

LNF-73/12  
6 Aprile 1973

G. De Franceschi, E. Etim and S. Ferrara: DEEP INELASTIC  
STRUCTURE FUNCTIONS FROM COLLINEAR  $SU(1, 1)$   
SYMMETRY. -

Laboratori Nazionali di Frascati del CNEN  
Servizio Documentazione

LNF-73/12  
6 Aprile 1973

G. De Franceschi, E. Etim and S. Ferrara: DEEP INELASTIC STRUCTURE FUNCTIONS FROM COLLINEAR  $SU(1,1)$  SYMMETRY; -

ABSTRACT. -

We exhibit a model of deep inelastic processes, based on conformal symmetry arguments, in which the structure functions for the scattering and annihilation channels both scale and are related by the Gribov-Lipatov relation. Analytic continuation is however not valid in general.

2.

In this note we present a model in which the structure functions, upon diagonalization of the forward Compton amplitude with respect to representations of the conformal group<sup>(1)</sup>, are given as generalized Born terms.

In the transition form factors  $\langle p | J_\mu(0) | n \rangle$  figuring in the sum

$$(1) \quad W_{\mu\nu}(p, q) = \sum_n (2\pi)^4 \delta^4(p+q-p_n) \langle p | J_\mu(0) | n \rangle \langle n | J_\nu(0) | p \rangle$$

the states  $|p\rangle$  and  $|n\rangle$  are classified according to zero and continuous mass representations respectively of the conformal group  $SU(2,2)$ <sup>(2)</sup>. The structure functions associated with a single irreducible representation in the set spanned by  $n$  turn out to be non-trivial as a continuous spectrum of resonances is contained in any such representation. Scaling of the structure functions follows from the requirement that the field associated with the target is massless while their functional form is fixed by covariance under  $SU(1,1)$  which is the relevant part of the conformal group in the scaling limit. In configuration space this corresponds to projective transformations along a light-like line<sup>(3)</sup>.

Our classification of the intermediate states is in some sense analogous to a previous attempt by Del Giudice et al.<sup>(4)</sup> to assign partons to representations of  $SU(1,1)$ . However under the larger group  $SU(2,2)$  there is only one linear trajectory  $l = 1 + |j|$  for zero mass states of helicity  $j$  and scale dimension  $l$ <sup>(5)</sup>. Many interesting consequences emerge from such a scheme.

i) The structure functions have a good threshold behaviour as  $\omega \rightarrow 1$

$$(2) \quad F_2(\omega) \xrightarrow[\omega \rightarrow 1]{\lambda} A(\omega-1)^{2\lambda+1}$$

where the exponent  $\lambda$  is the SU(1,1) quantum number of the continuum which builds up the inelastic processes

ii) Their Regge behaviour is universal i. e. independent of  $\lambda$

$$(3) \quad F_2(\omega) \xrightarrow[\omega \rightarrow \infty]{} \omega^{-2} \log^2 \omega$$

Apart from the log this is the same as found in dual resonance models<sup>(6)</sup>.

iii) The asymptotic behaviour of the excitation form factors is correlated with the threshold behaviour in the manner first suggested by Drell, Yan and West<sup>(7)</sup>.

iv) The structure functions of scattering ( $F_2(\omega)$ ) and annihilation ( $\bar{F}_2$ ) are related by the Gribov-Lipatov reciprocity<sup>(8)</sup> relation

$$(4) \quad \bar{F}_2\left(\frac{1}{\omega}\right) = -\omega^3 F_2(\omega)$$

v) The analytic continuation

$$(5) \quad \bar{F}_2(\omega) = F_2(\omega)$$

does not hold in general although the threshold theorem is satisfied<sup>(9)</sup>.

vi)  $F_2(\omega)$  so calculated for  $\lambda = 1$  fits the data points rather well in the interval  $1 \leq \omega < 10$ .

