COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-73/7
22 Febbraio 1973

M. Greco and Y. Srivastava: SOME RESULTS IN INCLUSIVE e^+e^- ANNIHILATION INTO HADRONS.

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ABSTRACT. -

e^+e^- annihilation into multihadrons is investigated through the energy-momentum conservation sum rules. Canonical scaling results for total cross-section and $\overline{F}_1$, $\overline{F}_2$ are obtained and new scaling results for 2-particle inclusive processes are derived. The high energy behaviour of the multiplicities is discussed in terms of the correlation functions, in analogy with the purely hadronic case. A sum rule is obtained connecting single and double pion production.

Finally, in a self-consistent way, using FESR and EVMD model we estimate phenomenologically the pion form factor and structure function. A striking prediction is that near $x \simeq 1$, pion structure function $F_2^\pi(x)$ may be an order of magnitude or more larger than that of the proton.

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Supported in part by a grant from the National Science Foundation.

2.

INTRODUCTION. -

Colliding beam inclusive processes have received much attention theoretically in the past few years in hopeful expectation of experimental results to follow shortly. In this work, we analyze n-particle inclusive process

$$e^+e^- \rightarrow \gamma(q) \rightarrow 1 + \ldots\ldots + n + \bar{X} \ ,$$

first via the energy-momentum sum rules. A brief account of such an application may be found in ref. (1). This enables us to shed some light on general grounds about scaling properties of multi-hadron production processes, the question of multiplicities and correlations. We check explicitly that canonical scaling for total cross-section and single hadron production follows and then obtain the precise scaling behaviour of individual amplitudes related to the two-particle production processes. We also find that hadronic multiplicities tend to a finite limit at infinite energies, a result repeatedly obtained in various models[2, 3, 4]. We then derive a sum rule which, e.g., relates π^+ structure function to an integral over the difference of $(\pi^+\pi^-)$ and $(\pi^+\pi^+)$. 2-particle inclusive structure functions. In the last section we estimate the pion form factor and structure function phenomenologically using Bloom-Gilman[5] type FESR arguments and an extended vector meson dominance (EVMD) model, which has been shown[6, 7] to be consistent with scaling in e^+e^- and deep-inelastic scattering. The analysis is in rough agreement with the data where available and is self-consistent. A striking result is that the pion structure function near $x \simeq 1$ ("elastic" limit) may be at least an order of magnitude larger than that for the proton.

I. - KINEMATICS. -

Consider the inclusive e^+e^- process for the production of n hadrons, in the one-photon approximation:

$$e^+(k_+)+e^-(k_-)\to\gamma(q)\to h(k_1)+h(k_2)+\ldots+h(k_n)+X. \tag{1.1}$$

Neglecting the electron mass, the differential cross-section can be written as

$$f^{(n)}_{e^+e^-}\equiv\frac{d\sigma^{(n)}}{\prod_{i=1}^{n}(d^3k_i/E_i)}=\frac{(2\pi)^2\alpha^2}{2s^3}L^{\mu\nu}H^{(n)}_{\mu\nu}, \tag{1.2}$$

where $q^2=s$, $L^{\mu\nu}$ is the leptonic tensor obtained by summing over their spins,

$$L^{\mu\nu}=4\left[k_+^{\mu}k_-^{\nu}+k_-^{\mu}k_+^{\nu}-(k_+k_-)g^{\mu\nu}\right], \tag{1.3}$$

and $H^{(n)}_{\mu\nu}$ is the hadronic tensor, which for identical hadrons (called π's) reads as follows:

$$H^{(n)}_{\mu\nu}=\frac{1}{2^n(2\pi)^{3n}}\sum_m\frac{1}{m!}(2\pi)^4\delta^4(q-k_1-k_2-\ldots-k_n-P_m)\,d\mathcal{P}_m \tag{1.4}$$
$$\langle 0|J_\mu|k_1\ldots k_n;m\rangle\langle k_1\ldots k_n;m|J_\nu|0\rangle.$$

Here:

$$d\mathcal{P}_m=\prod_{i=1}^{m}\left[\frac{d^3q_i}{(2\pi)^3 2q_i^0}\right], \tag{1.5}$$

is the m-particle phase space.

The extension of (1.4) for several species is clear and shall be discussed later.

 4.

The scaling variables are $x_i = \frac{2k_i . q}{s}$, which in the e^+e^- CM frame ("photon" rest frame) is simply $= \frac{2E_i}{\sqrt{s}}$. Hence x_i tells us about the fraction of energy carried off by the i^{th} hadron in the "decay" of a "photon" at rest.

