COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF - 73/6
22 Febbraio 1973

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(Paper presented at "The 1973 Particle Accelerator Conference", San Francisco, March 5-7, 1973).

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

BEAM-BEAM LIMITS

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The forces due to beam-beam interaction set the ultimate limit on the performances of a storage ring; I shall try to review briefly the status of the knowledge and the experimental data on the beam-beam limits.

In the following I shall refer mainly to electron storage rings, with bunched beams and gaussian distribution of the charge in three dimensions (x and z transverse s longitudinal); I shall also assume (unless otherwise specified) the crossing of a weak beam I_w with a strong beam I_s, the difference in charge being such that the strong beam is not appreciably affected by the weak beam and can be considered as a fixed charge distribution.

Let us consider first the effect of the beam-beam interaction on the transverse motion; an useful parameter is the linear Q-shift per crossing given by:

$$\xi_{x,z} = \frac{i_s}{\sigma_x \sigma_z} \; \frac{R}{\gamma} \left(\frac{\beta}{H}\right)_{x,z} \qquad (1)$$

where i_s is the current per bunch in units of the Alfvén current $I_o = (ec/r_e) = 17.000$ A;

σ_x, σ_z are the transverse r.m.s. beam dimensions;

R is the mean radius of the ring;

γ is the energy of the particles in rest mass units;

H is a form factor of the beam $H_{x,z} = 1 + \frac{\sigma_{x,z}}{\sigma_{z,x}}$;

$\beta_{x,z}$ is the amplitude function at the crossing.

It has to be noted that the Q-shift depends on the amplitude of oscillation of the particle: ξ is the maximum value (for amplitude $\rightarrow 0$), and it is positive in the case of e^+e^- collision, while for large amplitudes the Q-shift tends to 0; ξ is therefore also a measure of the Q spread per crossing introduced by the beam-beam interaction.

Various types of numerical computations have shown that the parameter ξ determines the beam-beam limit, and its maximum allowed value is between 0.05 and 0.10, provided that particularly bad operating points be avoided; the total Q-shift per revolution is expected to be limited to values certainly not higher than the distance to the closest half-integer or integer.

More recent computations on the stochasticity limit due to the localized non-linear forces of the beam-beam interaction have shown that the criterion of the resonance area equal to the area of the Q_x, Q_z diagram is fulfilled when $\xi \simeq (0.09 \div 0.05)$, depending on the beam shape, the larger values being for flat beams(1).

As the luminosity that can be achieved depends on the trasverse current density, eq. (1) shows that, for a given maximum value of ξ, the luminosity can be increased if the amplitude function is made small: "low β" insertions(2) with β as low as 5 or 10 cm have been designed and operated.

I want to remark here that the beam-beam forces are proportional to the transverse current density, not to ξ; it is only the effect on the transverse motion that depends on ξ. This means that if the beam-beam force has a longitudinal component, the use of the "low β", allowing much bigger forces before running into troubles with the transverse motion, may cause the longitudinal beam-beam instability to become the dominant limit.

The experimental data on the maximum values of ξ achieved on existing rings are listed in table I; they refer to the crossing of two beams of very similar current, and, therefore, should not be directly comparable with the computations on the stochasticity limits. I may add that tests done on ADONE show that the beam-beam limit at high energy (E > 1 GeV) seems to be about the same for the two cases of weak-strong and strong-strong beams. Another remark on the ξ measurement: it is an indirect method, which

TABLE I

	n° of crossings m	ampl. funct. at the crossing $\beta_{x,z}$(m)	Max. Energy E (GeV)	Max. Q-shift per Xing ξ_m
Stanf.-Princeton	1	~ 2, 1	0.55	$\sim 10^{-2}$
VEPP-II	2	~ 2	0.7	$\sim (0.5 \div 1) \times 10^{-2}$
ACO	2	2, 0; 4, 0	0.52	4×10^{-2}
ADONE	6	8, 9; 3.2	1.5	8×10^{-2}
CEA-Bypass(a)	1	0.046; 0.28	2.0	$\sim (2.5 \div 5) \times 10^{-3}$
SPEAR	2	1, 0; 0.37	2.6	8×10^{-2}
ISR-CERN (pp)(b)	8	22, 6; 14, 9	28	$\sim (5 \div 8) \times 10^{-4}$

(a) - In the CEA-bypass the beams interact at a distance over a large fraction of the ring; the Q-shift per crossing takes into account therefore only a part of the beam-beam force.

(b) - The operation of the ISR-CERN is current limited, therefore the Q-shift per crossing is not at the beam-beam limit.

requires the measurement of the luminosity L and of the current of the weaker beam I_w, together with the knowledge of the imperturbed amplitude function at the crossing and of the beam aspect ratio; from the luminosity formula and eq. (1) one obtains:

$$(2) \qquad \xi_{x,z} = \frac{L}{I_w} \frac{2r_e^2}{\gamma c} I_o \left(\frac{\beta}{H}\right)_{x,z}$$

Eq. (2) is valid under the assumption of gaussian distribution, but the result does not critically depends on the assumption; it has the advantage that it does not require the measurement of quantities that are very difficult to determine accurately (like the beam cross section).

