Laboratori Nazionali di Frascati

B. Bartoli, F. Felicetti and V. Silvestrini: ELECTROMAGNETIC
STRUCTURE OF THE HADRONS

Electromagnetic Structure of the Hadrons.

B. BARTOLI, F. FELICETTI and V. SILVESTRINI

Laboratori Nazionali del CNEN - Frascati

Introduction.

Nature has provided us with good probes of the electromagnetic structure (E.M.S.) of stable particles: as we know, these probes are the charged leptons (μ, e).

By means of a high-energy lepton l the structure of a target particle T, considered as a whole, can be studied by performing elastic-scattering experiments of l on T. This line of investigation has been pursued during the last 15 years, and has provided a large quantity of information about the E.M.S. of the proton, the neutron and the pion.

Complementary information can be obtained through the production of a $T\overline{T}$ pair by lepton-antilepton annihilation. Experiments in this category have become possible only in the last few years through the operation of e^+e^- storage rings at Orsay, Novosibirsk and Frascati.

As a consequence of some rather general hypotheses, the amplitudes for both the elastic-scattering and the pair production processes can be written in terms of a small number of form factors, which summarize the information on the E.M.S. of the target hadron T.

The form factors are analytic functions of one single variable q^2, the square of the four-momentum transferred from the projectile lepton to the target hadron. The form factors are explored for essentially negative values of their argument in the scattering reactions, and for positive values in the pair production experiments (*).

If T were really elementary, the above categories of experiments would provide all the possible information on the E.M.S. of T. This is however not true, and we know that the impact can excite T, or produce particles on it, or break it into its constituents. Additional information on the E.M.S. of T can be obtained by studying these processes—the inelastic-scattering processes. While in the elastic scattering the kinematics is completely known once the momentum of the final lepton (or of the recoil target particle) is measured, in inelastic processes the complete knowledge of the final-state configuration cannot be achieved by detecting a single particle. The study of inelastic processes based on a detailed study of the final state becomes therefore very difficult both experimentally and theoretically.

However, some relevant insight into the E.M.S. of T can be achieved also through that partial knowledge of the final state which is obtained by measuring only the momentum of the final-state lepton. These experiments—the so-called inclusive experiments—have been an important field of investigation during the last few years. The corresponding process in the timelike region is of course the production of many-hadron systems from e^+e^- interactions.

The following four categories of experiments are the object of this purely experimental review article:

    a) elastic-scattering experiments of leptons on nucleons and on virtual pions,

    b) inelastic-scattering experiments of leptons on nuclear targets (inclusive),

    c) hadron-antihadron pair production from e^+e^- interactions,

    d) production of multihadron systems in e^+e^- collisions.

(*) Throughout this paper, the following convention is used: $q^2 = q_\mu q^\mu = E^2 - |\boldsymbol{p}^2|$, so that positive q^2 are timelike, and negative q^2 are spacelike.

In addition, we will review the experimental evidence for the validity of the hypotheses which allow us to express the results of the experiments in terms of structure functions of the hadrons involved in the reactions.

I. Spacelike region.

1. - Validity of the underlying hypotheses in the spacelike region.

The hypotheses which allow one to express the elastic- and inelastic-scattering amplitudes in terms of structure functions of the target hadrons are essentially the following [1-3]:

  i) the scattering amplitude is well described by the first-order, one-photon–exchange (1-PE) Feynman diagrams;

  ii) charged leptons behave as pointlike Dirac particles;

 iii) the photon propagator is given by $1/q^2$, the inverse of its four-momentum squared.

The validity of more general hypotheses—Lorentz invariance, parity and time-reversal symmetry, invariance under rotations in the isospin space and gauge invariance—will be assumed to hold in electromagnetic interactions without discussing their experimental foundation.

Let us instead review the experimental support to the above hypotheses i), ii) and iii) in the spacelike region.

1'1. *One-photon–exchange hypothesis.* – Due to the smallness of the electromagnetic coupling constant $\alpha = e^2/\hbar c \simeq 1/137$ the first-order one-photon–exchange contribution $A_{1\gamma}$ (proportional to α) to the scattering amplitude in lepton scattering is expected to dominate over the second-order (proportional to α^2) and higher-order contributions.

Despite the technical difficulties [4], the contribution of higher-order diagrams can be calculated in detail in the case of lepton-lepton scattering described by pure QED. This contribution, which depends in part on the features of the experimental apparatus, does not exceed in general a few percent; it can therefore be treated as a correction (radiative corrections), and the 1-PE contribution to the scattering amplitude can be accurately extracted from the experimental data. In the case of lepton-hadron scattering, the problem is treated in a similar way: at each order, the knowledge of the hadron structure which is needed for the calculation is derived from the results of the lower-order approximation.

Some doubts, however, have been raised about the correctness of this procedure. In particular, it has been pointed out that the interference term $2A_{1\gamma}\,\mathrm{Re}\,A_{2\gamma}$ (*) between the 1-PE term and the contribution $A_{2\gamma}$ from the graph
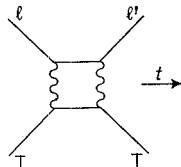
when evaluated with the above procedure could result in large uncertainty, since possible resonances could enhance $A_{2\gamma}$ [5, 6]. Since the magnitude of $A_{2\gamma}$ has been determined by several experimental investigations, we can consider these as tests of the correct evaluation of $A_{2\gamma}$ with the standard radiative correction techniques. These experiments are based on cross-section measurements, on the comparison of ℓ^--proton (ℓ^--p) with ℓ^+-proton (ℓ^+-p) elastic-scattering cross-sections and on polarization measurements.

1'1.1. Cross-section measurements. Rosenbluth plots. Lorentz invariance, gauge invariance and the 1-PE hypothesis lead to the form of the electron-nucleon elastic-scattering cross-section known as the Rosenbluth formula [3]:

$$(1.1.1) \qquad \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = A\left[\frac{G_E^2 + \tau G_M^2}{1 + \tau}\,\mathrm{ctg}^2\,(\theta/2) + 2\tau G_M^2\right],$$

where

$$(1.1.2) \qquad A = \left(\frac{e^2}{2E_0}\right)^2 \frac{1}{\sin^2(\theta/2)\left(1 + (2E_0/M)\sin^2(\theta/2)\right)} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{NS}}\mathrm{tg}^2\frac{\theta}{2},$$

$$(1.1.3) \qquad \tau = \frac{-q^2}{4M^2};$$

θ is the electron scattering angle, E_0 the energy of the incident electron and M the nucleon mass. The form factors G_E and G_M are real functions of q^2 only, related to the Dirac and Pauli form factors F_1 and F_2 (see Sect. 2).

Plots of $R = (1/A)\mathrm{d}\sigma/\mathrm{d}\Omega$ vs. $\mathrm{ctg}^2(\theta/2)$ at fixed q^2 are known as Rosenbluth plots and are straight lines of slope $(G_E^2 + \tau G_M^2)/(1+\tau)$ and intercept $2\tau G_M^2$. Deviations of the experimental points from the Rosenbluth plot would imply the break-down of one of the hypotheses, the weakest of which is the 1-PE approximation.

Experimental tests of the Rosenbluth formula have been extensively made [7]. We report an example in Fig. 1.

(*) Because of gauge invariance and current Hermiticity $A_{1\gamma}$ is real.
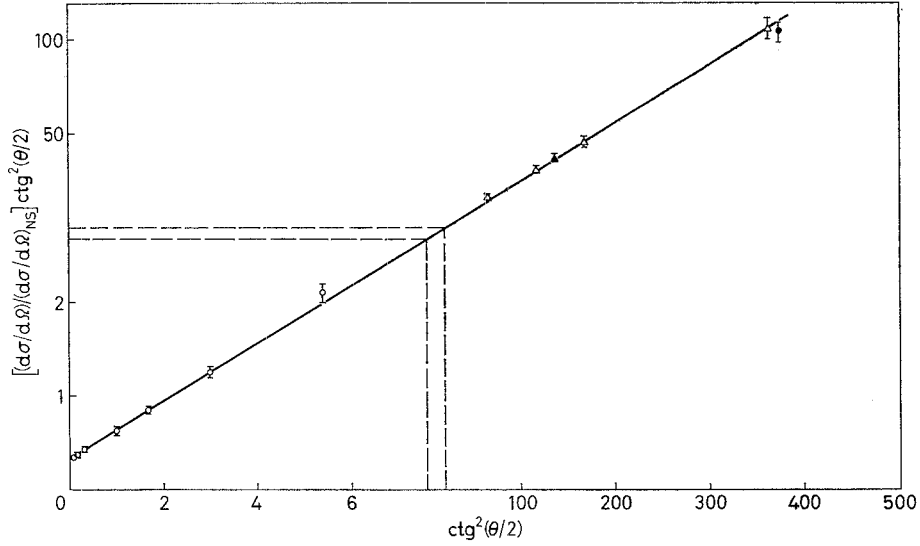
Fig. 1. – Typical Rosenbluth straight-line plot for electron-proton elastic scattering at $-q^2 = 0.39$ (GeV/c)^2. The quantity $R = [(\mathrm{d}\sigma/\mathrm{d}\Omega)/(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{NS}}]\,\mathrm{ctg}^2\,(\theta/2)$ is plotted against $\mathrm{ctg}^2\,(\theta/2)$: o JANSSENS et al. [7], △ BEHREND et al. [7], ▲ BARTEL et al. [7], ● ALBRECHT et al. [7].

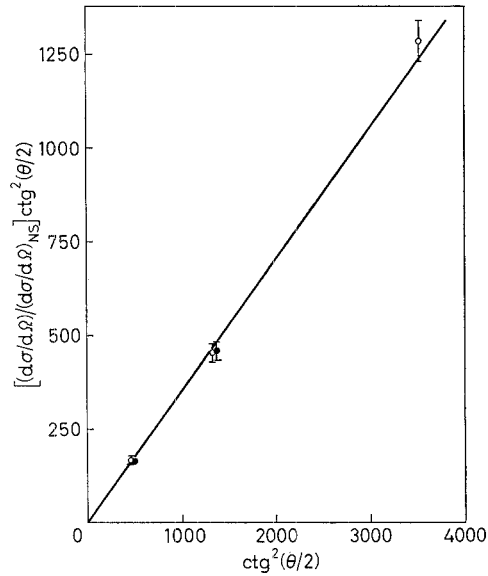Fig. 2. – Typical Rosenbluth straight-line plot for muon-proton elastic scattering at $-q^2 \simeq 0.39$ (GeV/c)^2. The quantity $R = [(\mathrm{d}\sigma/\mathrm{d}\Omega)/(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{NS}}]\,\mathrm{ctg}^2\,(\theta/2)$ is plotted against $\mathrm{ctg}^2\,(\theta/2)$: o $\mu^- + \mathrm{p} \to \mu^- + \mathrm{p}$ [8], ● $\mu^+ + \mathrm{p} \to \mu^+ + \mathrm{p}$ [8].