Since we will need later explicit expressions for the total cross-section, single and double inclusive cross-sections in terms of the scalar invariants, we write them down below.

i) Total cross-section $\sigma^{tot}_{e^+e^-}(s)$: $n=0$.-

This is obtained when we sum over all the produced hadrons. $H^{(o)}_{\mu\nu}(q)$ is proportional to the photon "propagator" and we normalize it as follows:

$$H^{(o)}_{\mu\nu}(q) \equiv \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{s}\right)\left(\frac{s}{6\pi}\right) D_o(s) . \qquad (1.6)$$

Then,

$$\sigma^{tot}_{e^+e^-}(s) = \frac{4\pi\alpha^2}{3s} D_o(s) . \qquad (1.7)$$

This normalization is chosen because it yields $D_o(s)=1$ for the production of an "elementary" spin 1/2 object of charge 1. Thus, in the quark model $D_o(s)$ simply counts the sum of the squared charges of the quarks.

ii) Single inclusive cross section $f^{(1)}$: $n=1$.-

The tensor $H^{(1)}_{\mu\nu}(q, k_1)$ is related in the following way to the usual scaling functions $\bar{F}_1$ and $\bar{F}_2$ (8).

$$H^{(1)}_{\mu\nu}(q, k_1) = \frac{1}{(2\pi)^2} \left\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{s}\right) \bar{F}_1(s, x_1) + \right. \qquad (1.8)$$

$$(1.8) \qquad + \frac{2}{x_1 s}\left(k_{1\mu} - \frac{k_1 q}{s} q_\mu\right)\left(k_{1\nu} - \frac{k_1 q}{s} q_\nu\right)\bar{F}_2(s, x_1)\Bigg\} .$$

Then, the single inclusive differential cross-section may be written as

$$(1.9) \qquad \frac{d\sigma^{(1)}}{dx_1 dz_1} \simeq \left(\frac{\pi \alpha^2}{2s}\right)\sqrt{x_1^2 - 4\mu^2/s}\left\{2\bar{F}_1 + \frac{1}{2x_1}(x_1^2 - 4\mu^2/s)(1 - z_1^2)\bar{F}_2\right\},$$

where μ is the pion mass and z_1 is the cosine of the angle made by the pion with respect to the e^+e^- beam.

iii) Double inclusive cross-section $f^{(2)}$: $n=2$.-

The tensor $H^{(2)}_{\mu\nu}(q, k_1, k_2)$ may be decomposed using four gauge-invariant tensors that are available:

$$H^{(2)}_{\mu\nu}(q, k_1, k_2) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{s}\right)G_1 + \frac{1}{\mu^2}\left(k_{1\mu} - \frac{k_1 q}{s} q_\mu\right)\left(k_{1\nu} - \frac{k_1 q}{s} q_\nu\right)G_2$$

$$(1.10) \qquad + \frac{1}{\mu^2}\left(k_{2\mu} - \frac{k_2 q}{s} q_\mu\right)\left(k_{2\nu} - \frac{k_2 q}{s} q_\nu\right)G_3$$

$$+ \frac{1}{2\mu^2}\left[\left(k_{1\mu} - \frac{k_1 q}{s} q_\mu\right)\left(k_{2\nu} - \frac{k_2 q}{s} q_\nu\right) + \left(k_{2\mu} - \frac{k_2 q}{s} q_\mu\right)\left(k_{1\nu} - \frac{k_1 q}{s} q_\nu\right)\right]G_4$$

The invariants G_1, G_2, G_3 and G_4 depend upon 4 Lorentz scalars, which may be chosen to be s, $x_1 = \frac{2k_1 q}{s}$, $x_2 = \frac{2k_2 q}{s}$ and the "missing-mass" variable, $M_x^2 = (q - k_1 - k_2)^2$.

In terms of G_i's we can write down the double differential cross-section using eqs. (1.2), (1.3) and (1.10). We will only need partially integrated cross-sections for later purposes:

$$(1.11) \qquad \frac{d\sigma^{(2)}}{dx_1 dx_2 dz} = \frac{(2\pi)\alpha^2}{3s}\sqrt{x_1^2 - 4\mu^2/s}\sqrt{x_2^2 - 4\mu^2/s}\left\{3Y_1 + Y_2 + Y_3 + zY_4\right\}$$

where $z = \cos(\hat{\underset{\sim}{k}}_1 \cdot \hat{\underset{\sim}{k}}_2)$ and we have defined Y_i's as follows :

$$G_1 = \frac{1}{(2\pi)^3}\left(\frac{4}{s}\right) Y_1 \quad ; \qquad G_i = \frac{1}{(2\pi)^3}\left(\frac{16\mu^2}{s^2}\right)(x_i^2 - 4\mu^2/s)^{-1} Y_i \quad ;$$

(1.12) $$G_4 = \frac{1}{(2\pi)^3}\left(\frac{16\mu^2}{s^2}\right)\frac{1}{\sqrt{x_1^2-4\mu^2/s}\ \sqrt{x_2^2-4\mu^2/s}}\, Y_4 \ . \qquad (i = 2, 3)$$

Y_i's are dimensionless and will turn out to "scale", ie. become independent of s for large s and fixed x_1, x_2, z.

