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A. Bietti and G. Parisi :

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ON THE POSSIBILITY OF OBTAINING A FINITE VALUE FOR THE PROTON-NEUTRON MASS DIFFERENCE

A. BIETTI
Istituto di Fisica, Università di Roma
Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italy

and

G. PARISI
Laboratori Nazionali del C.N.E.N., Frascati, Italia

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We discuss the possibility of evaluating the p-n mass difference unambiguously, taking in account a possible violation of Bjorken scaling law in the deep inelastic e-p, e-n scattering. We find that even a very small amount of violation is sufficient in order to obtain the correct value for the p-n mass difference.

It is one of the best known experimental facts that the proton-neutron mass difference is finite and the neutron is heavier than the proton: on the other hand the state of theoretical calculations on this subject is far from being satisfactory [1].

In fact, if we try to evaluate the p-n mass difference in a conventional point-like nucleon theory, we find an infinitely heavier proton [2]. Such a difficulty could in principle be avoided by introducing a counter term in the Lagrangian in order to make the p-n mass difference finite, but this procedure turns out to be very ambiguous and the calculation of the mass difference looses therefore any reliability.

A possible outcome from this difficulty was then the hope that the presence of strong interactions could damp the elastic and inelastic form factors [3, 4] at large values of the virtual photon momentum, in order to have a convergent result for the mass difference.

Unfortunately the interpretation of the recent SLAC experiments [5] on the deep inelastic e-p and e-n scattering in terms of the Bjorken scaling law [6] creates serious difficulties towards a hope of a finite result for $\delta m^{\text{p-n}}$. In fact, if we assume the validity of the Bjorken scaling [6] for large enough values of the squared momentum transfered, the p-n mass difference turns out to be at least logarithmically divergent [7].

On the other hand, quite recently [8, 9], it was realized that a standard renormalizable field theory with anormalous dimensions [10] is incompatible with the Bjorken scaling limit: in other words only really free fields, with canonical dimensions, can give Bjorken scaling.

This circumstance enables us to take into account a possible violation of scaling in the deep inelastic e-p and e-n scattering. In fact, the present experimental data, at least for the e-p case, are consistent with the possibility that scaling is true only as a first approximation and violations due to the strong interactions should be present even at moderate energies [11].

The aim of this note is to show that if we have a violation of the Bjorken scaling law we are able to compute unambiguously the proton neutron mass difference simply because the presence of anomalous dimensions in the operators involved makes the integral over the "photon" momentum convergent, or else if it diverges, an unambiguous prescription for extracting the finite part can be given.

We will then find that even a very small amount of violation of scaling is sufficient in order to obtain the correct mass difference value.

Following Zee [1], we can write for the self mass the Cottingham formula

$$\delta m^{\text{p-n}} =$$

$$-\frac{1}{2\pi}\int_0^\infty \frac{\mathrm{d}q'^2}{q'^2}\int_0^q \mathrm{d}\nu\,(q'^2-\nu^2)^{1/2} g^{\mu\nu} T_{\mu\nu}^{\text{p-n}}(q'^2, i\nu)$$

$$T_{\mu\nu}^{\text{p-n}} = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) T_1^{\text{p-n}}(q^2,\nu)$$

$$+\frac{1}{m^2}\left(p_\mu - \frac{pq}{q^2}q_\mu\right)\left(p_\nu - \frac{pq}{q^2}q_\nu\right) T_2^{\text{p-n}}(q^2,\nu)$$

and is the off-shell Compton amplitude on proton or neutron, where p_μ and q_μ are respectively the four momenta of the nucleon and the virtual photon and $\nu = (pq)$. As it is customary [12], we consider dispersion relations in ν: once substracted for T_1 and unsubtracted for T_2.