The Rosenbluth behaviour has also been tested in μ-p scattering [8], as shown in Fig. 2.

Agreement has always been found within the errors. It is worth recalling, however, that only the real part of a possible 2-PE amplitude $A_{2\gamma}$ gives an α^3 contribution to the cross-section, and in addition even a nonvanishing Re $A_{2\gamma}$ does not necessarily destroy the Rosenbluth behaviour [6].

I'1.2. Comparison of ℓ^+-p and ℓ^--p cross-sections. While the squared moduli of both the $A_{1\gamma}$ and the $A_{2\gamma}$ amplitudes are obviously even in the charge of the incident lepton ℓ, the interference term is odd:

(1.1.4) $$\frac{d\sigma^\pm}{d\Omega} = A_{1\gamma}^2 + |A_{2\gamma}|^2 \pm 2A_{1\gamma}\, \text{Re}\, A_{2\gamma};$$

σ^+ and σ^- are the cross-sections for $e^+(\mu^+)$ and $e^-(\mu^-)$ scattering in the same kinematical situation.

The comparison of the $e^+(\mu^+)$ and $e^-(\mu^-)$ scattering cross-sections has been performed by many authors [9-15].
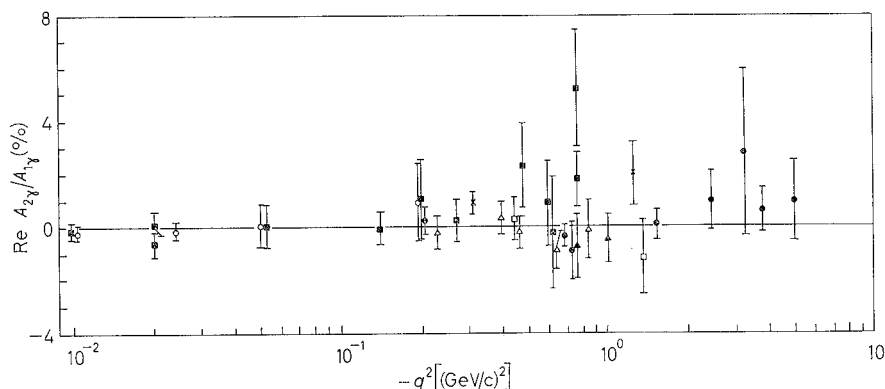

Fig. 3. – Comparison of the e^--proton and e^+-proton cross-sections. The ratio of the real part of the 2-photon–exchange amplitude to the 1-photon–exchange amplitude Re $A_{2\gamma}/A_{1\gamma} = \frac{1}{2}((\sigma^+ - \sigma^-)/(\sigma^+ + \sigma^-))$ is plotted vs. $-q^2$: ● ref. [9], ○ ref. [10], ■ ref. [11], □ ref. [12], × ref. [13], △ ref. [14], ▲ ref. [15].

The summary of all the experimental results available is given in Fig. 3 and 4, in which

$$R = \frac{\text{Re}\, A_{2\gamma}}{A_{1\gamma}} = \frac{1}{2}\frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

is plotted vs. $-q^2$. For electrons, R appears to be smaller than $\sim 1\%$ for $0.01 < -q^2 < 1$ (GeV/c)^2, and smaller than $\sim 2\%$ up to $-q^2 \simeq 5$ (GeV/c)^2. In
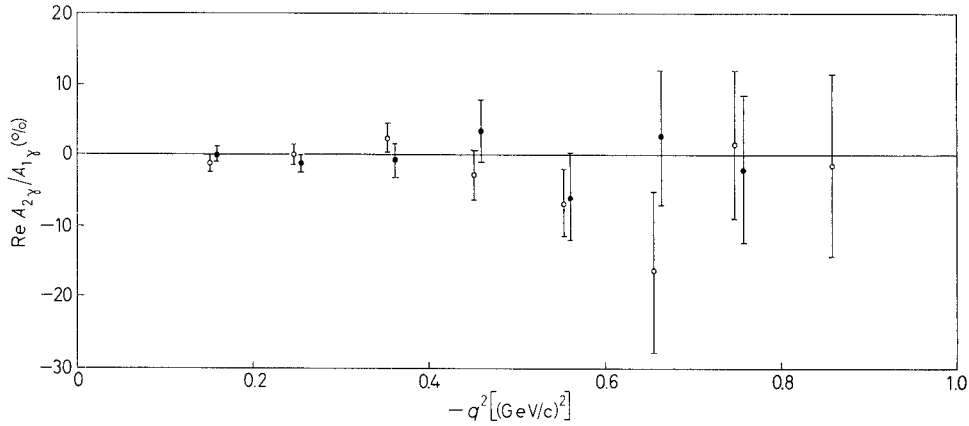
Fig. 4. – Comparison of the μ^--proton and μ^+-proton cross-sections. The ratio of the real part of the 2-photon–exchange amplitude to the 1-photon–exchange amplitude $\mathrm{Re}\, A_{2\gamma}/A_{1\gamma} = \frac{1}{2}((\sigma^+ - \sigma^-)/(\sigma^+ + \sigma^-))$ is plotted vs. $-q^2$. Data at two different primary-muon energies are shown: open circles 6 GeV, black circles 11 GeV (ref. [8]).

the case of muons [8], only $-q^2 \lesssim 1$ $(\mathrm{GeV}/\mathrm{c})^2$ has been explored with a lower accuracy.

1'1.3. Polarization measurements. In the 1-PE approximation, gauge invariance and the Hermiticity of the e.m. current restrict the elastic ℓ-p scattering cross-section to have no spin dependence. In other words, both
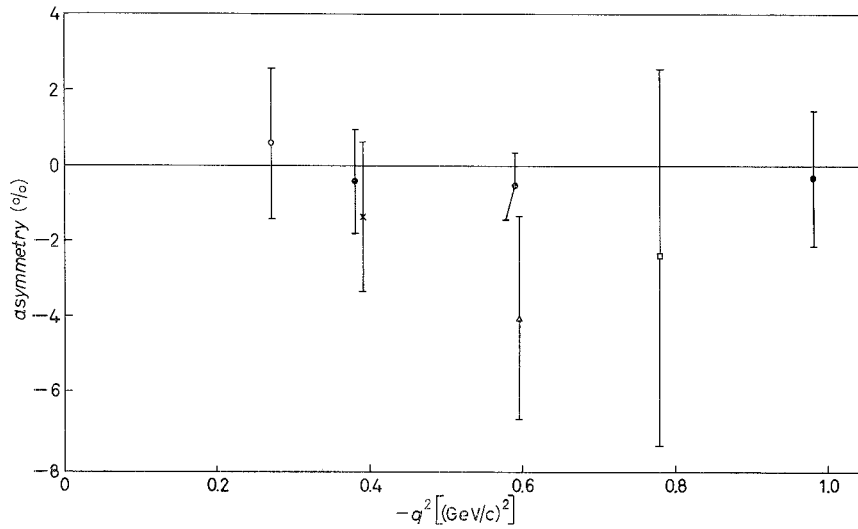

Fig. 5. – Values of the recoil-proton polarization or of the asymmetry in the scattering from polarized protons for elastic electron-proton scattering: □ ref. [17], △ ref. [18], × ref. [19], ○ ref. [20], ● ref. [21].

the polarization P of the recoil proton and the asymmetry A in the scattering of leptons from a polarized target must vanish.

Due to the presence of an imaginary part of a two-photon amplitude, however, A and P could be different from zero. If time-invariance holds, then $A = P$ are linear homogeneous functions of the imaginary part of the two-photon–exchange spin amplitudes [16].

Both A and P have been experimentally investigated in e-p scattering [17-21]. The results are summarized in Fig. 5.

No evidence for A or P different from zero appears in the explored q^2 range $(0.2 < -q^2 < 1\,(\text{GeV}/c)^2)$. The experimental accuracy is typically $(1 \div 3)\,\%$.

1'2. Tests of the photon propagator and of the pointlike leptons. – We now consider the experimental tests of the two hypotheses, ii) of the pointlike leptons and iii) of the $1/q^2$ photon propagator. These experiments go usually under the name of tests of the validity of QED (quantum electrodynamics).

Here, we review only those which are relevant in the spirit of testing the leptons as good probes of the e.m. structure of hadrons, *i.e.* we are interested in the experimental proof of the hypothesis that the vertex function and photon propagator in the graph

(1.2.1)

(ℓ and ℓ', the initial and final leptons, being both on their mass shell) can be written according to the rules of pure QED, *i.e.* γ_μ and $1/q^2$ [22] respectively.

The most general modifications of QED allowed in this case by gauge and Lorentz invariance are

(1.2.2) $\gamma_\mu \to \gamma_\mu F_1(q^2) + k\sigma_{\mu\nu}q^\nu F_2(q^2)\,,$

(1.2.3) $\dfrac{1}{q^2} \to \dfrac{M(q^2)}{q^2}\,.$

The restrictions on F_1, F_2 and M required by general principles [23] are not relevant in this context.

The $g-2$ experiments [24] allow us to conclude that $kF_2(q^2) \equiv 0$ (*) with, from our point of view, absolute precision. Thus, the most general form of

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(*) The measurement of a static quantity like $g-2$ gives information also on the values of the form factors at $q^2 \neq 0$. For details see, for instance, K. OKAMOTO: *Nucl. Phys.*, **4** B, 226 (1967).