We can also define the "correlation tensors" as :

(1.13) $$H^{(2)}_{\mu\nu}(q; k_1, k_2) = \frac{1}{T} H^{(1)}_{\mu\lambda}(q; k_1)\, H^{(1)}_{\lambda\nu}(q; k_2) + C^{(2)}_{\mu\nu}(q; k_1, k_2) \ ,$$

where $T = \frac{1}{3} g^{\alpha\beta} H^{(o)}_{\alpha\beta}(q)$ is essentially the trace of $n=0$ (total) H-function. With this definition it can be easily checked that

(1.14) $$\frac{d\sigma^{(2)}}{dx_1 dz_1 dx_2 dz_2} = \frac{1}{\sigma^{tot}}\left(\frac{d\sigma^{(1)}}{dx_1 dz_1}\right)\left(\frac{d\sigma^{(1)}}{dx_2 dz_2}\right) + $$

$$+ \left(\frac{d\,\sigma^{(2)}_{corr.}}{dx_1 dz_1 dx_2 dz_2}\right) \ ,$$

where the "correlated" part of the cross-section ($\sigma^{(2)}_{corr}$) is obtained through eqn. (1.2) with $H^{(2)}_{\mu\nu}$ replaced by $C^{(2)}_{\mu\nu}$.

2. - ENERGY-MOMENTUM CONSTRAINTS ON SCALING AND MULTIPLICITIES. -

These are obtained most simply from eq. (1.4) by using the identity of particles on the right hand side of that equation. Thus, we have

$$(q-k_1-k_2\ldots k_n)^{\alpha} H^{(n)}_{\mu\nu}(q;k_1\ldots k_n) = \frac{1}{2^n(2\pi)^{3n}} \sum_{m'} \frac{1}{m'!} k^{\alpha}_{n+1} (2\pi)^4 \times$$

$$\times \delta^4(q-k_1\ldots -k_n-k_{n+1}-P_{m'}) \langle 0|J_{\mu}|k_1\ldots k_n, k_{n+1};m'\rangle \, d\mathcal{P}_{m'} \times \tag{2.1}$$

$$\times \langle k_1\ldots k_n, k_{n+1};m'|J_{\nu}|0\rangle \, d\mathcal{P}_{n+1} = \int \left(\frac{d^3k_{n+1}}{E_{n+1}}\right) k^{\alpha}_{n+1} H^{(n+1)}_{\mu\nu}(q;k_1\ldots k_n, k_{n+1}).$$

For many types of particles, one simply sums over all types on the right hand side of (2.1). Let us explore (2.1) for n = 0, 1 and 2 in some detail.

i) n = 0 :

This relates the total cross-section to the single inclusive cross-section. In terms of the scaling quantities, $D_o(s)$, $\overline{F}_1$ and $\overline{F}_2$, we obtain through (1.6), (1.8) and (2.1) :

$$q^{\alpha}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{s}\right)D_o(s)=\frac{3}{(2\pi)s}\int\left(\frac{d^3k_1}{E_1}\right)k_1^{\alpha}\left\{\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{s}\right)\bar F_1+\right.$$

(2.2)

$$\left.+\frac{2}{x_1 s}\left(k_{1\mu}-\frac{k_1 q}{s}q_\mu\right)\left(k_{1\nu}-\frac{k_1 q}{s}q_\nu\right)\bar F_2\right\}$$

Eq. (2.2) leads to only <u>one</u> non-trivial condition:

(2.3) $$D_o(s)=\frac{3}{4}\int_{2\mu/\sqrt{s}}^{1}dx_1\,x_1\sqrt{x_1^2-4\mu^2/s}\left\{\bar F_1+\frac{1}{6x_1}(x_1^2-4\mu^2/s)\bar F_2\right\}.$$

As shown in ref. (1), (2.3) then leads to scaling of $\bar F_{1,2}$ provided i) $D_o(s)$ scales (i.e. the total cross-section $\sim 1/s$) and ii) $\bar F_i$'s have a uniform asymptotic behaviour (for details see ref. (1)).

The hadronic multiplicity is defined as

(2.4) $$\langle n(s)\rangle=\frac{3}{4D_o(s)}\int_{2\mu/\sqrt{s}}^{1}dx_1\sqrt{x_1^2-4\mu^2/s}\left\{2\bar F_1+\frac{1}{3x_1}\left(x_1^2-\frac{4\mu^2}{s}\right)\bar F_2\right\}$$

It is clear that $\bar F_1\sim x_1^{-2-\alpha}$ and $\bar F_2\sim x_1^{-3-\alpha}$ $(\alpha<1)$ are the highest (power) singularities allowed as $x_1\to 0$, otherwise (2.3) would fail to converge. In this case $\langle n\rangle\sim(\sqrt{s})^{\alpha}$ ($\langle n\rangle\sim\ln s$ for $\alpha=0$) as s approaches infinity. We shall return to the question of multiplicities later.

