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G. Parisi: A SIMPLE TEST OF THE TRASVERSALITY
HYPOTHESIS IN e^+e^- ANNIHILATION INTO HADRONS. -

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G. Parisi^(x): A SIMPLE TEST OF THE TRASVERSALITY HYPOTHESIS
IN e^+e^- ANNIHILATION INTO HADRONS.

ABSTRACT. -

We derive kinematical inequalities for the angular distribution of hadrons produced in e^+e^- collisions: we find that a $1 + \cos^2\theta$ inclusive distribution implies that all produced particles are coplanar.

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2.

Simple theoretical models, like the parton model⁽¹⁾, predict that the spin of the elementary fields is connected to the angular distribution of hadrons produced in e^+e^- annihilation at asymptotic energies.

From general theorems it follows that in the one photon approximation the inclusive differential cross section for the process $e^+e^- \rightarrow h + \text{anything}$ has the form⁽¹⁾

$$(1) \quad \frac{d\sigma}{dE_h d\Omega_h} = f(E_h) \left[1 + \cos^2\theta - 2R(E_h) \cos^2\theta \right]$$

where E_h is the energy of the observed hadron and θ is the angle between the electron-positron beam and the direction of h . Positivity of the cross section implies that $0 \leq R \leq 1$.

The direct determination of R is rather difficult because of the experimental bias which may distort the θ distribution.

In this letter we show that some information on R can be gained without doing any analysis in θ . In particular strong constraints on the final states follow from the hypothesis of only spin zero or spin 1/2 constituents.

For each event $e^+e^- \rightarrow N$ hadrons, we denote by θ_{ri} the angle between a generical straight line r and the direction of flight of the i -th produced hadron. We make the positions

$$(2) \quad A = \frac{1}{N} \min_r \sum_{i=1}^N \cos^2\theta_{ri}$$
$$B = \frac{1}{N} \max_r \sum_{i=1}^N \cos^2\theta_{ri}$$

where the minimum and the maximum are taken over all the possible values for r .

The mean values of A, B and R satisfies the inequality:

$$(3) \quad 0 \leq \left\langle \frac{2A}{1+A} \right\rangle \leq R \leq \left\langle \frac{2B}{1+B} \right\rangle \leq 1$$

We note that $A=0$, $B=1$ implies that all the particles are produced parallel to each other; $A=0$, $B \neq 1$ implies that all the particles belong to a plane; in this last case if $R=0$ the angle φ between the normal to the plane and the beam direction shows a $\sin^2 \varphi$ dependence.

If we are interested in the contribution to $\langle R \rangle$ of particles having energies greater than E_t , the sum in (2) should run only on this kind of particles.

The proof is simple if we consider how the inclusive distribution is obtained from the exclusive one. We prove that the contribution of an arbitrary N particles final state $|H\rangle$ to (3) satisfies the inequality.

If J_μ is the conserved electromagnetic current in the c. m. frame the vector $j_\mu^H = \langle 0 | J_\mu | H \rangle$ has only spatial components. If j_μ^H points in the direction r , an explicit computation reveals that the contribution of $|H\rangle$ to $\langle R \rangle$ is

$$(4) \quad R = 2 \frac{\sum_1^N \cos^2 \theta_{ri}}{\left[N + \sum_1^N \cos^2 \theta_{ri} \right]}$$

which clearly satisfies eq. (3), q. e. d.

A simple way to determine the values of A and B is the following: we denote by x_α^i the director cosines of the i-th particles.

We construct the 3 x 3 matrix

$$M_{\alpha\beta} = \frac{1}{N} \sum_i x_\alpha^i x_\beta^i$$

4.

Its eigenvalues λ^j satisfies the relation:

$$(5) \quad \sum_1^3 \lambda^j = 1 \quad A = \min \lambda_j \quad B = \max \lambda_j$$

REFERENCES. -

- (1) - S. D. Drell, D. J. Levy and Tung Mow Yan, Phys. Rev. Letters 22, 744 (1969); Phys. Rev. 187, 2159 (1969).