For simplicity we consider Compton scattering off a scalar target of 4-momentum  $p$  as in Fig. (1). The kinematical variables are standard

$$\omega = 2pq / -q^2, \quad S = -q^2(\omega - 1) > 0$$

4.

with  $1 \leq \omega < \infty$  for scattering and  $0 \leq \omega \leq 1$  for annihilation and  $S$  the invariant mass of the resonances exchanged between the blobs in Fig. 1. In the short distance limit the form of these latter is fixed by  $SU(1,1)$  symmetry and using standard reduction techniques one gets as  $x_- \rightarrow 0$  and  $x^2 \rightarrow 0$

$$\begin{aligned} & \text{Im} \int d^4 x e^{iqx} \langle p | (J_-(x_+, x_-, \vec{x}_T) \psi_-^{(n)}(0)) | 0 \rangle = \\ & \text{Im} \int dx_+ dx_- e^{i/2(q_+ x_- + q_- x_+)} \frac{1}{(x_-)} = 1/2(d_n + 1) \frac{1}{(x_+)}^{-1/2(\tau_n - 1)} \\ & \int_0^1 du u^{(1-\tau_n)/2} (1-u)^{(\tau_n-3)/2} e^{iu x_+ p_- / 2} \quad (x_+ = t + z) \\ & = (q_+^2)^{\frac{1}{2}(\tau_n - 3)} \frac{1 + \tau_n}{(q_+)} \omega^{(1-\tau_n)/2} (\omega - 1)^{\tau_n - 2} x \end{aligned}$$

(6)

$$x {}_2F_1\left(\frac{\tau_n - 1}{2}, \frac{\tau_n - 1}{2}, \tau_n - 1, 1 - \frac{1}{\omega}\right) \theta(\omega - 1)$$

where  $\omega = p_- / q_-$ . We have used the  $SU(1,1)$ <sup>(10)</sup> light cone expansion to recover the vertex function in coordinate space.  $\psi^{(n)}(x)$  is a conformal weight zero field<sup>(5)</sup> of spin  $n = 1/2(d_n - \tau_n)$  and scale dimension  $l_n = 1/2(d_n + \tau_n)$ . Making use of (6) in eq. (1) one gets for a given conformal quantum number  $\lambda = 1/2(\tau_n - 3)$

$$(7) \quad F_2^{(\lambda)}(\omega) = \frac{C_\lambda}{\omega(\omega - 1)} Q_\lambda^2\left(\frac{\omega + 1}{\omega - 1}\right)$$

where  ${}_2F_1(\dots)$  has been expressed in terms of the Legendre function  $Q_\lambda(\dots)$ <sup>(11)</sup>. The physical meaning of  $\lambda$  is that it determines the

asymptotic power growth of the spectrum of resonances<sup>(12)</sup>

$$(8) \quad \varrho(S) \xrightarrow[S \rightarrow \infty]{} S^{2\lambda + 1}$$

From eq. (6) the electromagnetic excitation form factor behaves for large  $q^2$  as<sup>(10, 13)</sup>

$$(9) \quad G(q^2) \xrightarrow[q^2 \rightarrow \infty]{} (q^2)^{-(1+\lambda)}$$

which on comparison with eq. (8) reproduces the Drell, Yan and West relation<sup>(7)</sup>. From eq. (7) on the other hand one obtains for each  $\lambda$ , eqs. (2) and (3) together with

$$(10) \quad F_2^{(\lambda)}\left(\frac{1}{\omega}\right) = (-1)^{2\lambda+1} \omega^3 F_2^{(\lambda)}(\omega)$$

while for the annihilation structure function one finds

$$(11) \quad \bar{F}_2^{(\lambda)}(\omega) = (-1)^{2\lambda} F_2^{(\lambda)}(\omega)$$

Eqs. (9) and (10) together give

$$(12) \quad \bar{F}_2^{(\lambda)}\left(\frac{1}{\omega}\right) = -\omega^3 F_2^{(\lambda)}(\omega)$$

independently of  $\lambda$  whence eq. (4) upon summation over this latter. From the above results it follows that the analytic continuation in eq. (5) is not true in general so that the stronger reciprocity relation

$$(13) \quad F_2\left(\frac{1}{\omega}\right) = -\omega^3 F_2(\omega)$$

6.

does not hold. This would be so under the additional assumption that the set of contributing representations correspond to integer  $\lambda$ . It is interesting to point out that eq. (7) is a particular realization of the following formula of Gribov and Lipatov<sup>(8)</sup>.

$$(14) \quad F_2(\omega) = \frac{\omega-1}{\omega^2} \varphi\left(\left(\frac{\omega-1}{\omega+1}\right)^2\right)$$

where  $\varphi(x)$  is positive for  $x > 0$  and regular at  $x=0$ . The right hand side of eq. (7) becomes an elementary function for  $\lambda = n > 0$ <sup>(11)</sup>. An interesting case is  $n=1$  which gives.

$$(15) \quad F_2(\omega) = \frac{C}{\omega(\omega-1)} \left[ \frac{\omega+1}{\omega-1} \ln \omega - 2 \right]^2$$

A plot of eq. (15) is shown in Fig. 2 together with the SLAC data points<sup>(14)</sup>. The free parameter  $C$  has been chosen near 3.  $C \simeq 3$ .