ii) $n=1$:

Now we obtain a relationship between $\bar F_1, \bar F_2$ and the Y_i's belonging to two-particle production cross-section. There emerge 5 relations:

(2.5a) $$\left(1-\frac{x_1}{2}\right)\left[\bar F_1+\frac{x_1^2-4\mu^2/s}{2x_1}\bar F_2\right]=\frac{1}{2}\int dx_2\,x_2(x_2^2-4\mu^2/s)^{1/2}\int dz\ \times$$

$$\times\left\{Y_1+Y_2+z^2Y_3+zY_4\right\},$$

$$(2.5b)\quad -\sqrt{x_1^2-4\mu^2/s}\left[\bar{F}_1+\frac{x_1^2-4\mu^2/s}{2x_1}\bar{F}_2\right]=\int dx_2(x_2^2-4\mu^2/s)\int dz\, z \quad x$$
$$x\left\{Y_1+Y_2+z^2Y_3+zY_4\right\},$$

$$(2.5c)\quad 2(1-\frac{x_1}{2})\left[3\bar{F}_1+\frac{x_1^2-4\mu^2/s}{2x_1}\bar{F}_2\right]=\int dx_2\, x_2\sqrt{x_2^2-4\mu^2/s}\int dz \quad x$$
$$x\left\{3Y_1+Y_2+Y_3+zY_4\right\},$$

$$(2.5d)\quad -\sqrt{x_1^2-4\mu^2/s}\left[3\bar{F}_1+\frac{x_1^2-4\mu^2/s}{2x_1}\bar{F}_2\right]=\int dx_2(x_2^2-4\mu^2/s)\int dz\, z\ x$$
$$x\left\{3Y_1+Y_2+Y_3+zY_4\right\},$$

$$(2.5e)\quad -\sqrt{x_1^2-4\mu^2/s}\left[\bar{F}_1+\frac{x_1^2-4\mu^2/s}{2x_1}\bar{F}_2\right]=\int dx_2(x_2^2-4\mu^2/s)\int dz \quad x$$
$$x\left\{zY_1+zY_2+zY_3+\frac{1}{2}(1+z^2)Y_4\right\}.$$

Eqs. (2.5) then tell us that the scaling of $\bar{F}_1$ and $\bar{F}_2$ implies that Y_i's scale as well (ie become independent of s for large s and fixed x_1, x_2 and z) under assumptions of uniform asymptotic behaviour-similar to those employed in the earlier sector.

We can go further to put some constraints on the multiplicities. Using eqs. (2.5d) in conjunction with (1.11), we can rewrite eq. (2.4) in the following symmetric form:

$$(2.6)\quad \langle n(s)\rangle=\frac{3s}{(4\pi\alpha^2)D_o(s)}\int\frac{dx_1}{\sqrt{x_1^2-4\mu^2/s}}\ \frac{dx_2}{\sqrt{x_2^2-4\mu^2/s}}\Big[(x_1^2-4\mu^2/s)+$$
$$+(x_2^2-4\mu^2/s)\Big]\left(\frac{-1}{2}\right)\int dz\, z\left(\frac{d\sigma^{(2)}}{dx_1\,dx_2\,dz}\right)$$

Actually in eqn. (2. 6) only the correlated part of the cross-section ($\sigma^{(2)}_{corr}$) is left upon integration over z. Thus, the behaviour of $\langle n(s)\rangle$ is directly tied to the behaviour of $(d\sigma^{(2)}_{corr})/(dx_1 dx_2 dz)$.

For purely hadronic case it is found phenomenologically (as well as in the Regge model) that the normalized correlation functions $\rho^{12}(y_1, y_2)$ behaves as

$$\rho^{12}(y_1, y_2) \sim e^{-\frac{|y_1 - y_2|}{\zeta}} \sim \left(\frac{1}{s_{12}}\right)^{1/\zeta}$$

where the correlation length ζ is approximately two. While this result is an asymptotic one valid for large rapidity differences it seems however to work well also for small rapidity differences[9].

In our case we can catalog the behaviour of $\langle n(s)\rangle$ by assuming for the correlation functions a similar form

$$\left(\frac{d\sigma^{(2)}}{\frac{d^3k_1}{E_1}\,\frac{d^3k_2}{E_2}}\right)_{corr} \sim \frac{1}{\left(x_1 x_2 - z\sqrt{x_1^2 - 4\mu^2/s}\sqrt{x_2^2 - 4\mu^2/s}\right)^{\alpha}}$$

where α is an undetermined constant.

