Laboratori Nazionali di Frascati

LNF-72/120

G. Parisi : BJORKEN SCALING AND THE PARTON MODEL.

Estratto da : Phys. Letters 42B, 114 (1972)

BJORKEN SCALING AND THE PARTON MODEL

G. PARISI*

Laboratori Nazionali del C.N.E.N., Frascati, Italy

Received 17 July 1972

In the framework of renormalized quantum field theory we prove under a technical assumption that Bjorken scaling in deep inelastic electron proton scattering implies the validity of the parton model.

The scaling properties of deep inelastic electron proton scattering [1, 2] may be explained supposing the existence of real point-like constituents of the hadrons which have been named partons by Feynman [3]. This hypothesis has however far reaching consequences on the structure of the final states, on total cross section for e^+e^- into hadrons [4–6] etc.

Many authors [7, 8] have proposed alternative model based on the structure of the light cone singularities of the Wilson expansion of the product of two currents [9]. This light cone approach is less predictive than the parton hypothesis: all detailed predictions are lost.

In this letter we prove that in the framework of standard renormalizable quantum field theory the validity of the parton model follows from the onset of Bjorken scaling [2] in deep inelastic electron proton scattering.

The argument is the following: all the results of the parton model may be deduced from the assumption that at very high energy or at light like distances the fundamental fields behave as free fields and that they do not interfere among themselves in their temporal evolution [1, 8]. Of course this may happen also in a non-free theory.

In all other models the product of two currents near the light cone has a singularity of the same strength as in free theory, but the fundamental fields do not behave as free at light like distances.

The inconsistency of this second possibility may be proved using a theorem recently obtained by Ferrara et al. [10]. They have shown that Bjorken scaling implies the presence of an infinite number of operators with increasing spin which are conserved in the light like region.

In this letter we prove that the presence of this infinite number of conservation laws implies that the theory is free on the light cone and therfore the parton model is true.

Before entering into the details of the proof we sketch some consequences of our results: the validity of Bjorken scaling implies that the operatorial form of the light cone singularities is the same as in free field theory. All the detailed predictions of the parton model, following only from the structure of the light cone singularities, must be satisfied: the total cross section for e^+e^- annihilation into hadrons must be proportional to the squared charges of the partons [5]; both the Adler result for axial current anomaly and the Bjorken sum rule for deep inelastic electron-proton scattering on polarized targets must be satisfied [11] etc.

All these sum rules follow from the assumption of scaling in deep inelastic electron proton scattering. If only one of these sum rules is experimentally false, Bjorken scaling cannot be asymptotically true.

We stress than this last possibility is not in contradiction with present experimental data: if there is a breakdown of Bjorken scaling only at very high energies, the derivation of the parton model sum rules is no more valid.

The proof is based on the following assumptions:

a) We are in the framework of standard Lagrangian quantum field theory: the interaction Lagrangian is a polynomial in the field and is renormalizable.

b) The Callan Symanzik [12] function $\beta(g)$ has a zero at $g_c \neq 0$.

c) The Wilson expansion of two operators on the light cone exists and the strength of the singularity is

* This research has been partially sponsored by C.N.R. G.N.S.M.

fixed by the dimension of the operator involved; the coefficient of the leading term is mass independent.

d) The theory is conformal invariant at short distances.

Assumption b) is true in any theory which is asymptotically scale invariant at short distances [12]; if the non trivial zero of function $\beta(g)$ does not exist, any hope of understanding Bjorken scaling in renormalizable quantum field theory would vanish: the possibility that all the logarithms of the perturbation theory sum up to a simple power which may be very small or zero, is strictly connected to the existence of the zero of $\beta(g)$.

Assumption c) has been proved in perturbation theory by Zimmerman [13] and the conformal invariance of the correlation functions at short distances follows from the other hypothesis in any theory whose kinetical energy is conformal invariant [14, 15].

We now study the correlation functions of the fields in the zero mass limit. The physical relevance of this limit is deeply connected to the fact that the operatorial form of the light cone singularities does not depend on the mass and it is the same both in massive and zero mass theories [13].

The existence of the zero of function $\beta(g)$ implies that the zero mass theory renormalized a la Gell-Mann and Low does exist and it is exactly scale and conformal invariant.

We now prove that if deep inelastic e-p scattering scales a la Bjorken in the massive world, the corresponding zero mass theory is free and therefore the parton model is true.

From the F.G.G.P. theorem [10] it follows that there exists an infinite number of operators of increasing dimension of twist equal to two, which are exactly conserved in the zero mass theory. Unfortunately O'Raifertaight's theorem [17] and its generalizations cannot be applied in absence of a mass gap. Nevertheless one can still prove that the existence of this infinite number of conserved quantities implies free field theory.

We consider one of the conserved twist two operators which behaves as a conformal scalar [18, 19]: $[K_\lambda, O_{\mu_1,,\mu_N}(0)]$. We remember that K_λ is the generator of the special conformal transformations and $O_{\mu_1,,\mu_N}(0)$ is a symmetric traceless tensor irreducible under the Lorentz group.