(1.2.2) becomes

(1.2.4) $$\gamma_\mu \to F_1(q^2)\gamma_\mu \, .$$

For spacelike values of q^2, $F_1(q^2)$ and $M(q^2)$ have been measured, in the case of electrons, by means of the e^-e^- (MØLLER) [25] and e^+e^- (BHABHA) [26] elastic scattering. The Feynman graphs which describe these processes to first order are

BHABHA   MØLLER
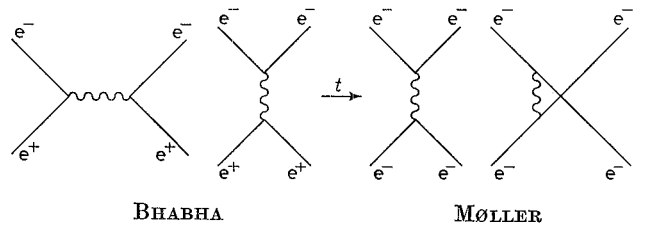
The amplitude corresponding to each graph is obviously proportional to $M(q^2)F_1^2(q^2)$. Since q^2 is not the same for the two graphs contributing to each process, the ratio $R = \sigma_{\mathrm{exp}}/\sigma_{\mathrm{th}}$, in terms of which the experimental data are conveniently expressed, is *not* proportional to $M^2(q^2)F_1^4(q^2)$. However, in the kinematical regions explored by the experiments, one graph (the one corresponding to the lower value of the momentum transfer squared q_t^2 which is spacelike also in the Bhabha scattering) usually dominates the other so that R is approximately proportional to $M^2(q_t^2)F_1^4(q_t^2)$. In Fig. 6, R (for both Møller
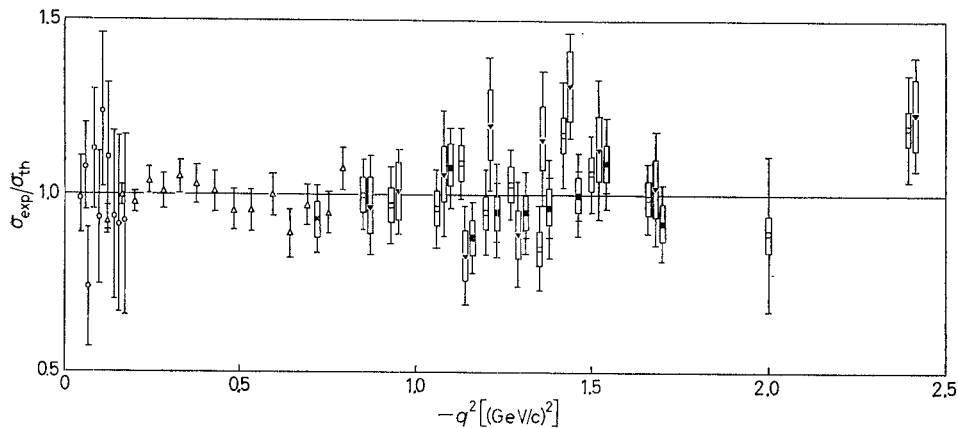
Fig. 6. – Comparison of experimental data on e^-e^- and e^+e^- elastic scattering with QED predictions as a function of the spacelike momentum transfer $- q^2$. The results are expressed in terms of the ratio $\sigma_{\mathrm{exp}}/\sigma_{\mathrm{th}}$. Statistical errors are shown as bars, while boxes represent the systematic uncertainty: □ BORGIA *et al.* [30], ▼ BARTOLI *et al.* [31], ○ BARBER *et al.* [27], × AUGUSTIN *et al.* [32], △ BARBER *et al.* [28], ■ ALLES-BORELLI *et al.* [29].

and Bhabha scattering) is shown as a function of $-q_t^2$. In the Stanford I [27] and II [28] experiments the absolute value of the cross-section was not measured; therefore the average value of R has been normalized to 1. In the Frascati data [29-31] and in the Orsay point [32], in addition to the statistical error (bars), the systematic uncertainty is also displayed (boxes) including the overall normalization uncertainty in each experiment.

The above data from e^--e^- and e^+-e^- storage ring experiments allow us to conclude that the form factor of the electron is tested to be equal to 1 to within $\sim (1 \div 2)\%$ (F_1^4 within $(4 \div 8)\%$) up to $-q^2 \lesssim 1.5$ (GeV/c)^2; and within $\sim 5\%$ (F_1^4 within $\simeq \pm 20\%$) up to about 2.5 (GeV/c)^2. $M(q^2)$, appearing in R only squared, is measured with about twice as large a relative error.

In the case of muons, no scattering measurement on lepton targets has been performed. However, the muon structure in the spacelike region has been investigated by comparing the muon-proton elastic cross-section $d\sigma_\mu/d\Omega$ with the electron-proton elastic scattering $d\sigma_e/d\Omega$ in the same kinematical situation.

In the ratio $\Sigma_{\mu,e} = d\sigma_\mu/d\sigma_e$ the proton structure contribution cancels out (and the same is true for a possible modification of the photon propagator $M(q^2)$), so that $\Sigma_{\mu,e}$ turns out to be proportional to $F_\mu^2(q^2)/F_e^2(q^2)$. The result of this comparison [33, 34] in the case of lepton-proton elastic scattering is shown in Fig. 7. The ratio $F_{\mu,e} = F_\mu(q^2)/F_e(q^2)$ is displayed as a function of
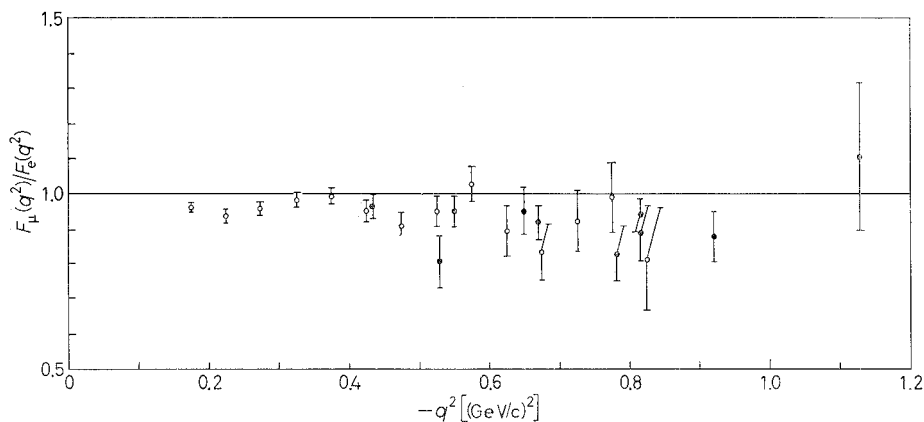

Fig. 7. – Comparison of muon-proton and electron-proton elastic scattering. The ratio $F_{\mu,e} = F_\mu(q^2)/F_e(q^2)$ is displayed as a function of $-q^2$. $F_\mu(q^2)$ and $F_e(q^2)$ are the vertex modification functions: o CAMILLERI *et al.* [34], • ELLSWORTH *et al.* [33].

$-q^2$ $(0.1 \lesssim -q^2 \lesssim 1$ (GeV/c)$^2)$. $F_{\mu,e}$ does not appear to have any appreciable q^2 dependence within the errors; its absolute value, however, appears to be $\simeq 0.96$. This 4% discrepancy, six standard deviations outside the statistical errors, corresponds to an $\sim 8\%$ difference in the cross-sections. The au-

thors [33, 34] do not exclude a possible systematic normalization error of this size in either the muon or the electron experiments.

Similar tests have also been performed by comparing μ-p with e-p inelastic-scattering experiments. These inelastic processes allow us to compare the muon with the electron structure even better than elastic-scattering experiments. In fact a larger category of kinematical situations can be explored in this case. In particular the large values of the inelastic cross-sections allow us to have a statistically significant measurement of $F_{\mu,e}$ up to $-q^2 \simeq 3$ $(\text{GeV}/c)^2$. The result is [35, 36] that $F_{\mu,e}$ can be fitted with a straight line $N(1-q^2 \cdot K^2)$ with $N = 0.946 \pm 0.042$ and $K^2 = (0.021 \pm 0.021)$ $(\text{GeV}/c)^{-2}$.

Again the slope of $F_{\mu,e}$ is compatible with zero, the absolute value (although smaller than one) being consistent with 1 within the errors.

2. – Nucleon form factors.

The most general form of the matrix element of the e.m. current of a nucleon between two states of four-momentum p and p' $(p^2 = p'^2 = M^2)$ is [1, 2]

$$(2.1) \qquad \langle p'|J_\mu|p\rangle = \langle p'|F_1(q^2)\gamma_\mu + k\sigma_{\mu\nu}q^\nu F_2(q^2)|p\rangle \,,$$

where $q_\mu = p'_\mu - p_\mu$ is the difference between the final and initial nucleon four-momenta; k is the anomalous magnetic moment $\mu - 1$. F_1 and F_2 (the so-called Dirac and Pauli form factors) are restricted by the requirement of the Hermiticity of the e.m. current to be real for $q^2 < 0$ (spacelike region).

As a consequence of the hypotheses discussed in the previous Section, the cross-section for lepton-nucleon scattering turns out to be a rather simple expression of E (energy of the incident lepton), θ (scattering angle), F_1^2 and F_2^2. By performing at least two cross-section measurements at the same value of q^2, but at different angles and energies, F_1^2 and F_2^2 can in principle be evaluated, although it is quite a complicated numerical procedure [1]. This procedure becomes quite simple if we solve in terms of two other functions, G_E^2 and G_M^2. G_E and G_M, the so-called « electric » and « magnetic » form factors, are related to F_1 and F_2 as follows [37]:

$$(2.2) \qquad \begin{cases} G_E = F_1 - \tau k F_2 \,, & F_1 = \dfrac{\tau G_M + G_E}{1 + \tau} \,, \\[2ex] G_M = F_1 + k F_2 \,, & F_2 = \dfrac{G_M - G_E}{(1 + \tau)k} \end{cases} \qquad \left(\tau = \dfrac{-q^2}{4M^2}\right).$$

As usual, from here on we will attach when needed a second index p or n to the form factors according to whether they refer to the proton or to the neutron.

The isovector and isoscalar form factors are also conveniently introduced:

(2.3)
$$\begin{cases} G_{MV} = G_{Mp} - G_{Mn}, \\ G_{MS} = G_{Mp} + G_{Mn}, \\ G_{EV} = G_{Ep} - G_{En}, \\ G_{ES} = G_{Ep} + G_{En}. \end{cases}$$

In terms of G_E and G_M the elastic-scattering cross-section assumes the simple form (1.1.1), and G_E^2 and G_M^2 are simply related to the slope and intercept of the Rosenbluth plots. It is worth noticing, however, that as $\tau = -q^2/4M^2$
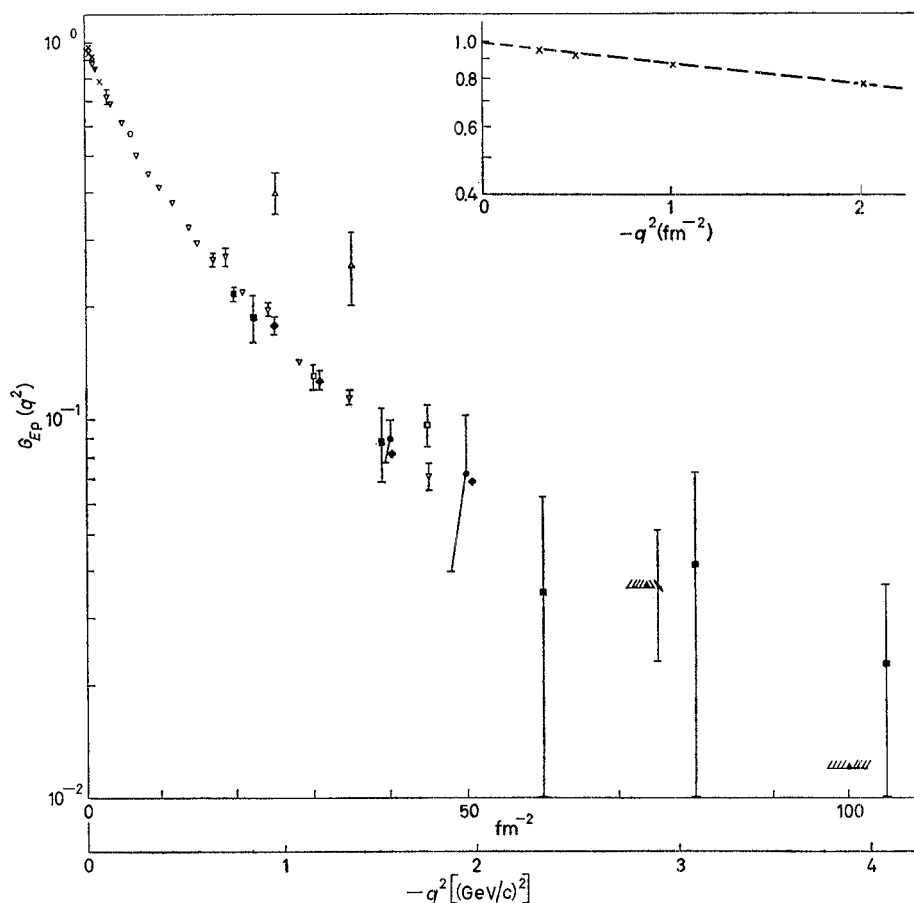