One easily finds that for

$$\begin{array}{lll} \alpha = 2 & \langle n(s)\rangle & \sim \sqrt{s} \\ \alpha = 1 & \langle n(s)\rangle & \sim \ln s \\ \alpha < 1 & \langle n(s)\rangle & \text{finite.} \end{array}$$

Thus if an analogy with strong interactions holds then α is equal to $1/\zeta = 1/2$, in which case the multiplicities will be finite. Conversely to obtain growing $\langle n(s)\rangle$ one has to invoke correlations which behave much more violently than in hadronic interactions.

3.- A SIMPLE CHARGE SUM RULE.-

Relations analogous to (2.1) hold for any additive quantum number, e.g. charge, strangeness, beryon number. We have

$$(3.1)\quad (0-e_1-e_2-\dots-e_n)H^{(n)}_{\mu\nu}(q,k_1,\dots,k_n)=\sum_{e_{n+1}} e_{n+1}\int\left(\frac{d^3k_{n+1}}{E_{n+1}}\right)\times$$
$$\times H^{(n+1)}_{\mu\nu}(q;k_1,\dots,k_{n+1}),$$

where e stands for the generalized "charge". Let us consider in detail eq. (3.1) for $n=0$ and $n=1$.

i) $n=0$:

This is almost trivial, since it simply states that

$$(3.2)\qquad H^{(h)}_{\mu\nu}(q;k)=H^{(\bar{h})}_{\mu\nu}(q;k),$$

where $\bar{h}$ denotes the antiparticle of h.

ii) $n=1$:

This sector leads us to an interesting sum rule on particle production. First, we notice that any two-particle production function, $H^{(h_1,h_2)}_{\mu\nu}(q;k_1,k_2)$, satisfies the relation

$$(3.3)\qquad H^{(h_1,h_2)}_{\mu\nu}(q;k_1,k_2)=H^{(h_1,h_2^{x})}_{\mu\nu}(q;k_1,k_2),$$

where h_1 is "neutral" and h_2^{x} is the "charge conjugate" to h_2.

Examples:

$$H^{(\pi^0,\pi^+)}_{\mu\nu}=H^{(\pi^0,\pi^-)}_{\mu\nu};\qquad H^{(\pi^+,k^+)}_{\mu\nu}=H^{(\pi^+,k^-)}_{\mu\nu};$$

$$H^{(k^+,p)}_{\mu\nu}=H^{(k^+,\bar{p})}_{\mu\nu}=H^{(k^-,p)}_{\mu\nu},\ \text{etc.}.$$

Consider now π^+ production. Eq. (3.1) reads:

$$- H_{\mu\nu}^{(\pi^+)}(q;k) = \sum_{h'} e'_h \int (\frac{d^3k'}{E'}) H_{\mu\nu}^{(\pi^+, h')}(q;k,k'). \qquad (3.4)$$

The sum on the right simplifies considerably, since the contribution from strangeness and baryon number carrying h's cancel in pair upon using eq. (3.3). Thus, (3.4) reduces to:

$$H_{\mu\nu}^{(\pi^+)}(q;k) = \int (\frac{d^3k'}{E'}) \left[H_{\mu\nu}^{(\pi^+\pi^-)}(q;k,k') - H_{\mu\nu}^{(\pi^+\pi^+)}(q;k,k') \right] \qquad (3.5)$$

This is the sum rule, which for the general case reads:

$$H_{\mu\nu}^{(h)}(q;k) = \int (\frac{d^3k'}{E'}) \left[H_{\mu\nu}^{(h\bar{h})}(q;k,k') - H_{\mu\nu}^{(hh)}(q;k,k') \right] \qquad (3.6)$$

provided h is not completely neutral (e.g. π^0 or η^0).

By defining the "correlation functions" as before

$$H_{\mu\nu}^{(hh')}(q;k,k') = \frac{1}{T} H_{\mu\lambda}^{(h)}(q;k) H_{\lambda\nu}^{(h')}(q;k') + C_{\mu\nu}^{(hh')}(q;k,k'), \qquad (3.7)$$

we find that

$$H_{\mu\nu}^{(h)}(q;k) = \int (\frac{d^3k'}{E'}) \left[C_{\mu\nu}^{(h\bar{h})}(q;k,k') - C_{\mu\nu}^{(hh)}(q;k,k') \right] \qquad (3.8)$$

It is amusing to notice that an integrated version of (3.8) implies that

$$N^{(h)} = N^{(h\bar{h})} - 2\, N^{(hh)}, \qquad (3.9)$$

where N's stand for the average number of particles produced per collision. The factor of 2 in the last term arises due to the identity of particles.

4. - ESTIMATES OF $F_\pi(s)$ AND $F_2^{(\pi)}(x)$ FROM FESR AND EVMD. -

We devote this section to making some estimates about the pion form factor $F_\pi(s)$ and the pion structure function $F_2^{(\pi)}(x)$, using various model-dependent inputs, and finite energy sum rules.