$$T_1(q^2,\nu) = T_1(q^2,0) + \nu^2 \int_{\nu_0^2}^{\infty} d\nu'^2 \frac{W_1(q^2,\nu')}{\nu'^2(\nu'^2-\nu^2)}$$

$$T_2(q^2,\nu) = \int_{\nu_0^2}^{\infty} d\nu'^2 \frac{W_2(q^2,\nu')}{\nu'^2-\nu^2}$$

with $\nu_0 = -q^2/2$ and $W_i(q^2,\nu) = \pi^{-1}\,\text{Im}\,T_i(q^2,\nu)$. If we now believe to Bjorken scaling law for photon squared momenta larger than a certain q_0^2, we can assume that $W_i(q^2,\nu) \equiv W_i(x)$ with $x = -q^2/2\nu$ so that the contribution to $\delta m^{\text{p-n}}$ coming from the scaling q^2 region will be

$$\delta m_\infty^{\text{p-n}} = m_\infty^{\text{p}} - m_\infty^{\text{n}} = \frac{3\pi e^2}{4} \int_{-q_0^2}^{\infty} \frac{dq'^2}{q'^2}$$

$$\times \left[-q'^2 T_1^{\text{p-n}}(q'^2,0) + \int_0^1 dx (F_2^{\text{p-n}}(x) + 2xF_1^{\text{p-n}}(x))\right]$$

where $F_1 = m^2 W_1$ and $F_2 = \nu W_2$. From this formula it is already evident the possibility of a logarithmically divergent $\delta m_\infty^{\text{p-n}}$. In order to evaluate the subtraction constant $T_1(q^2,0)$ we can follow the very reasonable suggestion given by Jackiw et al. [13]: the amplitude

$$T_l(q^2,\nu) = T_1(q^2,\nu) + \frac{\nu^2}{m^2q^2} T_2(q^2,\nu) \qquad (1)$$

satisfies an unsubtracted dispersion relation.

In such a case we obtain, using also the condition $F_2(x) = 2xF_1(x)$, which follows from the assumption, compatible with experiment, at least for e-p scattering, that in the deep inelastic region the longitudinal cross section σ_L vanishes, (this condition also avoids a quadratic divergence for δm_∞)

$$\delta m_\infty^{\text{p-n}} = \frac{3\pi e^2}{4} \int_{-q_0^2}^{\infty} \frac{dq'^2}{q'^2} \left[K^{\text{p-n}} - 2\int_0^1 dx F_2^{\text{p-n}}(x)\right] \qquad (2)$$

with

$$K^{\text{p-n}} = \lim_{-q^2\to\infty} \frac{-q^2}{m^2} \int_0^1 \frac{dx}{x^2}$$

$$\times \left[F_l^{\text{p-n}}(x,q^2) - \frac{4m^2x^2}{q^2} F_2^{\text{p-n}}(x,q^2)\right]$$

where $F_l = (-1/2\pi)m^2 x\,\text{Im}\,T_l$. ($T_l$ is given in eq. (1).) It is even more clear now that, unless we have the very peculiar circumstance of a cancellation between the two members in the integral in eq. (2), we will end up with a logarithmically divergent result for $\delta m_\infty^{\text{p-n}}$.

However, the possibility that Bjorken scaling fails cannot too easily be rejected:

a) It is very hard to build up, in presence of fermions, a consistent model which doesn't reduce itself to a free non interacting theory [8], and shows Bjorken scaling.

b) If we do a careful analysis of the data, we find that large violations of scaling can be present [11].

In contrast to Bjorken scaling let us then assume that each of the following integrals (for instance considering the F_2 structure function) has a simple q^2 dependence [14]

$$\int_0^1 dx\, x^{N-2} F_2^{\text{p-n}}(x,q^2) \xrightarrow[-q^2\to\infty]{} \sum_i C_{N,i}^{\text{p-n}} \left(\frac{-q^2}{m^2}\right)^{\alpha_{N,i}} \qquad (3)$$

Bjorken scaling is clearly equivalent to the hypothesis that $\alpha_{N,i} = 0$ for every N and i. The meaning of $\alpha_{N,i}$ is the anomalous dimension of the operators of spin N which contribute to the leading light cone singularity [15] in the product of two currents. i is an isotropic or SU(3) index: there are more then one contributing operator for each spin; the structure functions for e-p and e-n scattering will be a combination of isoscalar and isovector terms (the isoscalars are the dominating contributions).

