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A VECTOR MESON DOMINANCE APPROACH TO SCALE INVARIANCE

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A model of scale invariance for both deep inelastic scattering and e^+e^- annihilation incorporating an infinite number of vector mesons is proposed. Structure functions are calculated explicitly in good agreement with experiments. Precocious scaling arises naturally in the model.

The scaling of inelastic structure functions observed at SLAC [1] can be understood in the framework of parton models [2] or through light cone dominance of current commutators [3]. Such approaches however cannot be easily extrapolated to low (eventually zero) momentum transfers q^2. In this region on the other hand the vector meson dominance model is qualitatively able to accomodate the data. It has recently been suggested [4] that the VMD predictions for the total photon absorption and Compton scattering cross section on nucleons can be improved if higher mass vector mesons (actually infinite) are included in the scheme. Similar improvements are also found in radiative meson decays [5]. If an extended vector meson dominance (EVMD) model can also account for the scaling properties of inelastic structure functions, one would have a simple theory covering a wide range of momentum transfers. Extensions along these lines have recently been attempted [6].

In this letter we show that EVMD aided by some simple assumptions satisfactory describes the observed scaling of the inelastic structure functions.

In a model with infinite vector mesons coupled to the photon the total e^+e^- annihilation cross section into hadrons is given by

$$\sigma_{e\bar{e}\to h}(s) = \frac{16\pi^2\alpha^2}{s^{\frac{3}{2}}} \sum_n \frac{m_n^3}{f_n^2} \frac{m_n\Gamma_n}{(s-m_n^2)^2 + m_n^2\Gamma_n^2}\,, \tag{1}$$

where $\sqrt{s}$ is the total c.m. energy and the coupling of the photon to the nth vector meson is em_n^2/f_n. If asymptotically $\sigma_{e\bar{e}\to h}(s)$ scales as $1/s$ then

$$\frac{1}{\sqrt{s}} \sum_{n=\bar{n}}^{\infty} \frac{m_n^3}{f_n^2} \frac{m_n\Gamma_n}{(s-m_n^2)^2 + m_n^2\Gamma_n^2} = \text{const.} \times \theta(s-m_{\bar{n}}^2) \tag{2}$$

implying for large n

$$m_n^2/f_n^2 \to \text{const.}\,. \tag{3}$$

The interpretation [4] of the Frascati data [7] in terms of a broad ρ' at a mass of about 1.6 GeV implies $(f_\rho/f_{\rho'})^2 \simeq \frac{1}{4}$, suggesting that eq. (3) is already satisfied. We therefore assume for simplicity that eq. (3) is true for all n. An immediate consequence of this is then the absolute prediction of the asymptotic annihilation cross section. From eq. (1) and making use of the mass formala $m_n^2 = m_0^2(1+4n)$ of Veneziano-type for even daughters one finds, with $f_\rho^2/4\pi \simeq 2.54$ [8]:

$$\sigma_{e\bar{e}\to h}(s) \xrightarrow[s\to\infty]{} \frac{\pi}{f_\rho^2/4\pi}\,\sigma_{\mu\bar{\mu}}(s) \simeq 1.25\,\sigma_{\mu\bar{\mu}}(s)\,. \tag{4}$$

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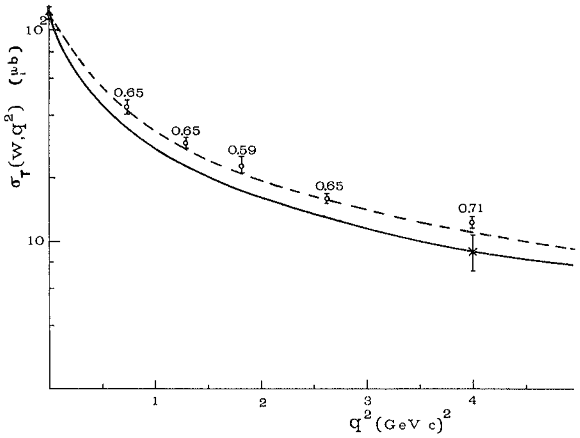

Fig. 1. Total transverse cross section $\sigma_T(\nu, q^2)$ for fixed W. $W = 4$ GeV; —— σ_T; - - - $1.2\ \sigma_T$; σ_T; $\sigma_T + \epsilon\sigma_S$ (ϵ as indicated).

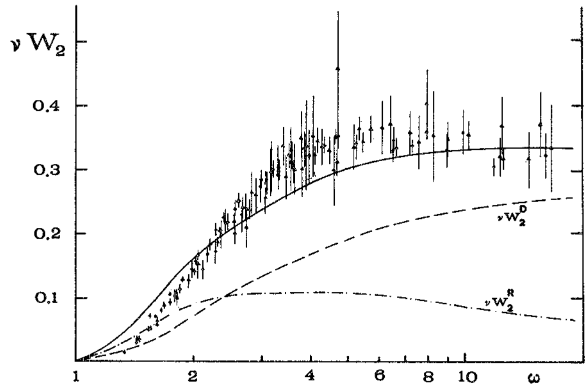

Fig. 2. Diffractive (D) and non-diffractive (R) contributions to the transverse part of $\nu W_2(\omega)$. See the main text for details.