We accept the usual arguments[8] which imply, that

(4.1)
$$F_2^{(\pi)}(x) \underset{x\to 1}{\sim} C_\pi (1-x)^2 ,$$

where C_π is a constant which we try to estimate below using Bloom--Gilman type FESR. The latter implies that for large s,

(4.2)
$$\frac{1}{s}\int_{\mu^2}^{M_o^2} dM_x^2\, F_2(x,s) \approx \int_{x_o}^{1} F_2^{scaling}(x)\,dx$$

where $x_o = 1 - (M_o^2 - \mu^2)/s$ and M_o is some fixed mass. Saturating (4.2) with the elastic term alone, we obtain that

(4.3)
$$|F_\pi(s)|^2 \underset{s\ large}{\sim} \frac{C_\pi}{3}\left(\frac{M_o^2}{s}\right)^3 \approx \frac{C_\pi}{3}\left(\frac{m_\varrho^2}{s}\right)^3 ,$$

where we have set $M_o \approx m_\varrho$, which seems quite reasonable as it corresponds to a 3 final state with $\langle m^2_{\pi\pi}\rangle \approx m_\varrho^2$.

Now we obtain the large s behaviour of $F_\pi(s)$ using a scaling model of electromagnetic interactions in which the photon is coupled to a continuum of hadronic states. This model has been shown[7] to be successful in describing the main features of different processes involving photons over a wide range of the mass q^2.

The process under consideration is then visualized as the production of an infinite string of vector mesons, V_n which then decay into $\pi^+\pi^-$. (See Fig. 1). So, we have for the form factor

(4.4) $$F_\pi(s) = \sum_{n=0}^{\infty} \frac{m_n^2}{f_n} \frac{1}{(s-m_n^2)+im_n\Gamma_n} g_{n\pi\pi} = \sum_{n=0}^{N} + \sum_{n=N+1}^{\infty} \equiv \Sigma + \Sigma' .$$

The term having (1/s) behavious coming from Σ is asymptotically cancelled by the continuum if we are to obtain $(1/s)^{3/2}$ behavious as required by our threshold condition (4.1). Furthermore Σ is evaluated by fitting the low energy experimental data on $F_\pi(s)$ (Orsay, Novosibirsk and Frascati), while Σ' is to evaluated by appealing to asymptotic considerations.

In the earlier work(7) to obtain scaling it was shown that (for large n):

(4.5) $$m_n^2 = m_\varrho^2 (1+2n), \quad \frac{m_n}{f_n} = \text{constant} = b \simeq \frac{m_\varrho}{f_\varrho}, \quad \frac{\Gamma_n}{m_n} = \text{constant} = \gamma$$

Under these hypothesis, from $\Sigma' \sim (\frac{1}{s})^{3/2}$ it follows:

(4.6) $$g_{n\pi\pi} \sim g \frac{1}{n^2}$$

and therefore

(4.7) $$\frac{\Gamma_{n\pi\pi}}{\Gamma_n} \sim n^{-4} \sim \left(\frac{m_\varrho^2}{m_n^2}\right)^4 .$$

The Σ' sum can be transformed into an integral form:

(4.8) $$\Sigma' = \frac{2b}{m_\varrho} \left(\frac{m_\varrho^2}{s}\right)^{3/2} g \int_{y_0}^{\infty} \frac{dy}{y^{3/2}\left[1-y(1-i\gamma)\right]}$$

where $y_0 = \frac{m_\varrho^2}{s}(1+2(N+1))$. The integral (4.8) diverges at the lower limit, giving rise to a term $\sim 1/s$. We finally obtain:

(4.9) $$\Sigma'(s) = \frac{2bg}{m_\varrho}\left(\frac{m_\varrho^2}{s}\right)^{3/2}\left\{\frac{2}{\sqrt{y_o}} - \pi(1+\gamma^2)^{1/4}(\sin\varphi + i\cos\varphi)\right\}$$

where $\varphi = \frac{1}{2}\,\text{tang}^{-1}\gamma$ and $\gamma^2 \ll 1$.

The coupling constant g is determined through Σ, which has been evaluated taking into account of ϱ (760), $\varrho'(1250)$[x] and $\varrho''(1600)$. The parameters used were:

i) $F_\pi(s \sim m_\varrho^2)$ is from Orsay fit[10], which includes $\varrho - \omega$ interference. Consistently with the data, $\frac{g_{\varrho\pi\pi}}{f_\varrho} \simeq 1.2$

(4.10) ii) $\varrho'(1250)$ $\Gamma_{\varrho'} \simeq 0.130$ GeV; $\frac{g_{\varrho'\pi\pi}}{f_{\varrho'}} \simeq -0.05$

iii) $\varrho''(1600)$ $\Gamma_{\varrho''} \simeq 0.350$ GeV; $\frac{g_{\varrho''\pi\pi}}{f_{\varrho''}} \simeq -0.1.$

The signs in ii) and iii) have been chosen to agree with the experimental data which lie above the ϱ tail. The fit to the cross sections from colliding beams is shown in Fig. 2. The Frascati data includes kaon production as well, thus making any comparison of our formulae with the higher energy data doubtful. An estimate of kaon production which includes only the contribution of the ϱ, ω and φ tails is also shown in that figure.

