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# ON THE MEAN LIFE-TIME OF THE PERSISTENT CURRENT IN A TWO-DIMENSIONAL SUPERCONDUCTOR

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Using an approximate form of the time dependent Gizburg-Landau equation we find that in a two dimensional system quasi-persistent currents are present, their mean life-time exponentially larger.

It is well known that rigorous theorems [1] state that in a two dimensional system the onset of very large fluctuations prevents the appearance of a second order phase transition characterized by long range order. These theorems imply that really persistent currents cannot be present in a two dimensional superconductor; however persistent currents have been observed in two-dimensional clean Al films: their mean life-time was greater than ten hours and compatible with infinity [2].

In order to understand how no contradictions arise between the experimental and the theoretical results, we study the approach to equilibrium using a time-dependent Ginzburg-Landau equation [3], where we take care of the effect of fluctuations in some approximate way [4, 5]. In this model we find, as expected, that in the infinite time limit the mean value of the order parameter $\langle \psi \rangle$ is zero; however, if at some time long range order is present, $\langle \psi \rangle$ decays with a mean life-time which is exponentially increasing with decreasing temperature.

For typical films used [3] the theoretical mean life-time increases by a factor $\sim 10$ each milliKelvin

We start from the following form of the time dependent Ginzburg-Landau equation for the order parameter:

$$\left\{\tau \frac{\partial}{\partial t} - \frac{\hbar^2 (\nabla - \mathrm{i}eA)^2}{2m} + a(T - T_c^*) + b\langle |\psi(t)|^2 \rangle\right\} \psi(x, t) = f(x, t) \qquad (1)$$

The function $f(x, t)$ is a random force with a stochastic autocorrelation; eq. (1) has the form of a Langevin equation.

The quantity $\langle |\psi(t)|^2 \rangle$ can be written as

$$\langle |\psi|^2 \rangle = \frac{1}{2\pi} \int_{p^2 < 1/Q^2} \mathrm{d}^2 p \, G(p, t) \qquad (2)$$

where $G(p, t)$ is the Fourier transform of the order parameter correlation function $G(x, t) = \langle \psi(x, t), \psi(0, t) \rangle$ and $Q$ is an ultraviolet cutoff, having the dimension of a length.

Using standard methods [3] one finds that, if the mean value of the order parameter is different from zero in the infinite time limit, in the same limit a $1/p^2$ singularity must be present in the propagator; the integral in eq. (2) is not convergent, a mean value of the order parameter which is asymptotically different from zero is thus not consistent.

If we suppose that at the initial time the mean value of the order parameter is different from zero and we study the time evolution of the system, we find that at a very large time the correlation function has the form:

$$\frac{1}{p^2} \left\{ 1 - \exp \left[ -\frac{\hbar^2 p^2}{2m\tau} t \right] \right\}.$$

Substituting in eq. (2) one obtains:

$$\langle |\psi|^2 \rangle = \frac{bKTm}{2\pi d \hbar^2} \ln \frac{t\hbar^2}{2m\tau Q^2}. \qquad (3)$$

This quantity is really divergent for $t = \infty$ but may be relatively small also after times that are very large on the human scale. The time evolution of the order parameter turns out to be approximately

$$\langle \psi \rangle = \left[ \frac{a}{b} (T_c^* - T) - \frac{KTm}{2\pi d \hbar^2} \ln \frac{t\hbar^2}{2m\tau Q^2} \right]^{1/2}$$

the mean life-time being equal to

$$\frac{2m\tau Q^2}{\hbar^2} \exp \left\{ \frac{2\pi a\hbar^2}{bKTm} d(T_c^* - T) \right\}.$$

The conclusions are that the system has a simple Ginzburg-Landau behaviour for finite times; the only effect of the fluctuations being a shift in the transition temperatur [6].

Although more theoretical and experimental work should be done to understand in all details the behaviour of a system whose relaxation time may go to infinity, we believe that we have identified a possible mechanism which accounts for persistent currents in superconducting thin films: all "no-go" theorems are based on a mild logarithmical divergence of an integral and this is a mathematical and not a physical infinity. Rigorous theorems can be applied only in exact equilibrium situations and they are of no practical interest if the relaxation time may go to infinity.

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