At the present Frascati energies, $2.0 \leqslant \sqrt{s} \leqslant 2.5$ GeV, eq. (4) gives $26 \geqslant \sigma_{e\bar{e}\to h} \geqslant 17$ nb to be compared with an experimental average [7] of about 25 nb.

In going to electroproduction, one finds for the transverse virtual photon cross section $\sigma_T(\nu, q^2)$:

$$\sigma_T^D(\nu, q^2) = \frac{4\pi\alpha}{f_\rho^2}\left(1 - \frac{1}{\omega}\right)\sigma_\rho^D(\nu)\sum_{n=0}^{\infty}\frac{1+4n}{(1+q^2/m^2+4n)^2}\frac{\sigma_n^D(\nu)}{\sigma_\rho^D(\nu)} + \text{isoscalars}\,, \tag{5a}$$

$$\sigma_T^R(\nu, q^2) = \frac{4\pi\alpha}{f_\rho^2}\left(1 - \frac{1}{\omega}\right)^{\frac{1}{2}}\sigma_\rho^R(\nu)\sum_{n=0}^{\infty}\frac{1+4n}{(1+q^2/m^2+4n)^2}\frac{\sigma_n^R(\nu)}{\sigma_\rho^R(\nu)} + \text{isoscalars}\,, \tag{5b}$$

corresponding to a separation of the vector meson nucleon total cross section $\sigma_n(\nu)$ into a diffractive (D) and non-diffractive (R) parts respectively. The Bjorken variable ω is defined as $\omega = 2M\nu/q^2$ ($q^2 \geqslant 0$). In eqs. (5a, b) terms coming from the interference of different vector mesons have been neglected*.

To proceed further from eqs. (5a, b) one needs the purely strong interaction cross sections $\sigma_n(\nu)$. It is clear that a constant total cross section equal for all n will make eqs. (5a, b) diverge. Finite and scaling structure functions result if for each n

$$\sigma_n^D(\nu)/\sigma_\rho^D(\nu) = (1+cn)^{-1}\,, \tag{6}$$

where c is determined at $q^2 = 0$ from the diffractive contribution to the total γp cross section. Eq. (6) is consistent with naive dimensional counting in which the vector meson mass ($m_n^2 \sim n$) sets the scale. Similar conclusions have also been drawn by Rittenberg and Rubinstein [11].

Using the parametrizations

$$\sigma_{\gamma p}(\nu) = 98.7 + 64.9/\sqrt{\nu}\ (\mu\text{b})\,,\ \text{see ref. [9]}\,, \tag{7a}$$

$$\sigma_{\rho p}(\nu) = 23 + 11.2/\sqrt{\nu}\ (\text{mb})\,,‡ \tag{7b}$$

* This is consistent with the results of ρ and ω photo- and electroproduction on protons [9, 10].

‡ Eq. (7b) follows from the use of the quark model or from ρ photoproduction data. See for instance ref. [9].

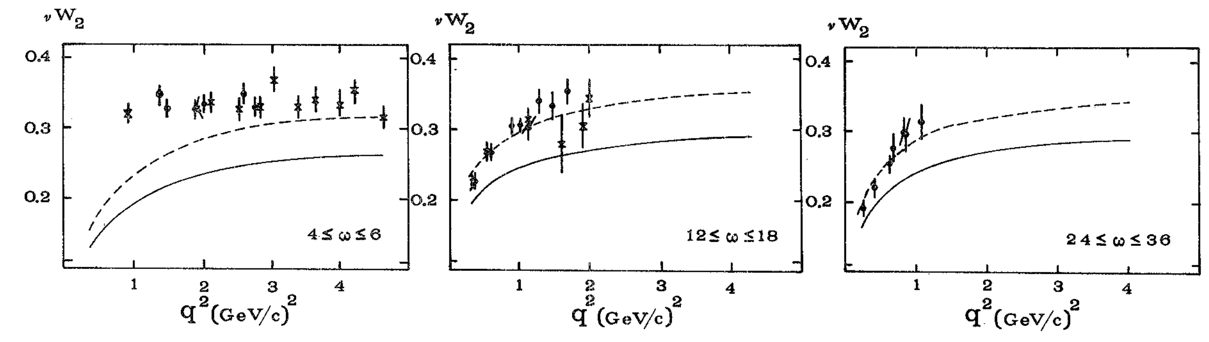

Fig. 3. $\nu W_2(q^2)$ for various ranges of ω. Theoretical predictions (full and dashed lines as in fig. 1) correspond to the values a) $\omega = 5$, b) $\omega = 15$ and c) $\omega = 30$.

one finds $c \simeq 1.4$. For the non-diffractive part of the cross sections $\sigma_n^{\mathrm{R}}(\nu)$ we have on the other hand and consistent with Regge theory:

$$\sigma_{np}(\nu) = \sigma_n^{\mathrm{D}}(1+b\sqrt{m_n/2M\nu}) \,, \tag{8}$$

where for simplicity b is chosen to be the same for all n. The parameter b is given from $\sigma_\rho^{\mathrm{R}}(\nu)$ [eq. (7b)]; the consistency of our choice is easily checked by calculating $\sigma_{\mathrm{T}}^{\mathrm{R}}(\nu, q^2=0)$ from eq. (5b) and comparing with (7a).