The results of table I cover a quite large range of energies and operating conditions. While the operating point of ACO (slightly below an integer resonance) does not allow values of ξ higher than 0.04 with two crossings per revolution, the behaviour of ADONE and SPEAR seems to be in agreement with the computations on the stochasticity limit. The important fact is that the SPEAR operation has proved that the assumption of a limit on ξ is valid also when the amplitude function is very small (as long as it is larger than the bunch length).

The energy dependence of the maximum value of ξ observed on ADONE[3] in the two strong beams operation ($\xi_m \propto \gamma^{1.5}$ for energies lower than about 1 GeV) is not as yet understood; measurements in the weak-strong case indicate that a similar dependence of ξ is observed below much lower energies ($0.5 \div 0.6$ GeV). The dynamics of a non linear system close to the stochasticity limit is not yet well known; models have been proposed where a relatively slow diffusion process depending on ξ, in competition with the radiation damping, gives an energy dependence of the maximum Q-shift[4]; assuming that the diffusion time constant is proportional to $m^{-1}\xi^{-2}$ (where m is the number of crossings per revolution), this interpretation is not in contrast with other experimental observations, as the threshold energy, below which the $\xi \propto \gamma^{1.5}$ regime should start, turns out to be 250 MeV for ACO (m=2) and 940 MeV for SPEAR (m=2), assuming 1 GeV for ADONE (m=6).

The extrapolation of these results to the case of proton beams (either proton-proton or proton-electron) is not obvious, as the lack of a damping mechanism enlarges the time scale of the dangerous phenomena by many orders of magnitude. It is probable that the beam-beam limit is set by slow diffusion processes, due to the non-linearity of the force, at values of ξ which might be an order of magnitude smaller than the stochastic limit[5].

Discussing the transverse beam-beam limit, it must be remembered that the beam-beam interaction can limit the current also when the beams are separated, as it usually happens at injection when one wants to store currents higher than those allowed by the beam-beam limit at the injection energy. In this case one has to take into account that the injected beam can be much longer than the regime dimension; in general it turns out that a "low β" in the direction of the beam separation must be avoided; the beam-beam separation required will depend also on the transverse emittance of the injected beam, besides its energy and maximum stored current.

It has been already mentioned that the beam-beam interaction can affect also the longitudinal motion; three different cases have been so far considered, namely:

a) when the beams cross at an angle the beam-beam force has a longitudinal component[6];

b) when the beams cross in a region where the off-energy function ψ is not zero the transverse beam-beam force changes the relationship between energy of the particle and its orbit length, affecting therefore the synchrotron motion[7];

c) the variation of the beam cross section at the interaction region, very pronounced when the β is low, gives rise to a longitudinal force[8].

These three effects change the synchrotron frequency (lowering it in the e^+e^- collisions); the first and the third ones give a limit on the beam current per bunch, while the second one gives a limit on the transverse density.

With certain assumptions (essentially that the beam dimensions are the ones given by radiation fluctuations) the relative change of the synchrotron frequency squared Δ, defined as:

$$(3) \qquad \Delta = \frac{Q_o^2 - Q_s^2}{Q_o^2}$$

where Q_o and Q_s are the unperturbed and the actual synchrotron frequencies, are given by:

$$\text{a)} \quad \Delta_a = \frac{2m}{\alpha} \frac{i_s}{\gamma \sigma_p^2} \frac{1}{H_z}$$

$$(4) \qquad \text{b)} \quad \Delta_b = \frac{2m}{\alpha} \frac{i_s}{\gamma \sigma_p^2} \frac{1}{H_z} \left\{ 1 + \frac{\sigma_{x\beta}^2}{\sigma_{xs}^2} \right\}^{-1}$$

$$\text{c)} \quad \Delta_c = \frac{2m}{\alpha} \frac{i_s}{\gamma \sigma_p^2} \frac{\sigma_s^2}{\beta_x \beta_z}$$

where: σ_p is the r.m.s. energy spread in the beam;
$\sigma_{x\beta}$, σ_{xs} are the r.m.s. betatron and synchrotron widths;
σ_s is the r.m.s. beam length;
α is the momentum compaction
m is the number of crossing per revolution.

Eq. (4a) and (4b) refer to a crossing angle $\delta_x \neq 0$ and to an off-energy function $\psi_x \neq 0$, both in the x direction. Which is the maximum allowed value for Δ is an open question; at $\Delta = 0.5$ one should get coherent instabilities[9], and, certainly, at $\Delta = 1$ incoherent instability (the synchrotron frequency becomes zero); on ADONE Δ_b is about 0.4 in normal operating conditions, without indications of longitudinal instability, while some preliminary results on low β tests, with higher values of Δ_b, might be interpreted as due to a longitudinal limitation.

Eqs. (1) and (4) show that the energy dependence of the maximum current per bunch is the same for the two beam-beam limits, transverse and longitudinal, in the assumption that they are defined by a value of ξ_m and Δ_m independent from energy and that each beam transverse dimension is proportional to γ; the maximum current per bunch is proportional to γ^3 and the luminosity turns out to be proportional to γ^4.

A last remark on the space charge compensation method proposed by the Orsay group(10); in the crossing of four beams (two positive and two negative) of equal current and charge distribution there are no beam-beam forces. The critical point of the method is the stability of the system of four beams, besides the practical question of their equality; at very high transverse density (corresponding to $\xi > 1$ for a single beam) the system is unstable for coherent motion(11), but, as far as I know, there are no quantitative evaluations of the transverse density at the stability limit.

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