## REFERENCES. -

- (1) - L. Castell, Phys. Rev. D2, 1161 (1970); S. Ferrara, R. Gatto, A. F. Grillo and G. Parisi, Nucl. Phys. B49, 77 (1972); A. A. Migdal, Landau Institute Preprint, Sept. 1972, to be published.
- (2) - L. Castell, Nuovo Cimento 46A, 1 (1966); H. Kastrup, Phys. Rev. 142, 1060 (1966).
- (3) - S. Ferrara, R. Gatto, A. F. Grillo and G. Parisi, University of Rome preprint n. 452, (January 1973) to be published.
- (4) - E. Del Giudice, P. Di Vecchia, S. Fubini and R. Musto, Nuovo Cimento 12A, 813 (1972).
- (5) - S. Weinberg, Phys. Rev. 134B, 882 (1964); G. Mack and I. T. Todorov, J. Math. Phys. 10, 2078 (1969); G. Mack and A. Salam, Ann. Phys. 53, 174 (1969).
- (6) - R. Casalbuoni and G. Gomis, Univ. of Florence Preprint January 1973 and ref. (4).
- (7) - S. D. Drell and T. M. Yan, Phys. Rev. Letters 24, 181 (1970); G. West, Phys. Rev. Letters 24, 1206 (1970).
- (8) - V. N. Gribov and L. N. Lipatov, Phys. Letters 37B, 78 (1971); Soviet J. Nucl. Phys. 15, 438, 675 (1972).
- (9) - R. Gatto, P. Menotti and U. Vendramin; Nuovo Cimento Letters 5, 754 (1972).
- (10) - S. Ferrara, R. Gatto and A. F. Grillo, Nucl. Phys. 34B, 349 (1971); S. Ferrara, A. F. Grillo and G. Parisi, Nuovo Cimento 12A, 952 (1972).
- (11) - I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series and Products. Academic Press, New York (1965); Eqs. (9) 134, 8 703 and 8 827.
- (12) - E. Etim; Phys. Letters 41B, 169 (1972); Nuovo Cimento 13A, 427 (1973).



8.

(13) - A. A. Migdal, Phys. Letters 37B, 98, 386 (1971).

(14) - E. D. Bloom, D. H. Coward, H. Destaebler, J. Drees, L. W. Mo  
and R. E. Taylor, Phys. Rev. Letters 23, 935 (1969):

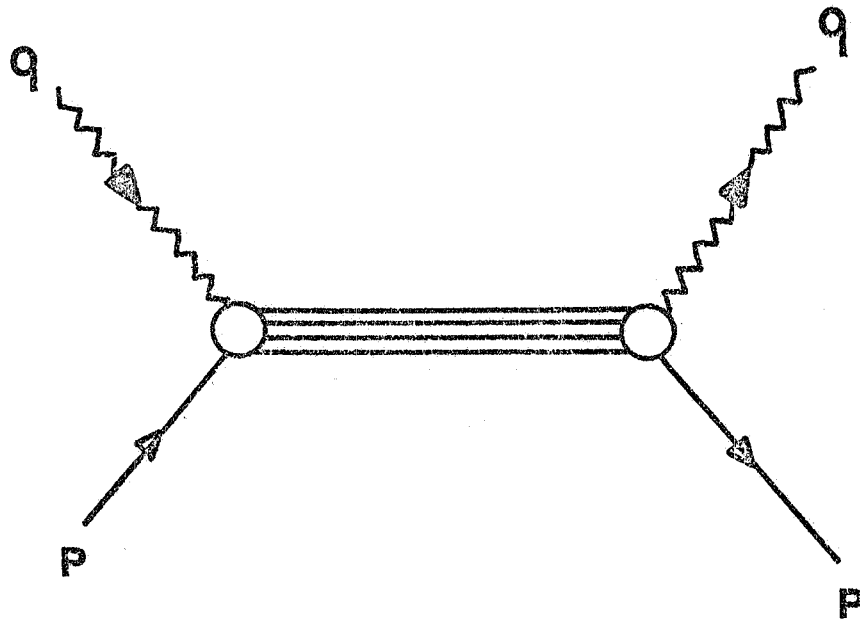


FIG. 1 - A typical diagram contributing to the structure function.

