changed to $(\alpha + 0.1)$ for shower sizes above $N_e = 7 \times 10^5$, the region where the break is found in the Moscow data. This led to frequencies shown by the dotted lines in Fig. 2. In the third case $\beta$ was changed to $(\beta + 0.05)$, again with the unmodified Moscow size spectrum, and the frequencies indicated by the dashed lines in Fig. 2 were obtained. The amounts of changes $\Delta\alpha$ and $\Delta\beta$ chosen follow from the physical assumptions of the respective models. If the composition varies from proton dominance below a certain $N_e$, to an average nuclear mass $A = N'_e/N_e$ at the end of the range of observations for which a mean shower size $N'_e$ was found, one expects roughly

(4) $$\Delta\alpha = (1-\alpha)(1 - \ln N_e / \ln N'_e)$$

In our case this implies a $\Delta\alpha = 0.05$ to $0.1$.

For changes in the interaction characteristics it can be shown that they will be mainly reflected in variations of the height of the shower maximum. The energetic muons recorded underground are mostly produced not far from that level. Then it follows from rather general arguments (cf. Bosia et al. (8)) that their lateral distribution must roughly correspond to

(5) $$n(r) \approx ((r+r_1)/r_o) \exp(-(r+r_1)/r_o); \qquad r_o = h\, p_o / E_\mu$$

in which h is the mean height of production, and $p_o$ the parameter of the CKP transverse-momentum distribution. Near the shower axis the power-law approximation (3) holds with

(6) $$\beta = -1 + \alpha\, r/(r+r_1) - (r/r_o)$$

so that $\Delta\beta$ is easily related to a change $\Delta h$ in the mean production level.

We have estimated the effects of the following interaction changes:

(i) a transition from a low-multiplicity law of secondary production to $n_s \alpha E^{1/2}$;

(ii) a change in the inelasticity of the primary collisions from

$k = 1/2$ to $k' \leq 1$; and

(iii) a change in the interaction cross section $\sigma$ by a fraction $\Delta\sigma/\sigma \simeq 0.1$ to $0.2$. It could be shown that in all cases $\Delta h \gtrsim (1-2)$ cascade units results, so that $\Delta\beta = 0.05$ is a conservative choice.

Now the graphs of Fig. 2(a) and (b) demostrate that only a change in the primary composition can account for the agreement between observed and calculated rates. The curves obtained with the unmodified spectrum turn to inconsistently high frequencies for high multiplicities. Moreover, also both changes in $\alpha$ and in $\beta$ applied to the "broken" Moscow spectrum already lead to a progressive disagreement at rising i. An unchanged spectrum and a change in $\alpha$ or in $\beta$ would evidently produce results which are quite incompatible with the experimental data.

We conclude, therefore, that our analysis leaves as the only tenable interpretation of the change in slope of the shower size distribution that of a genuine steepening of the primary energy spectrum above a few $10^{15}$ eV, probably as the result of astrophysical conditions affecting the propagation processes.

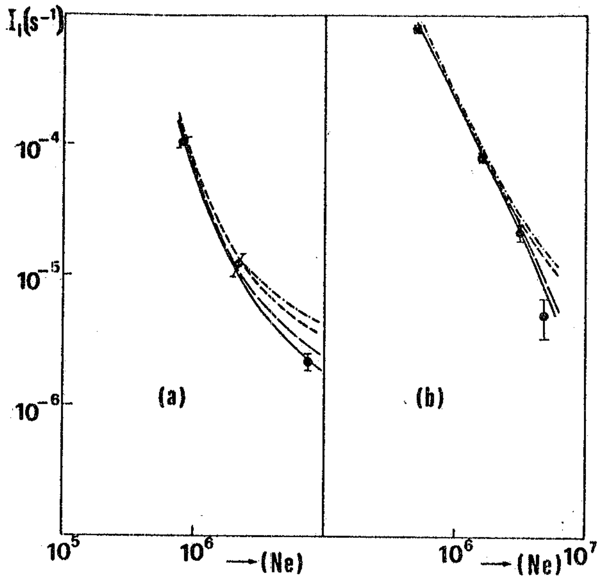

FIG. 2 - Comparison of observed frequencies I(i) with calculations based on the unchanged Moscow data (full lines), on the Moscow size spectrum and $\Delta\alpha = 0.1$, (dotted lines), or $\Delta\beta = 0.05$ (dashed lines), and on a constant-slope size spectrum (dashed-dotted lines). (b) refers to triggering mode $(M_1 \cdot M_2)$, (a) to $(M_1+M_2)/2$.

## REFERENCES. -

(1) - J. J. Braddick and G. S. Hensby, Nature 144, 1012 (1939).

(2) - K. Greisen, G. Cocconi and L. M. Bollingern, Phys. Rev. 82, 294 (1950); P. H. Barrett, L. M. Bollinger, G. Cocconi, Y. Eisenberg and K. Greisen, Revs. Mod. Phys. 24, 133 (1952).

(3) - E. Amaldi, C. Castagnoli, A. Gigli and S. Sciuti, Nuovo Cimento 9, 969 (1952).

(4) - S. N. Vernov, B. V. Khrenov and G. B. Khristiansen, Acta Phys. Hung, 29, Suppl. 4, 41 (1969).

(5) - H. Shibata, T. Kaneko and S. Kishimoto, J. Phys. Soc. Japan 29, 1427 (1970).

(6) - L. Bergamasco, C. Castagnoli, A. Piano, B. D'Ettorre Piazzoli, P. Picchi and R. Visentin, Lett. Nuovo Cimento 1, 1001 (1971).

(7) - K. Greisen, Ann. Rev. Nuclear Sci. 10, 63 (1960).

(8) - G. Bosia, L. Briatore and K. Sitte, Nuovo Cimento 12A , 1025 (1972).