Fig. 8. – Electric form factor of the proton $G_{Ep}(q^2)$ plotted against $-q^2$: ● ref. [38], \ ref. [39], ○ ref. [40], ■ ref. [41], □ ref. [42], △ ref. [43], ▼ ref. [46], ▽ ref. [45], ▲ ref. [44], × ref. [47], ◆ ref. [48]. The Figure on the top (right) is an expanded $-q^2$ scale at low q^2 (errors in the data points ±1%).

grows larger than 1, the cross-section becomes less and less dependent on G_E^2. As a consequence, at large values of $-q^2\ G_E^2$ is difficult to measure.

Elastic-scattering experiments of electrons on nucleons have been performed for more than 15 years.

A summary of the proton results is presented in Figs. 8-13 [52, 53].
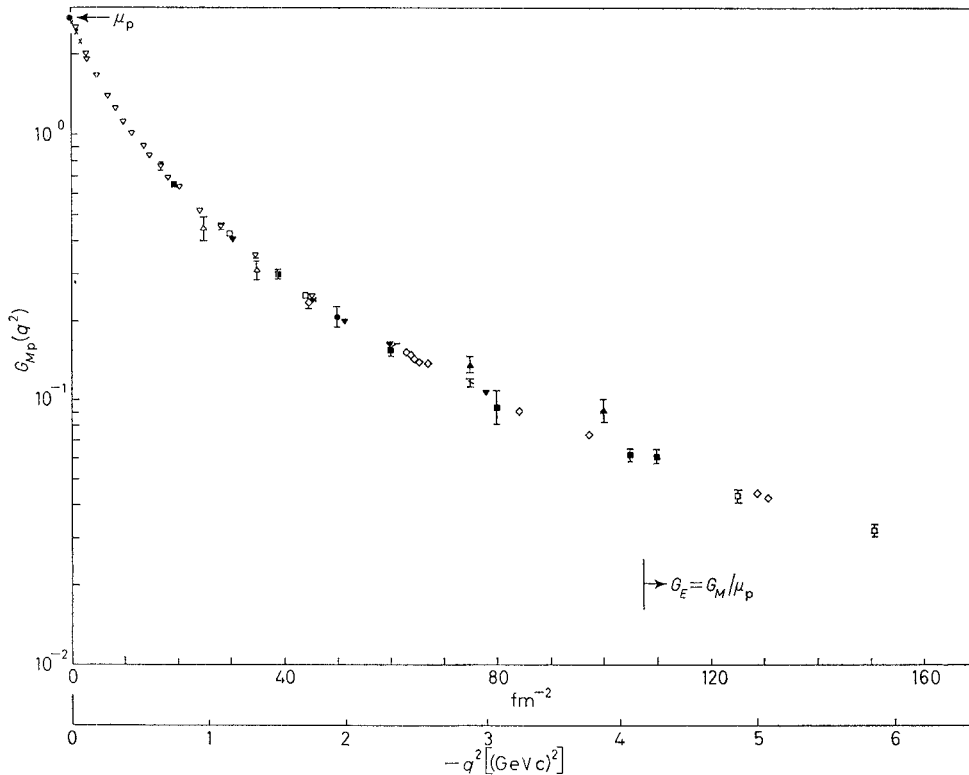
Fig. 9. – Magnetic form factor of the proton $G_{Mp}(q^2)$ plotted against $-q^2$ up to $-q^2 \leqslant 150$ fm^{-2}. Values of $G_{Mp}(q^2)$ for $-q^2 \geqslant 100$ fm^{-2} are obtained with the usual assumption $\mu_p G_{Ep}(q^2) = G_{Mp}(q^2)$: ● ref. [38], \ ref. [39], □ ref. [42], ■ ref. [41], ▼ ref. [49], ▽ ref. [45], ▲ ref. [44], / ref. [47], △ ref. [43], ◇ ref. [50].

In Fig. 8 $G_{Ep}(q^2)$ is shown. In Fig. 9, $G_{Mp}(q^2)$ for $-q^2 \leqslant 150$ fm^{-2} is presented. When $-q^2$ is larger than ~ 50 fm^{-2}, the errors in G_{Ep} become very large, and no measurement is available for $-q^2 \geqslant 100$ fm^{-2}. At higher values of $-q^2$, the G_{Ep} contribution to the cross-section becomes in fact so small that only G_{Mp} can be determined. This is usually done by assuming in this high-$(-q^2)$ region the same relation between G_{Mp} and G_{Ep} which experimentally approximately holds at lower momentum transfers, namely $G_{Ep} \simeq G_{Mp}/\mu_p$ (see the following). Actually, the cross-sections in the $-q^2$ region above ~ 150 fm^{-2}

depend so little on G_{Ep}, that the data points on G_{Mp} presented in Fig. 10 would remain the same within the errors unless G_{Ep} exceeds G_{Mp} by more than one order of magnitude. The fact that $G_{Ep}(q^2) = G_{Mp}(q^2)/\mu_p = 1$ for $-q^2 \to 0$ is
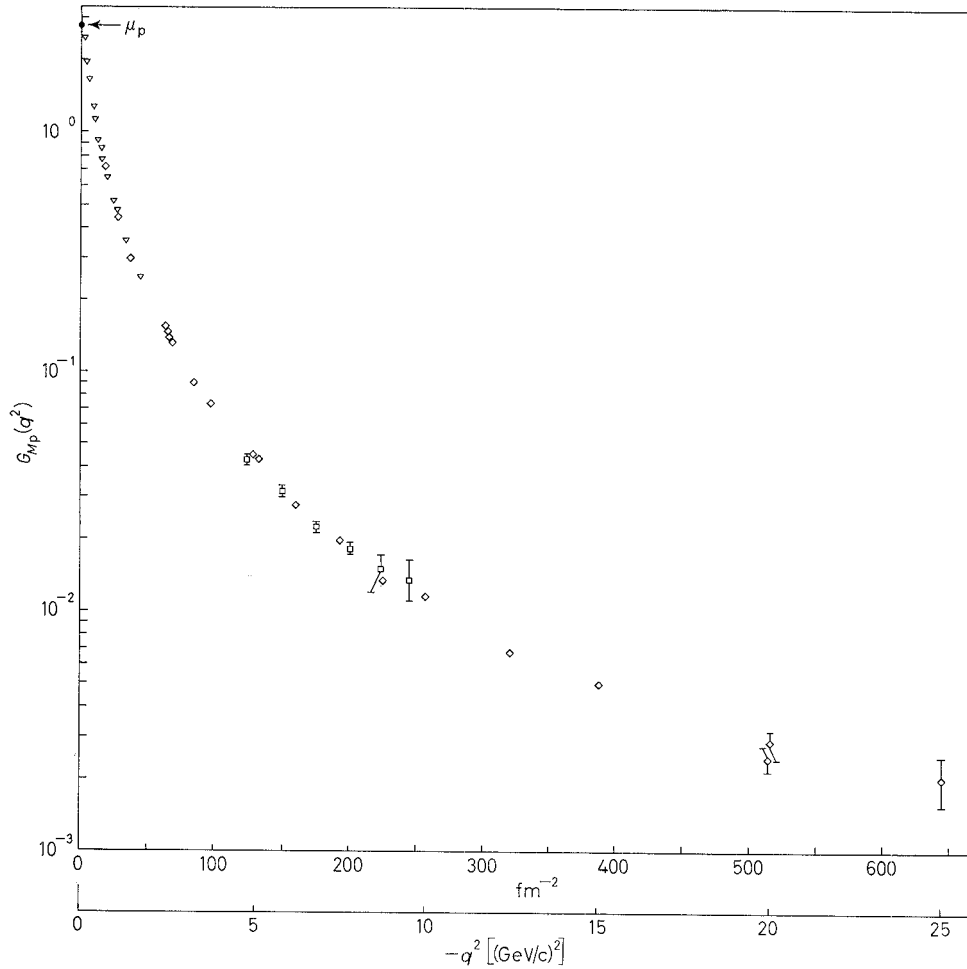

Fig. 10. – Magnetic form factor of the proton $G_{Mp}(q^2)$ plotted against $-q^2$ for all the measured $-q^2$ range. Values of $G_{Mp}(q^2)$ for $-q^2 \gtrsim 100 \text{ fm}^{-2}$ are obtained with the usual assumption $\mu_p G_{Ep}(q^2) = G_{Mp}(q^2)$: ∇ ref. [45], □ ref. [42], ◇ ref. [50].