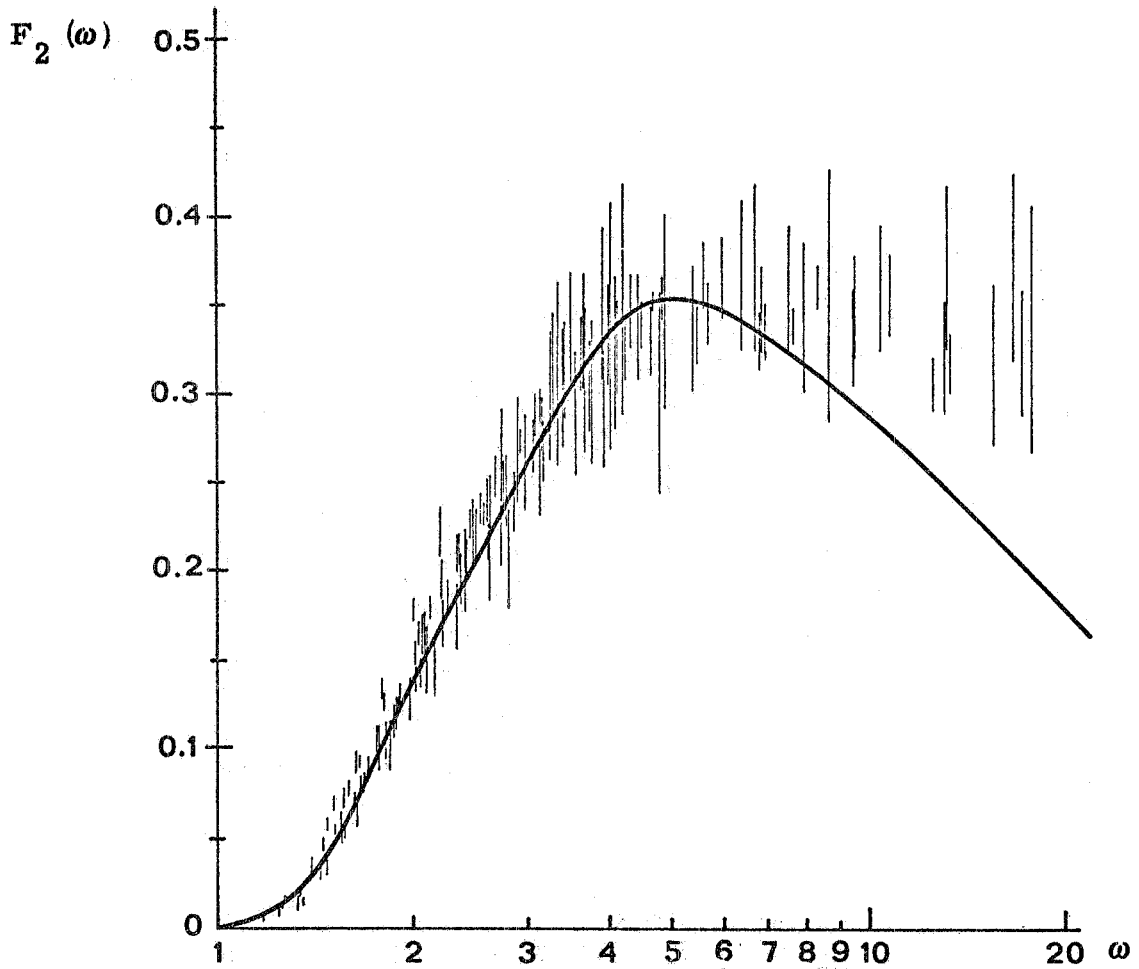


FIG. 2 - A plot of the calculated structure function for  $\lambda = 1$  (eq. (15)) as it compares with the data.