Using (4.4) and (4.10) we obtain for large s

(4.11) $$\Sigma(s) \rightarrow \left(\frac{m_\varrho^2}{s}\right)(0.7).$$

(x) - The implications of the existence of a $\varrho'(1250)$ meson are discussed in ref. (7).

Using eqs. (4.9) and (4.11), we find that the (1/s) term of $F_\pi(s)$ cancels, if $\frac{2g}{f_\varrho} \simeq -0.93$. We finally obtain

(4.12) $$|F_\pi(s)| \underset{s\ \mathrm{large}}{\approx} 3 \left(\frac{m_\varrho^2}{s}\right)^{3/2}$$

As a consistency check, we find that the above parametrization gives for F_π at s = 0

(4.13) $$F_\pi(s=0) \simeq \sum_{n=0}^{2} \frac{g_{n\pi\pi}}{f_n} + \frac{g}{f_\varrho \sqrt{2}} \sum_{n=3}^{\infty} \frac{1}{n^{5/2}} \approx 1.05 - 0.05 ,$$

consistent with 1.

Now we go back to relation (4.3) and obtain for C_π

(4.14) $$C_\pi \simeq 27 .$$

It will be noticed that this coefficient (C_π) is an order of magnitude larger than the corresponding coefficient (C_p) for the proton case obtained from SLAC[11] in the space like region

$$F_{2p}(x) \sim C_p (x-1)^3 , \qquad C_p = 1.274 .$$

We may carry this analysis a step further and obtain an estimate of pion multiplicity e.g., if we assume an explicit form for $F_2^\pi(x)$ for all $x (0 < x < 1)$.

We choose the simplest form consistent with its behaviour near $x \to 1$ which guarantees a finite multiplicity. We choose then

(4.15) $$F_2^\pi(x) = C_\pi (1/x - 1)^2$$

Then,

$$\langle n_{\pi^+} \rangle = \frac{1}{\sigma_{tot}} \int \frac{d\sigma}{dx} dx = \frac{\sigma_o}{\sigma_{tot}} \frac{C_\pi}{3} \simeq (0.9 - 1) ,$$

where we have used the experimental indication[12] that $\sigma_{tot} \simeq \simeq (2-2.5)\ \sigma_{\mu\bar{\mu}}$. If we further assume that the number of charged k's is roughly equal to that of charged π's, then we obtain

$$\langle n \rangle^{\text{charged}} \approx 4,$$

which agrees rather well with results[12,13] from Adone and at higher energies at CEA (2E=4 GeV), $\langle n \rangle^{\text{charged}} \approx 4.5 \pm 1$.

Also, the energy-momentum sum rule is approximately satisfied:

$$2 \simeq \frac{1}{\sigma_T} \sum_n \int x dx \left(\frac{d\sigma^h}{dx}\right) \approx 8\ \left(\frac{0.9}{4}\right),$$

where we have assumed i) equal contributions from each member of the pseudoscalar octet and ii) baryon contribution to be much smaller.

Thus, surprisingly anough, the above simple parametrization seems to agree rather well with the data when available and is moreover self-consistent, which makes us believe that it may provide a good first approximation

5. - CONCLUSIONS. -

In this paper, we have investigated e^+e^- annihilation into hadrons using the constraints derivable from the conservation laws. [illegible] We were also able to relate, e.g., single charged pion structure functions to an integral (over momentum) of the difference between $\pi^+\pi^-$ and $\pi^+\pi^+$ production structure functions. This may be of some use in constraining models of multipion production. Lastly, we have estimated semi-phenomenologi cally the form factor and the structure function for the pion using FESR and EVMD. A self-consistent approximation scheme seems to emerge. The surprising result (to us, at least) is that near $x \simeq 1$ the pion structure function may be an order of magnitude (or more) larger than the proton one. This result is certainly strange when viewed as a statement about the ratio of π over p Compton cross-section (for large photon mass). A naive analogy from hadronic total cross-section would yield a ratio 2 : 3 instead. However, if one views (near the "elastic" limit $x \simeq 1$) this ratio as some "zero frequency" limit, then the Thompson ratio $= m_p^2 : m_\pi^2 \approx 50 : 1$, is not unreasonably far from our estimate. Clearly the, this ratio is a crucial number about which experimental data is sorely needed.

ACKNOWLEDGEMENTS. -

One of us (M.G.) is grateful to A. Bramòn and E. Etim for valuable discussions. He is also pleased to acknowledge the kind hospitality of the Theoretical Study Division of CERN, where part of this work was done.