required by the interpretation of the form factors as Fourier transforms of the charge and magnetic moment distributions of the proton [6]: this picture is rigorous in the static limit $-q^2 \to 0$. On the contrary, at large momentum transfers the Fourier-transform interpretation is far from being rigorous [37], and an attempt to explain the rapid fall-off of the form factors for increasing $-q^2$ w th a hard core of the nucleon is therefore arbitrary.
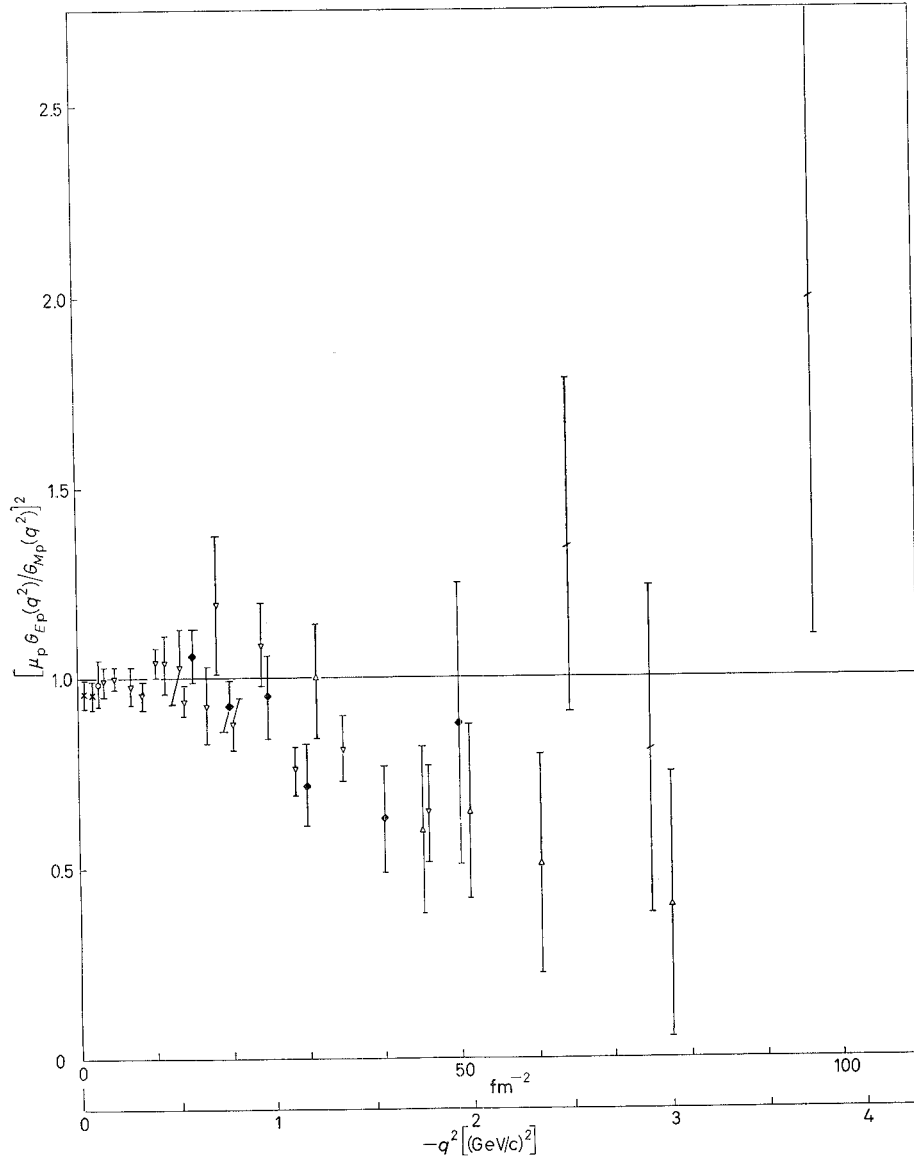
Fig. 11. – Comparison between the electric and magnetic proton form factors. The ratio $[\mu_p G_{Ep}(q^2)/G_{Mp}(q)]^2$ is shown against the momentum transfer $-q^2$: o ref. [52], × ref. [47], ▽ ref. [45], △ ref. [49], / ref. [51], ◆ ref. [48].

It is easily seen from Fig. 8 and 9 that $G_{Ep} \simeq G_{Mp}/\mu_p$. To what extent this famous relation (« scaling law ») really holds is better seen in Fig. 11, where the quantity $(\mu_p G_{Ep}/G_{Mp})^2$ is shown. We see that for $-q^2 \gtrsim 30$ fm^{-2} deviations $\gtrsim (20 \div 30)\%$ are experimentally found.
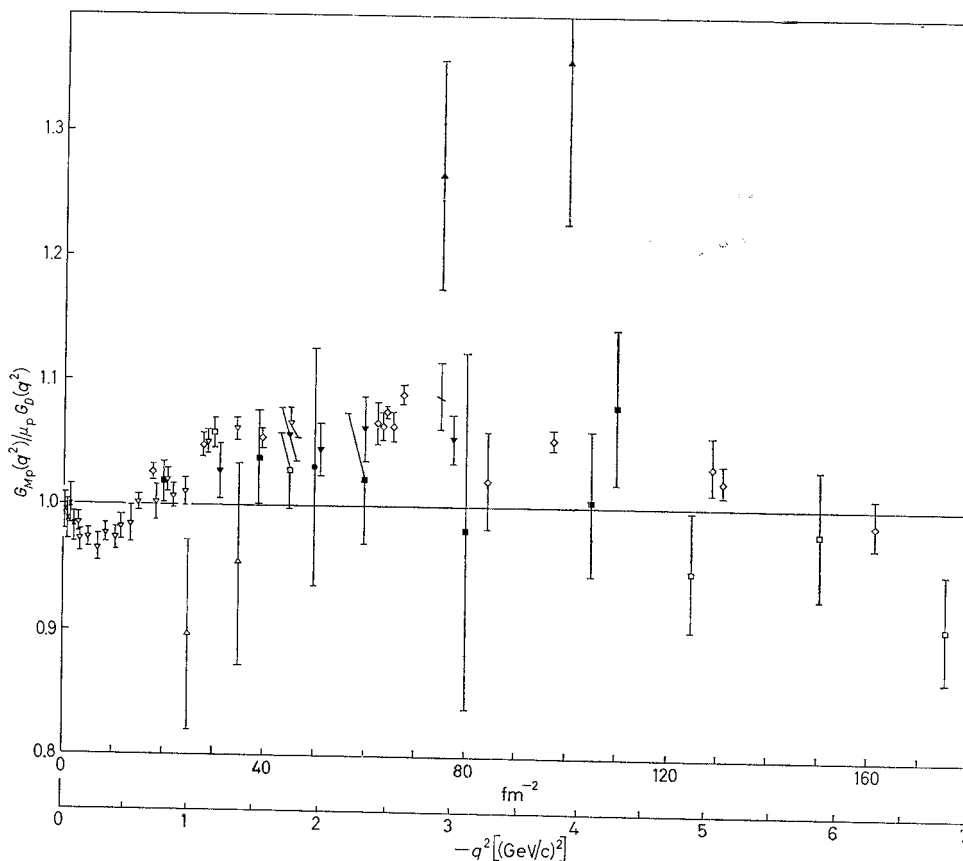
Fig. 12. – « Dipole fit » for the magnetic proton form factor. The ratio $G_{Mp}(q^2)/\mu_p G_D(q^2)$ is plotted against $-q^2$ for values of $-q^2 \leqslant 180\ \text{fm}^{-2}$: $\triangle$ ref. [43], $\bullet$ ref. [38], $\setminus$ ref. [39], $\blacksquare$ ref. [41], $\square$ ref. [42], $\blacktriangledown$ ref. [49], $\diamond$ ref. [50], $\blacktriangle$ ref. [44], $\times$ ref. [47], $\triangledown$ ref. [45].

The scaling law for the proton form factors is to be considered an approximate mnemonic rule. Its theoretical foundation (quark model, SU_6 [54]) is rather weak. In addition, in the timelike region, its validity near threshold would cause serious problems (see Sect. 2, Part II).

Another useful mnemonic rule (with *no* theoretical foundation) is the so-called dipole fit, *i.e.* $G_{Mp}/\mu_p \simeq G_D \equiv \left(1 - (q^2/0.71\ (\text{GeV}/c)^2)\right)^{-2}$. The comparison between G_{Mp}/μ_p and G_D is shown in Fig. 12, 13 where the quantity $G_{Mp}/\mu_p\,G_D$ is presented. The dipole fit appears to hold within $\simeq 30\%$.

The situation on the neutron form factors is much less clear. This is due to the fact that free-neutron targets are not available, so that the information on e-n scattering is extracted by means of a quite complicated and model-dependent procedure from e-deuteron (e-d) elastic and inelastic scattering. Some information at very low momentum transfers is also obtained by scattering low-energy (thermal) neutrons on high-Z atoms. An excellent review
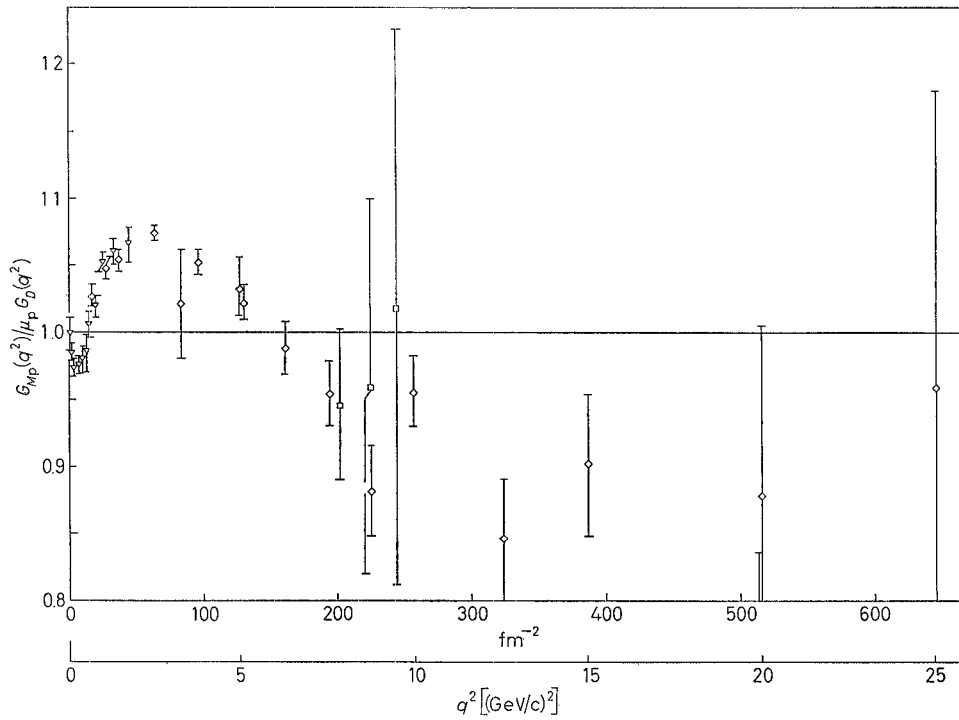
Fig. 13. – « Dipole fit » for the magnetic proton form factor. The ratio $G_{Mp}(q^2)/\mu_p G_D(q^2)$ is plotted against $-q^2$ for all the measured $-q^2$ range: □ ref. [42], ▽ ref. [45], ◇ ref. [50].

of the methods used to extract G_{En} and M_{Mn} from the experimental data can be found in ref. [6].

The difficulty in the experiments of scattering of neutrons on atoms (first performed by FERMI and MARSHALL [55]) is to separate the coherent scattering amplitude on the nucleus from the scattering amplitude of the neutrons on the atomic electrons.

Different atomic targets and analysis methods have been used in different experiments. The results, expressed in terms of $dG_{En}/dq^2|_{q^2=0}$ are in fair agreement with each other:

TABLE I.

Author	dG_{En}/dq^2 (GeV/c)$^{-2}$
KROHN I [56]	0.459 ± 0.02
KROHN II [56]	0.50 ± 0.01
MELKONIAN [57]	0.575 ± 0.019
HUGHES [58]	0.512 ± 0.019

dG_{En}/dq^2 is related to the root-mean-square charge radius of the neutron [6]

$$(2.4) \qquad \langle r_{ch}^2 \rangle = \frac{1}{6} \left| \frac{dG_{En}}{dq^2} \right|_{q^2=0} ,$$

which can thus be considered known—including uncertainties from the models—to within $\sim (5 \div 10)\%$.