Of course only the isovector term will be relevant in the computation of the p-n mass difference, and we have from eq. (3) that the integral in eq. (2) will be substituted by

$$\int_0^1 dx F_2^{\text{p-n}}(x,q^2) \xrightarrow[-q^2\to\infty]{} C_2^{\text{p-n}} \left(\frac{-q^2}{m^2}\right)^{\alpha_{2,\text{v}}} . \tag{4}$$

Just for give some numbers, let us now assume in eq. (3) that $K^{\text{p-n}} = 0$. This assumption amounts to say that $\sigma_L^p - \sigma_L^n = 0$ even in the non scaling region, and is not contradicted by the present experimental data [5], at least for the proton case. We then have

$$\delta m_\infty^{\text{p-n}} = -\frac{3\pi^2 e^2}{2} C_2^{\text{p-n}} \int_{-q_0^2}^{\infty} \frac{dq'^2}{q'^2} \left(\frac{q'^2}{m^2}\right)^{\alpha_{2,\text{v}}} .$$

If $\alpha_{2,\text{v}} < 0$ we get then a convergent result

$$\delta m_\infty^{\text{p-n}} = \frac{3\pi e^2}{2} C_2^{\text{p-n}} \frac{1}{\alpha_{2,\text{v}}} \left(\frac{-q_0^2}{m^2}\right)^{\alpha_{2,\text{v}}}$$

which furthermore is of the right sign in order to match the Born term contribution.

$C_2^{\text{p-n}}$ is defined as the value of eq. (4) when $-q^2 = m^2$. If $\alpha_{2,\text{v}}$ is small, this amounts to take the value of the integral in the case in which Bjorken scaling is supposed to be valid.

This can be taken from experimental analysis [5], and we have $C_2^{\text{p-n}} \approx 0.04\, m/(2\pi)^3$ *.

The Born term gives a contribution [4] to $\delta m^{\text{p-n}}$ of about +0.8 MeV. The inelastic (due to the resonances) contribution for small values of q^2 is almost negligible [4, 16]. Therefore in order to have a correct mass difference value $\delta m_\infty^{\text{p-n}}$ should be of the order of -2.1 MeV that is, assuming $-q_0^2 = 1\ \text{GeV}^2$, $\alpha_{2,\text{v}} \approx -0.035$. We see thus that even a very small violation of scaling is enough to give a correct mass difference value. The present experimental situation on e-p and e-n scattering unfortunately does not allow us to decide not only if $\alpha_{2,\text{v}}$ is negative or positive, but even if we can really assume $K^{\text{p-n}} = 0$.

If $K^{\text{p-n}} < 0$, as has been suggested by Lee [17] $|\alpha_{2,\text{v}}|$ could be slightly larger.

Another popular assumption [1] is to take $K^{\text{p-n}} \neq 0$ but $F_l(x,q^2) = 0$. This is also consistent with experiment, at last for e-p scattering, and we have now

$$\delta m_\infty^{\text{p-n}} = \frac{3\pi e^2}{2} C_2^{\text{p-n}} \int_{-q_0^2}^{\infty} \frac{dq'^2}{q'^2} \left(\frac{q'^2}{m^2}\right)^{\alpha_{2,\text{v}}} .$$

* Actually the integral is performed in the interval $0.08 \leqslant x \leqslant 1$, but the contribution for $x < 0.08$ can be shown to be essentially negligible.

In this case, in order to have the correct sign for $\delta m^{\text{p-n}}$ we should have $\alpha_{2,\text{v}} > 0$ and the integral then diverges. However, we can always extract unambiguously the finite part, using for example analytic renormalization techniques [18].

We see therefore that it is possible to get a definite and non ambiguous value for the p-n mass difference if we allow a small Bjorken scaling violation in deep inelastic scattering.

Of course this violation could be much larger in e-p or e-n scattering separately, because of the other contributions which vanish when we consider the difference between the structure functions of the two reactions.

It will be the task, we hope, of future experiments to detect or to exclude the presence of such violations of Bjorken scaling, which have been so strongly suggested [8] by field theoretical models.

After this paper was completed we learned that the possibility of obtaining a finite proton-neutron mass difference from a violation of the Bjorken scaling law was already discussed by Moffat and Wright [19] who used a Regge-pole model where the A2-pole contribution breaks the scale invariance. Our approach, however, is rather different, because it relates the Bjorken scaling breaking to the existence of anomalous dimensions in our case Bjorken scaling law should be violated also for the sum of the proton and neutron structure functions.

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