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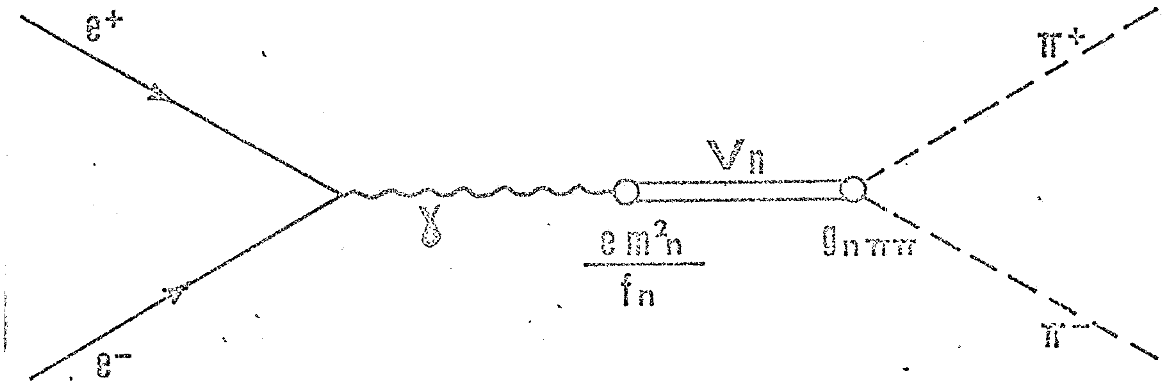

FIG. 1 - Diagram for $e^+e^- \rightarrow V_n \rightarrow \pi^+\pi^-$.

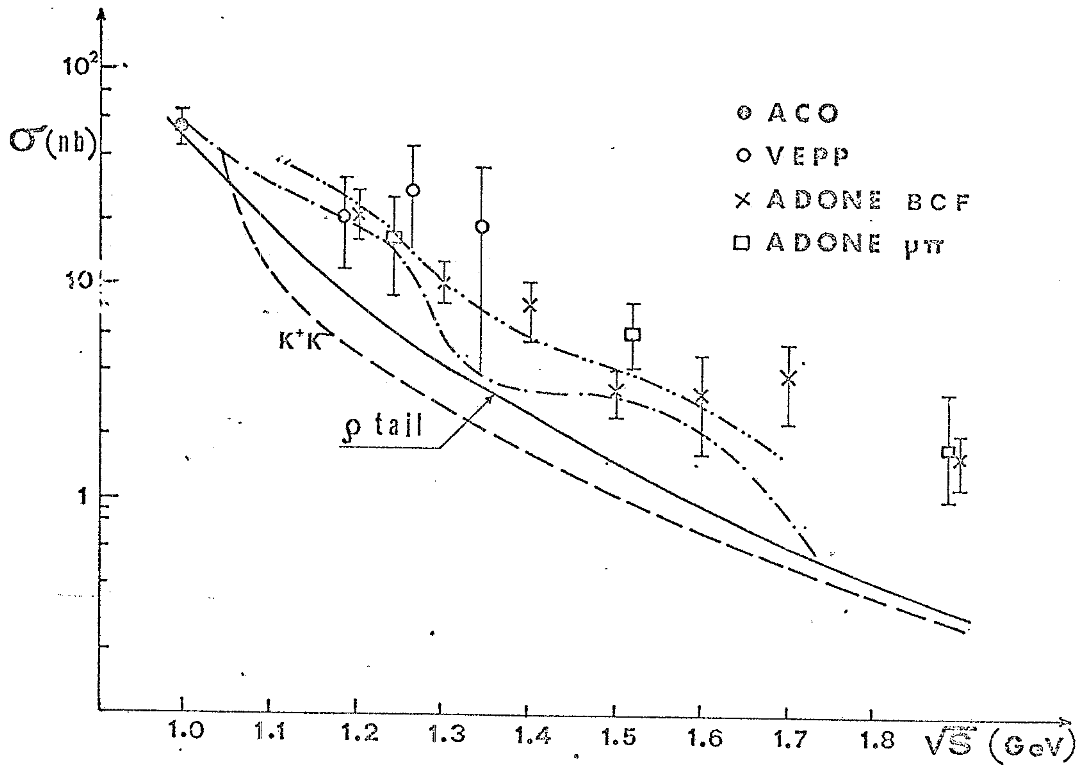

FIG. 2 - Total cross section $\sigma(s)$ for the process $e^+e^- \rightarrow \pi^+\pi^-$ as a function of the center of mass energy $\sqrt{s}$. The experimental data are taken from ref. (11). The different curves correspond to: ——— Extrapolation of the Orsay fit (Gounaris and Sakurai formula) which includes $\varrho - \omega$ interference; ---- ϱ, ω, and φ tails to k^+k^- production; -.-. $\varrho'(1250)$, $\varrho''(1600)$ contributions plus ———; -..-.. the sum of -.-. and ———.