At higher values of $-q^2$ $(0.3 < -q^2 \lesssim 20 \text{ fm}^{-2})$ G_{En} can be extracted from elastic e-d scattering measurements, once the electric proton form factor G_{Ep} and the deuteron stucture are known. In fact the e-d elastic-scattering cross-section can be written as [6]

$$(2.5) \qquad \frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[A(q^2) + B(q^2) \, \text{tg}^2 \frac{\theta}{2} \right] .$$

$A(q^2)$ and $B(q^2)$ contain the charge, magnetic and quadrupole form factors of the deuteron, in addition to the isoscalar (the deuteron has $T = 0$) nucleon form factors. For electron scattering angles θ smaller than $15°$ the term $B(q^2) \, \text{tg}^2(\theta/2)$ contributes less than 0.1% to the cross-section and can be neglected. Equation (2.5) takes then the simple form

$$(2.6) \qquad \frac{d\sigma}{d\Omega} \simeq \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} A(q^2) ,$$

so that cross-section measurements allow one to determine $A(q^2)$. Notice however that for $-q^2 \simeq 15 \text{ fm}^{-2}$, $A(q^2)$ is as small as $\sim 10^{-4}$, so that the experiments become very difficult.

In turn

$$(2.7) \qquad A(q^2) = \frac{G_{Ed}^2(q^2)}{1+\tau} + \frac{8}{9} \frac{\tau^2 G_{Qd}^2(q^2)}{1+\tau} + \frac{2}{3} \tau^2 \frac{G_{Md}^2}{1+\tau} ,$$

where

$$(2.8) \qquad \begin{cases} G_{Ed}(q^2) = 2G_{Es}(q^2) F_{Ed}(q^2) , \\[2mm] G_{Qd}(q^2) = 2G_{Es}(q^2) F_{Qd}(q^2) , \\[2mm] G_{Md}(q^2) = 2 \dfrac{M_d}{M} G_{Ms}(q^2) F_{Md}(q^2) . \end{cases}$$

F_{Ed}, F_{Qd} and F_{Md}, which describe the deuteron structure, can be calculated from the nonrelativistic deuteron wave functions for the S and D states. Several models are available for this purpose (HAMADA and JOHNSTON [59], MC GEE [60], FESHBACH and LOMON [61], etc.). Relativistic corrections have been evaluated by GROSS [62].
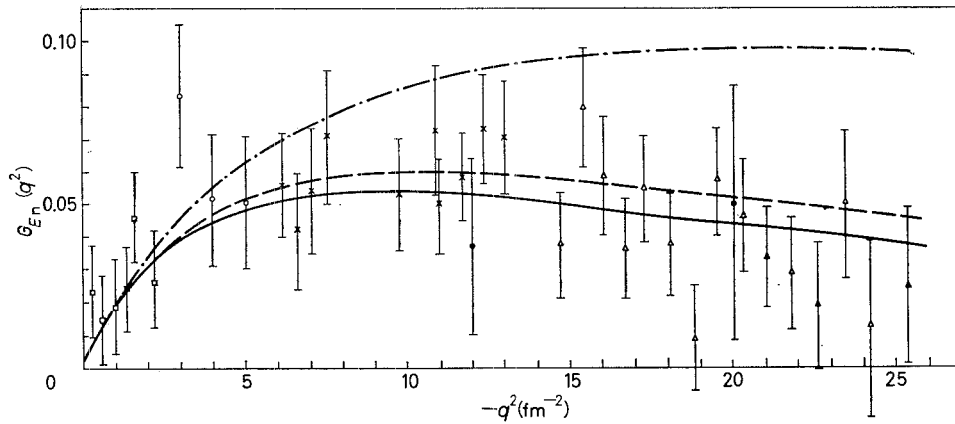
Fig. 14. – The neutron electric form factor $G_{En}(q^2)$ derived from elastic electron-deuteron scattering measurements. The Feshbach-Lomon deuteron wave function was used. The dashed line is $G_{En} = \mu_n \tau G_{Ep}$, which corresponds to the assumption $F_{1n} = 0$; the dash-dotted curve is $G_{En} = (\mu_n \tau/(1 + 4\tau)) G_{Ep}$; the solid curve is the best fit to the data points with a curve $G_{En} = (\mu_n \tau/(1 + b\tau)) G_{Ep}$ (b, the free parameter, turns out to be 5.6): ■ ref. [66], × ref. [63], □ ref. [65], △ ref. [69], ○ ref. [67], ● ref. [68].

The experimental values of $A(q^2)$ are compared with the different models. It turns out that no model fits the data if G_{En} is put equal to zero. This leads to $G_{En} \neq 0$, the actual value of G_{En} depending however on the wave function of the deuteron used in the analysis.

A review of the results for G_{En} obtained with this method can be found in ref. [63]. In Fig. 14 the results obtained with the Feshbach-Lomon model are shown [63]. Relativistic corrections are included [62]; the four-pole fit [64] for the electric proton form factor is used (to extract G_{En} from G_{ES}, G_{Ep} must
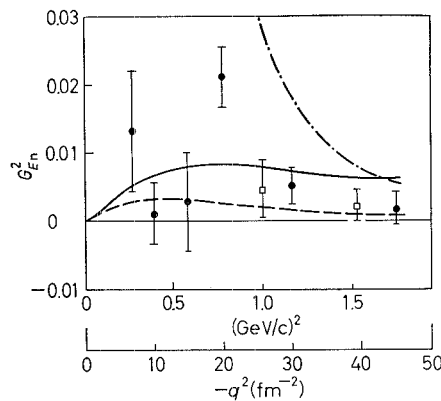

Fig. 15. – G_{En}^2 as determined from inelastic electron-deuteron scattering is plotted against $-q^2$: ● ref. [72], □ ref. [73], ——— $\tau^2 G_{Mn}^2$, —— —— $(\tau^2/(1 + 4\tau)^2) G_{Mn}^2$, —·—·—— G_{Ep}^2.
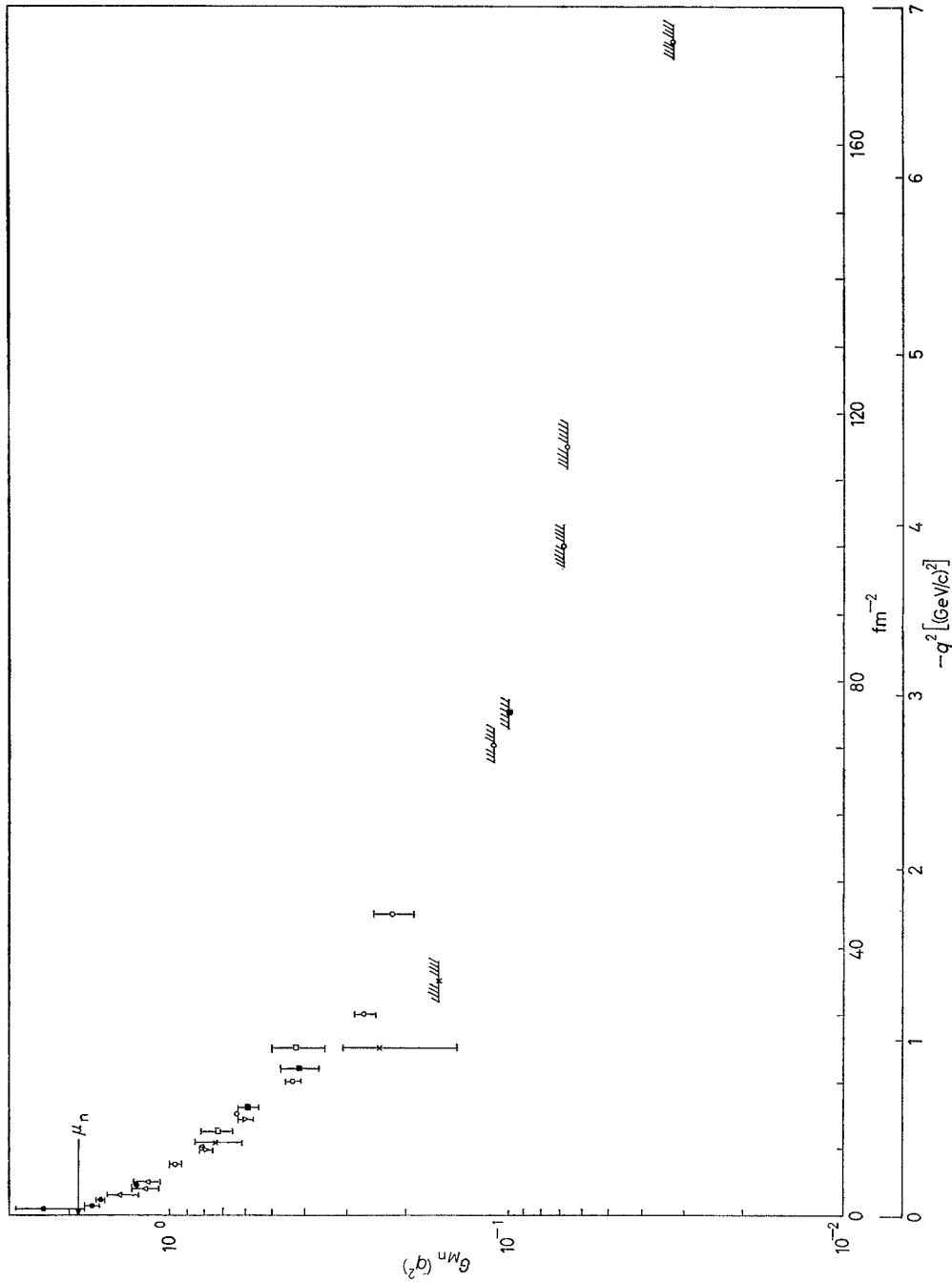
Fig. 16. – G_{Mn} as determined from inelastic electron-deuteron scattering. The symbols ▨ represent upper limits: ● ref. [74], ○ ref. [76], □ ref. [75], ■ ref. [77], ▽ ref. [78], × ref. [79], △ ref. [67].

be known, see (2.3)). The data points are from ref. [63, 65-69]. The dashed curve is $G_{En} = \mu_n \tau G_{Ep}$ [70], which corresponds to the assumption $F_{1n} = 0$; the dash-dotted curve is $G_{En} = (\mu_n \tau/(1 + 4\tau)) G_{Ep}$ [71], while the solid curve is the best fit to the data points with a curve $G_{En} = (\mu_n \tau/(1 + b\tau)) G_{Ep}$, where the parameter b turns out to be $b = 5.6$. At higher values of q^2 ($-q^2 \geqslant 15$ fm^{-2}) the neutron form factors can be determined by means of inelastic electron-deuteron scattering experiments.