Making use of the above results in eqs. (5a,b) and summing the series one finds that $\sigma_{\mathrm{T}}^{\mathrm{D}}(\nu, q^2)$ and $\sigma_{\mathrm{T}}^{\mathrm{R}}(\nu, q^2)$ behave asymptotically as

$$\sigma_{\mathrm{T}}^{\mathrm{D}}(\nu, q^2) \underset{q^2\to\infty}{\sim} \frac{4\pi\alpha}{f_\rho^2}\left(1-\frac{1}{\omega}\right)\sigma_\rho^{\mathrm{D}}\frac{1}{c}\,\frac{m_\rho^2}{m_\rho^2+q^2} + \text{isoscalars} \,, \tag{9a}$$

$$\sigma_{\mathrm{T}}^{\mathrm{R}}(\nu, q^2) \underset{q^2\to\infty}{\sim} \frac{4\pi\alpha}{f_\rho^2}\left(1-\frac{1}{\omega}\right)^{\frac{1}{2}}\sigma_\rho^{\mathrm{R}}(\nu)\sqrt{\frac{m_\rho^2}{q^2}}\,k(c) + \text{isoscalars} \,, \tag{9b}$$

where $k(c)$ is equal to 1.1 for $c = 1.4$. In fig. 1 we plot the total transverse cross section $\sigma_{\mathrm{T}}(\nu, q^2)$ (full curve) for fixed missing mass squared $W^2 = 2M\nu - q^2 + M^2 = 16\ \mathrm{GeV}^2$. Since the experimental data also include the longitudinal contribution $\sigma_{\mathrm{L}}(\nu, q^2)$ we have taken this into account by an overall factor of about 1.2 [1] – this gives rise to the dashed curve.

From eqs. (9a, b) the transverse part of the structure functions scales as

$$\nu W_2^{\mathrm{D}}(\omega) \to \text{const} \times (1-1/\omega)^2 \,, \tag{10a}$$

$$\nu W_2^{\mathrm{R}}(\omega) \to \text{const} \times (1-1/\omega)^{\frac{3}{2}}\,\omega^{-\frac{1}{2}} \tag{10b}$$

in the Bjorken limit*. In fig. 2 we plot $\nu W_2^{\mathrm{R,D}}$ obtained in this limit and compare their sum with the SLAC–MIT data. From this figure one sees that for $\omega \to 1$ the agreement of this model with data is only qualitative, a clear indication that the extrapolation of the Regge-like behaviour of $\nu W_2(\omega)$ from the large to small values of $\omega(\omega \lesssim 2)$ is inadequate. For an improved threshold behaviour one would have to consider in greater detail resonant hadron cross sections. In fig. 3 we plot νW_2 against q^2 for different values of ω. The precocious onset of

* The isoscalar contributions are included in the constants and have been estimated from vector meson photoproduction data [9].

scaling evident from these plots is related to the fact that in eqs. (9) m_ρ^2 fixes the scale of the momentum transfer.

Analogous considerations hold for the neutron structure functions νW_{2n}:

$$\nu W_{2n}^{D} = \nu W_{2p}^{D}\,, \tag{11a}$$

$$\nu W_{2n}^{R} = \frac{5.2}{11.2}\nu W_{2p}^{R}\,, \tag{11b}$$

where the ratio $\frac{5.2}{11.2}$ comes from the experimental knowledge [9] of $\sigma_{\gamma n}(\nu)$ and the separation of the $I = 0$ and 1 contributions to the cross sections. From eqs. (11) one obtains a prediction for the ratio $\nu W_{2n}/\nu W_{2p}$ which at threshold gives 0.46 consistent with the present experimental indication [12].

The overall picture that emerges from the above results is one of a satisfactory model capable of describing the main features of high energy processes involving real, timelike and spacelike photons. From the requirement of scaling in e^+e^- annihulation and using dimensional arguments to fix the vector meson-nucleon total cross sections the structure functions can be calculated in closed form. The agreement with the data is generally good apart from some discrepancy near threshold. The longitudinal cross section σ_L has not been considered in all of the above because in any VMD model it turns out to graw logarithmically in q^2 if extrapolated linearly in q^2 from $q^2 = 0$. For this reason we have simply taken it into account by using the experimental ratio $R = \sigma_L/\sigma_T \simeq 0.2$ [1].

Finally we would like to compare some predictions of this model with those of the parton model. In e^+e^- annihilations into hadrons we would predict the absence of jets which in the parton model arises from the existence of a transverse momentum cut-off. No such cut-off exists in the present model since the hadrons are produced through higher and higher mass vector mesons. In electroproduction on the other hand the well-known $p_\perp$ cut-off operates as in the parton model.

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