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EXPERIMENTAL LIMITS ON THE VALUES OF ANOMALOUS DIMENSIONS

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We show how to use the data on deep inelastic e–p scattering to put bounds on the values of the anomalous dimensions of the operators involved in the Wilson expansion of the product of two currents near the lightcone. Anomalous dimensions of the order of unity are found not to be in contradiction with present experimental evidence.

It is known that Bjorken [1] scaling is violated in perturbation theory, because of the appearance of annoying $\log(q^2)$ [e.g. 2]. This fact may be completely irrelevant because perturbation theory becomes useless in strong interaction, however there are general theoretical arguments which suggest that Bjorken scaling is still not valid also in the full theory [3].

It remains true that in the large q^2 region the following integrals scale [4, 5]

$$\int_1^\infty F_2(\omega, q^2)\, \omega^{-N} \mathrm{d}\omega \to \sum_i C_N^i \left(\frac{M^2}{q^2}\right)^{\sigma_N^i/2},$$

$$N = 2, 4, \ldots, \qquad (1)$$

but Bjorken scaling can be true only if $\sigma_N^i = 0$ for each even integer N. The physical meaning of σ_N^i is the anomalous part of the dimension [3] of the operators of spin N which yield the leading term in the Wilson expansion of two currents near the light-cone [6]. The index i runs on the possible quantum numbers of the spin N operators. The near equality of the two Callan-Gross [7] intagrals on proton and on neutron [8] may suggest that the leading contribution comes from the stress energy tensor [9, 10]. We recall that the anomalous part of the dimension of the stress energy tensor is zero.

For sake of simplicity we assume that for each spin there exist only one operator, or, if many operators are relevant, an effective index can be defined; we consistently pose $\sigma_2 = 0$.

In this letter we assume that Bjorken scaling is violated i.e. all the $\sigma_N, N \neq 2$ are different from zero; we try to use the SLAC data on deep inelastic e–p scattering to obtain some bound on the value of σ_N.

In order to define the problem we choose a reasonable but arbitrary parametrization of the anomalous dimensions:

$$\sigma_N = A\left[1 - \frac{12}{(N+1)(N+2)}\right]. \qquad (2)$$

This type of parametrization is very similar to the results of recent calculations [5, 11, 12]. We try now to find what are the possible values for A.

In the large q^2 region the old scaling law $F_2(\omega,q^2) = F_2(\omega, K^2)$ is no more valid; there are more complicated relations which allow to compute $F_2(\omega, q^2)$ for all ω and q^2 once we know it as function of ω at one fixed value $q^2 = K^2$.

The new "scaling law", which may be obtained expanding the right hand side of eq. (1) in powers of $\log(q^2/K^2)$ and using the Faltung-theorem for the Mellin transforms*, is:

$$F_2(\omega, q^2) = F_2(\omega, K^2)\left[1 - \Delta(\omega, K^2)\frac{A}{2}\log\frac{q^2}{K^2} + \mathrm{O}\left(A^2 \log\frac{q^2}{K^2}\right)\right],$$

$$\Delta(\omega, q^2) = +1 - \int_{1/\omega}^{1} \mathrm{d}x(x^2 - x^3)\,\frac{F_2(\omega x, q^2)}{F_2(\omega, q^2)}. \qquad (3)$$

If the function $\Delta(\omega, q^2)$ was identically equal to a constant δ, $F(\omega, q^2)$ would be of the form $f(\omega)\,(M^2/q^2)^{-\delta A}$; the fact that Δ is not constant indicates that the dependence from q^2 in the asymptotic region is more complicated; the physical meaning of this function is clear: $\Delta(\omega, q^2)$ is proportional to the violation in each point of Bjorken scaling law. A

* The analyticy of the scattering amplitude in the complex Regge plane allows us to perform an analytical continuation of eq. (1) to complex N and to use the Faltung-theorem for the Mellin transform.

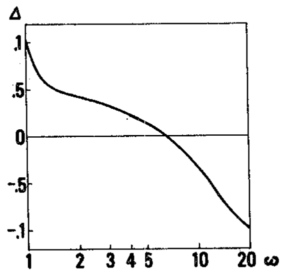

Fig. 1. Plot of the function $\Delta(\omega)$.

typical plot of Δ is shown in fig. 1.

We note that Δ goes to 1 for ω near to 1 and goes to -1 for ω going to infinity and it is much smaller in modulus than 1 in the central region $4 \leqslant \omega \leqslant 8$. This form of the function Δ is quite insensitive to changement of the parametrization used, but follows from the fact that $\int \Delta(\omega, q^2)\, F_2(\omega, q^2)\omega^{-2}\mathrm{d}\omega$ must be zero because $\sigma_2 = 0$.

It is clear that scaling in the central region of ω will be very good nearly independent of the amount of violation of scaling, provided $\sigma_2 = 0$. If scaling violations are present the function F should be decreasing with q^2 at small ω and increasing at large ω.

It is interesting to note that this type of behaviour of the function $F_2(\omega, q^2)$ has been experimentally observed [8], however at too low values of q^2 and ω to be clearly distinguishible from preasymptotic terms that should die at higher energy.

From the data in the high and low ω region we find that A may be as large as ≈ 0.8. If we use also the good "scaling" data at $\omega = 4$ we find $A \lesssim 0.5$. It is interesting that such large violations of the scaling law are not excluded by present data. Their presence may be an explanation of the fact that the "scaling limits" seem to be reached from above at small ω, and from below at large ω.

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