At sufficiently large momentum transfers the nucleons can be treated as approximately free (impulse approximation), but with a momentum distribution given by the deuteron ground-state wave function. (Again the knowledge of the deuteron wave function is needed [6].)

Inelastic e-d scattering experiments have been performed, detecting either only the scattered electron (noncoincidence experiments), or the scattered electron in coincidence with the proton or the neutron (coincidence experiments). Small corrections are needed in this case due to the tail of the deuteron wave function and the final-state interaction.
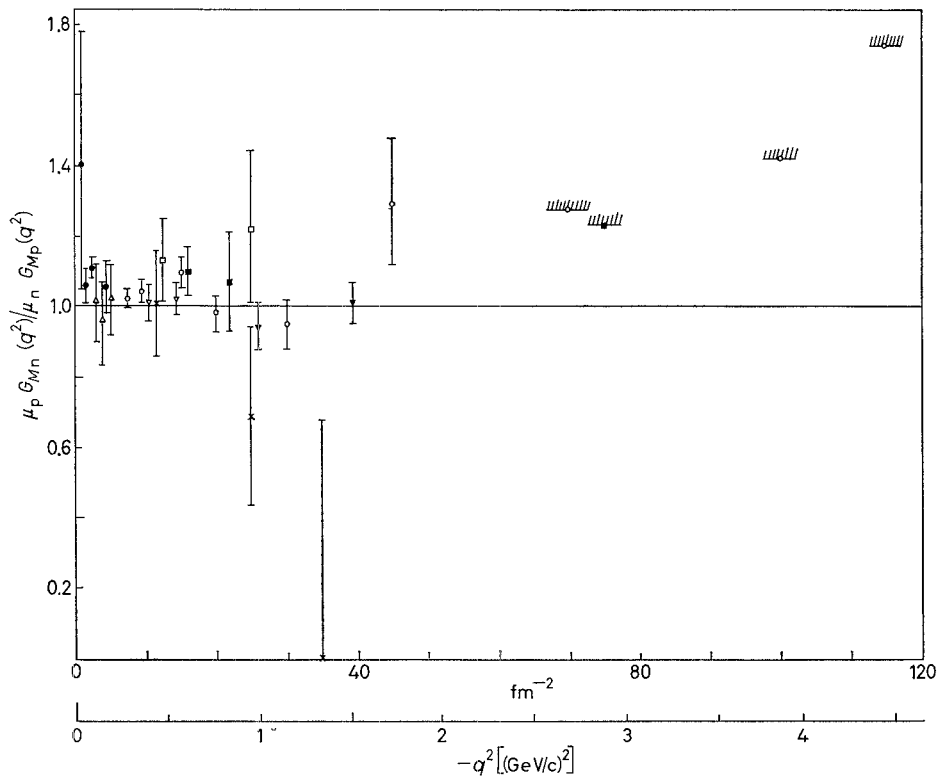

Fig. 17. – Test of the « scaling law »: $\mu_p G_{Mn}/\mu_n G_{Mp}$ is plotted as a function of $-q^2$. The symbols ▨▨▨ represent upper limits: ● ref. [74], △ ref. [67], ○ ref. [76], ▽ ref. [78], × ref. [79], □ ref. [75], ■ ref. [77], ▼ ref. [73].

In Fig. 15, 16, G^2_{En} and G_{Mn} as determined in inelastic e-d scattering measurements are shown. In Fig. 15 G^2_{En} is shown, and compared with the same models as in Fig. 14. In Fig. 16 G_{Mn} is shown. The data of some old experiments, not shown directly in the Figure, have been used in more recent work to improve the quality of the analysis, and are therefore quoted under the reference of the most recent analysis. The points at $-q^2 > 50$ fm^{-2} are upper limits.

In Fig. 17 the ratio $\mu_p G_{Mn}/\mu_n G_{Mp}$ is shown. We see that also the « scaling law » [54] $G_{Mn}/\mu_n = G_{Mp}/\mu_p$ approximately holds.

3. – Pion form factor.

The most general form of the matrix element of the e.m. current of a spin-0 particle B is

$$(3.1) \qquad \langle p'|J^B_\mu|p\rangle = F(q^2)(p'_\mu + p_\mu) \ .$$

As a consequence of (3.1) and of the usual hypotheses, the cross-section for the elastic scattering of electrons on B can be written as [6]

$$(3.2) \qquad \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \cdot F^2(q^2) \ ,$$

where $F(q^2)$, the form factor of the target particle, is a function of q^2 only and is real in the spacelike region.

Elastic-scattering experiments of electrons on free pions are however not possible, and elastic-scattering experiments of pions on electrons have not yet been performed. The information available on $F_\pi(q^2)$ in the spacelike region has been obtained by scattering electrons on virtual pions, namely through electroproduction experiments of pions on nucleons near threshold. In this case, however, the connection between experimental data and F_π is much more complicated than (3.2), and is in addition model dependent.

This is due to two facts:

a) in electroproduction processes (e.g. $\mathrm{e}^- + \mathrm{p} \to \mathrm{e}^- + \pi^+ + \mathrm{n}$) the contribution from the graph

(3.3)   « pion pole »
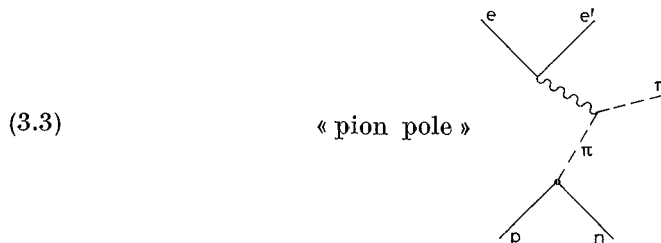
(which is relevant for the determination of F_π) is due to an off-mass-shell pion interacting with the nucleon;

b) the contribution from the graph (3.3) must be separated from the contribution of other graphs, e.g.

(3.4)

whose contributions are expected to be important in the electroproduction process near threshold.

The electroproduction cross-section—under the 1-PE assumption—can be written as [80]

$$(3.5) \qquad \frac{\mathrm{d}^3\sigma}{\mathrm{d}E' \mathrm{d}\Omega_e \mathrm{d}\Omega_\pi^*} = \Gamma_{\mathrm{ph}} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_\pi^*} ;$$

E' = energy of the scattered electron in the laboratory system,

$\mathrm{d}\Omega_e = \sin\theta_e \mathrm{d}\theta_e \mathrm{d}\varphi_e$, element of solid angle of the scattered electron in the laboratory system,

$\mathrm{d}\Omega_\pi^* = \sin\theta_\pi^* \mathrm{d}\theta_\pi^* \mathrm{d}\varphi_\pi^*$, element of solid angle of the produced pion in the c.m. system of the hadrons in the final state.

The factor Γ_{ph} contains the electrodynamics of the process (electron-photon vertex and photon propagator) and $\mathrm{d}\sigma/\mathrm{d}\Omega_\pi^*$ is the cross-section for pion photo-production by virtual (polarized) photons [80]:

$$(3.6) \qquad \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_\pi^*} = A + \varepsilon B + \varepsilon C \sin^2\theta_\pi^* \cos^2\varphi_\pi^* + \sqrt{2\varepsilon(1+\varepsilon)}\, D \sin\theta_\pi^* \cos\varphi_\pi^*,$$

where ε, the polarization of the virtual photon, is given by $\varepsilon = \left(1 + (2|\boldsymbol{p}|^2/|q^2|)\cdot \mathrm{tg}^2(\theta_e/2)\right)^{-1}$ with $\boldsymbol{p}$ the three-momentum of the photon in the laboratory system. The functions A, B, C and D contain the information on the structure of the hadrons involved in the process, i.e. they depend on the form factors at the vertices $[\gamma\pi\pi]$, $[\gamma\mathcal{N}\mathcal{N}]$, $[\gamma\mathcal{N}\Delta]$ ($\Delta = 1238\ \frac{3}{2}\ \frac{3}{2}$ resonance). The first term A is the differential cross-section for unpolarized transverse virtual photons, the second term is the contribution from virtual longitudinal photons, the third term from linearly polarized transverse photons, and the last term represents the interference between the transverse and the longitudinal amplitudes.

In order to extract from the experimental data the pion form factor $F_\pi(q^2)$, *i.e.* the [$\gamma\pi\pi$] vertex function, the contribution from the 1-pion–exchange pole diagram (3.3) must be isolated. For this purpose a detailed phenomenological theory of electroproduction is needed. Many different approaches have been tried by several authors. Dispersion relations [81-86], isobaric models [87], current algebra techniques and the PCAC assumption [88] are the main ingredients in the calculations. A rather complete list of references can be found in ref. [80]. The experimental information comes mainly from the reactions

(3.7) $$e+p \rightarrow e+n+\pi^+,$$
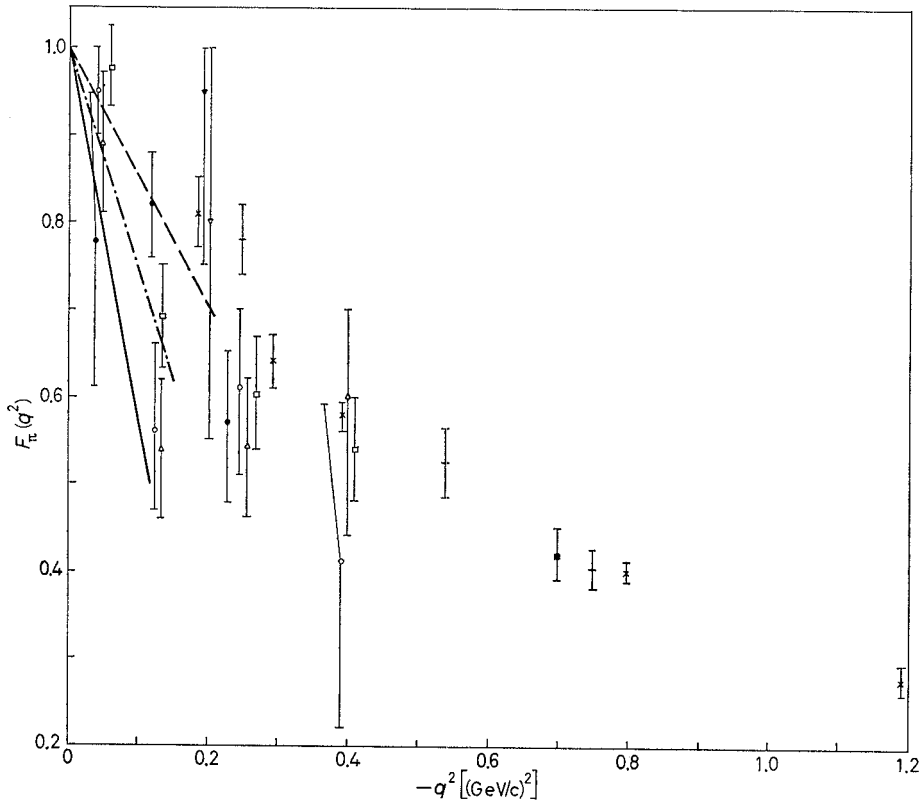
(3.8) $$e+p \rightarrow e+p+\pi^0,$$