For the sake of simplicity let us restrict ourselves to a $g(\phi^+\phi)^2$ theory. The equal time commutation relations of the operator $O_{\mu_1,,\mu_N}$ with the field ϕ are fixed from scale invariance:

$$[O_{0\mu_2,,\mu_N}(x), \phi(y)]\,\delta(x_0-y_0) = c_N\delta^4(x-y)[\partial_{\mu_2}\ldots\partial_{\mu_N}\phi(y) - \text{traces}] + \text{Schwinger terms.} \quad (1)$$

We stress that eq. (1) may be not valid in a two dimensional world where twist zero operators are present.

The constants c_N cannot be zero, otherwise also the three point function

$$\langle 0|O_{\mu_1,,\mu_N}(x)\phi^+(y)\phi(z)|0\rangle$$

vanishes identically [20, 21].

If there is no spontaneous breaking, the charge

$$Q_{\mu_2,,\mu_N}(t) = \int d^4x\,\delta(x_0-t)O_{0,\mu_1,,\mu_N}(x) \quad (2)$$

is time independent and a generator of an exact symmetry.

Although we are in a zero mass theory we are still able to prove that the Goldstone phenomenum cannot occur. We use a generalization of the Goldstone theorem due to Symanzik [22]: he has shown that, if a symmetry is spontaneously broken, the Wightman functions of the generating current satisfy the following clustering property

$$\lim_{\lambda\to\infty} \lambda^3\langle 0|O_{\mu_1,,\mu_N}(y+\lambda a)\phi(x^1)\ldots\phi^+(x^k)|0\rangle \neq 0 \quad (3)$$

where a is a spacelike vector.

This is impossible in a conformal invariant theory: from the general form of the conformal covariant N point Wightman function [19, 23], it follows that for generical points the right hand side of eq. (3) is always of order λ^{-2D+3} or less (D is the dimension of the operator $O_{\mu_1,,\mu_N}$).

We have thus proved that the operator $O_{\mu_1,,\mu_N}$ generates a "good" symmetry; and therefore the following equation must be valid:

$$O = \langle 0|[Q_{\mu_2,,\mu_N}\phi(x^1)\ldots\phi^+(x^k)]|0\rangle = C_N\left[\frac{\partial}{\partial x^1_{\mu_2}}\cdots\frac{\partial}{\partial x^1_{\mu_N}} + \cdots + (-1)^N\cdot\frac{\partial}{\partial x^k_{\mu_2}}\cdots\frac{\partial}{\partial x^k_{\mu_N}} + \cdots - \text{traces}\right]\langle 0|\phi(x^1)\ldots\phi^+(x^k)|0\rangle. \quad (4)$$

The Wightman functions must satisfy this infinite set of independent differential equations and one can see by explicit computaion that all the connected Wightman functions must be zero; the two point function being the only exception. The theory is therefore free.

We have thus proved that the existence of canonical light-cone singularities in the product of two currents in the massive case, implies that the relative zero mass theory is free; from the validity of the Bjorken scaling it follows that the operatorial form of light cone singularities is the same as in the free theory: the parton model is therefore the only possible realization of canonical light cone singularities in standard renormalizable quantum field theory.

The author is grateful to Professor K. Symanzik for useful suggestions, he also acknowledges interesting conversation with Professor G. Altarelli, R.A. Brandt, S. Ferrara, A.F. Grillo, G. Mack, D. Zwanziger.

References

[1] E.D. Bloom et al., Phys. Rev. Lett. 23 (1969) 930.
[2] J.D. Bjorken, Phys. Rev. 179 (1969) 1547.
[3] R.P. Feynmann, unpublished.
[4] S.D. Drell et al., Phys. Rev. Lett. 22 (1969) 744; Phys. Rev. 187 (1969) 2159.
[5] N. Cabibbo, G. Parisi, M. Testa, Lett. Nuovo Cim. 4 (1970) 35.
[6] N. Cabibbo et al., Lett. Nuovo Cim. 4 (1970) 569.
[7] R.A. Brandt and G. Preparata, Nucl. Phys. B27 (1971) 541.
[8] H. Fritzsch and M. Gell-Mann, Proc. 1971 Coral Gables Conf.
[9] K. Wilson, Phys. Rev. 179 (1969) 1499.
[10] S. Ferrara, R. Gatto, A.F. Grillo and G. Parisi, Phys. Lett. 38B (1972) 333.
[11] Crewter, Phys. Rev. Lett. 28 (1970) 1041 and references.
[12] C.G. Callan Jr., Phys. Rev. D2 (1970) 1541; K. Symanzik, Comm. Math. Phys. 16 (1970) 48 and 23 (1971) 49.
[13] W. Zimmerman. Lectures given at Bradais Summer School (1970).
[14] B. Schroer, Lett. Nuovo Cim. 2 (1971) 867.
[15] G. Parisi, Phys. Lett. 39B (1972) 643.
[16] M. Gell-Mann, F.E. Low, Phys. Rev. 95 (1954) 1300.
[17] L. O'Raifeartaigh, Phys. Rev. 139B (1965) 1052.
[18] G. Mack, A. Salam, Ann. Phys. 53 (1969) 174.
[19] S. Ferrara, R. Gatto, A.F. Grillo, Frascati Report LNF-71/79 to appear on Springer Tracts.
[20] A.A. Migdal, Phys. Letters 37B (1971) 98.
[21] S. Ferrara, R. Gatto, A.F. Grillo and G. Parisi, Lett. Nuovo Cim., to be published.
[22] K. Symanzik, Comm. Math. Phys. 6 (1967) 228
[23] A.M. Poliakov, JETP Lett. 12 (1970) 381.