Fig. 18. – The pion form factor $F_\pi(q^2)$ as determined from electroproduction experiments. In correspondence with each point we quote both the reference of the experiment (*e.g.*, MISTRETTA [91]) and of the model used for the analysis (*e.g.*, DOMBEY [89]). The slopes expected at $q^2 = 0$ for three different values of the pion radius $\sqrt{\langle r^2 \rangle}$ are also shown: MISTRETTA [91]: o ADLER [81], △ ZAGURY [82], □ DOMBEY [89]; AKER-LOF [93]: • ADLER [81]; SOFAIR [94]: ■ DEVENISH [85]; BROWN [95]: × BERENDS [84]; DRIVER [90]: – BERENDS [84]; AMALDI [92]: ▽ $G_{En} = 0$, ▼ $G_{En} = -G_{Mn}\tau/(1+4\tau)$, DE TOLLIS [83]; —— 0.63 $\sqrt{\langle r^2 \rangle}$ fm, —·—·— 0.81 $\sqrt{\langle r^2 \rangle}$ fm, —— 1.00 $\sqrt{\langle r^2 \rangle}$ fm.

in which one of the hadrons in the final state is detected in coincidence with the scattered electron. Reaction (3.8) is particularly relevant to a detailed phenomenological understanding of the electroproduction process since in this case the pole diagram (3.3) does not contribute; in particular the so-called « transition » form factor $G_{M\Delta}(q^2)$ to the first $\Delta(1238)$ resonance (namely the $\gamma \mathcal{N} \Delta$ vertex function) can be evaluated. This makes easier the interpretation of the more complicated data from reaction (3.7) in terms of $F_\pi(q^2)$.

In Fig. 18, a summary of the experimental information available on $F_\pi(q^2)$ is shown. In several cases the experimental data have been analysed using different theoretical models: this gives an idea of the dependence of $F_\pi(q^2)$ on the method of analysis. An additional uncertainty connected with the models, which does not show up as a difference in data points coming from different types of analysis, is the uncertainty in the foundation of hypotheses which are common to different models.

An important example of this kind has been pointed out by DOMBEY and READ [88, 96], and is related to our ignorance of the lower vertex of diagram (3.3). In current algebra techniques (PCAC), the pion field is related to the divergence of the axial weak current associated with the nucleon. This brings about the axial form factor of the nucleon G_A. Usually $G_A = G_{E_D}$ is assumed. If one however does not make this more or less arbitrary assumption (which is actually justified by chiral symmetry), at each value of q^2 a large band of possible values of $F_\pi(q^2)$ can fit the experimental data very well by properly choosing an associated value for G_A.

In Fig. 18 three slopes expected at $q^2 = 0$ for different pion radii $\sqrt{\langle r^2 \rangle}$ are also shown. It appears that at this stage $\sqrt{\langle r^2 \rangle}$ is still poorly determined by electroproduction measurements.

4. – The $\Delta(1238)$ transition form factor $G_{M\Delta}$.

When the invariant mass of the π-nucleon system in the pion electroproduction processes is near the $\Delta(1238)$ mass, the reaction is dominated by the production of the Δ-resonance.

This is especially true for reaction (3.8) to which diagram (3.3) does not contribute.

If we treat the Δ-resonance as a particle, then the cross-section can be written as [97]

$$(4.1) \qquad \frac{d\sigma}{d\Omega_e} = \sigma_{NS} \frac{|q^2|}{4M^2} \frac{1}{\varepsilon} \times \left[|G_{M\Delta}|^2 + |G_{E\Delta}|^2 + 2 \frac{|q^2|}{|p|^2} \varepsilon |G_{C\Delta}|^2 \right],$$

where

$$\sigma_{NS} = \frac{\alpha \cos^2(\theta_e/2)}{4E^2 \sin^4(\theta_e/2)[1 + 2(E/M)\sin^2(\theta_e/2)]},$$

$E =$ energy of the incident electron and $p =$ three-momentum of the virtual photon in the Δ rest frame.

$G_{M\Delta}(q^2)$, $G_{E\Delta}(q^2)$ and $G_{C\Delta}(q^2)$ correspond to the M_1, E_2 and C_2 (Coulomb octupole) which can excite the $J = \frac{1}{2}^+$ to $J = \frac{3}{2}^+$ transition. Both on theoretical [98] and experimental [99] grounds $G_{E\Delta}$ and $G_{C\Delta}$ are expected to be much smaller than $G_{M\Delta}$, and are usually neglected in the analysis of experimental data. In this case eq. (4.1) assumes a very simple form allowing one in principle to measure quite easily $|G_{M\Delta}|^2$. Notice in particular that, according to (4.1), $|G_{M\Delta}|^2$ can be measured in experiments in which only the scattered electron is detected.

In practice, however, there are problems connected with the contribution of nonresonant background and with the large width of the Δ-resonance. A detailed phenomenological theory of electroproduction is again needed, although the situation is much simpler than for the determination of the pion form factor F_π.

The information from coincidence experiments is in practice needed for the multipole analysis of the electroproduction data, in which case $G_{M\Delta}$ is simply connected to the q^2 behaviour of the M_1 multipole contribution [97].
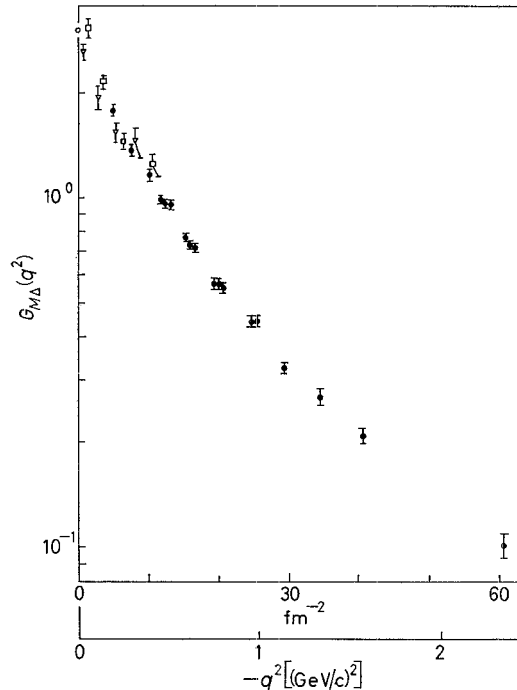

Fig. 19. – The transition form factor $G_{M\Delta}$ to the first nucleon isobar $\Delta(1238)$ as determined from electroproduction experiments. The point at $q^2 = 0$ comes from photoproduction experiments, see ref. [97]: o photoproduction, • BARTEL *et al.* [100], □ IMRIE *et al.* [101], ▵ ASH *et al.* [97].
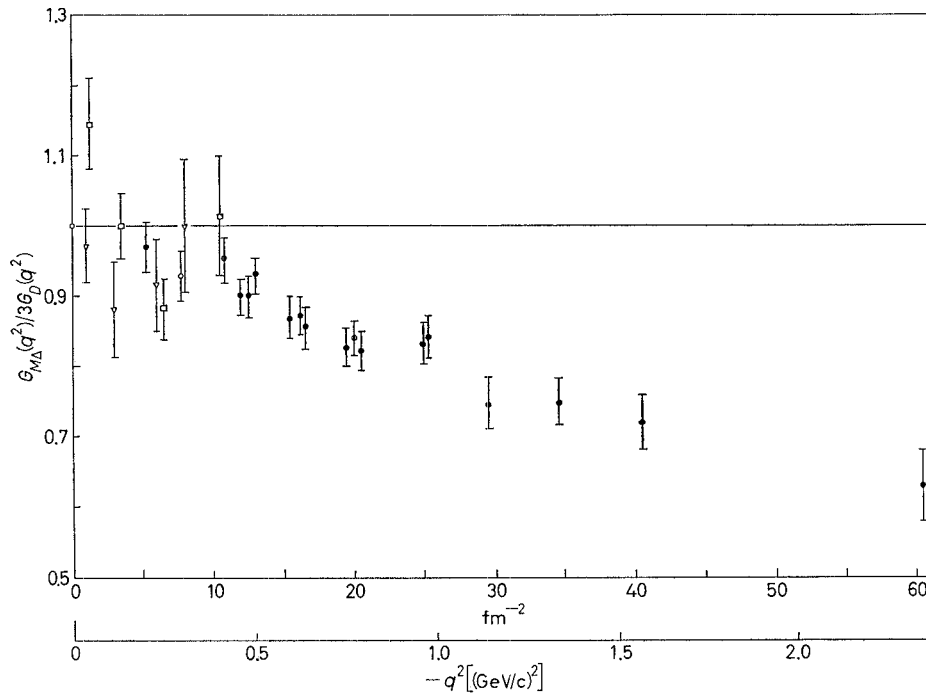
Fig. 20. - The ratio $G_{M\Delta}/3G_D$ of the transition form factor $G_{M\Delta}$ for the first nucleon isobar to three times the dipole fit is plotted as a function of $-q^2$: □ ref. [101], ∇ ref. [97], • ref. [100], o photoproduction, ref. [97].

The experimental situation for $G_{M\Delta}$ is summarized in Fig. 19. The value at $q^2 = 0$ determined from photoproduction data is 3.00 ± 0.01 [97]. With increasing $-q^2$, $G_{M\Delta}$ drops more rapidly than the proton form factors, as shown in Fig. 20, where the ratio of $G_{M\Delta}$ to three times the dipole fit is presented.

5. - Deep inelastic electron-nucleon scattering.

An extensive systematic investigation of electron-nucleon inelastic scattering has been carried out during the last few years at SLAC by a SLAC-MIT collaboration [102]. These data include measurements at large values of $|q^2|$ and correspondingly large energy transfers to the nucleon (deep inelastic region) and are of the inclusive type, i.e. only the scattered electron was detected. Some coincidence measurements—with the detection of part of final-state hadronic products—have also been performed [103], however these investigations are only now beginning and the results cannot yet properly be included in a review paper.

Also, since the inclusive-type data come essentially from one single experiment [104], we report here only a short summary of the results.

The cross-section for inclusive electron-nucleon inelastic scattering can be written as

$$(5.1) \qquad \frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \frac{\alpha}{4\pi^2}\,\frac{W^2 - M^2}{2M|q^2|}\,\frac{E'}{E}\,\frac{2}{1-\varepsilon}\,[\sigma_t + \varepsilon\sigma_s]\,.$$

The symbols have the same meaning as in (3.5), (3.6); W, the missing mass of the unobserved hadronic final state, is given by $W^2 = 2M(E - E') + M^2 - q^2$. The cross-sections for transverse and longitudinal polarized virtual photons, σ_t and σ_s, are functions of the invariants q^2 and W (or of q^2 and $\nu = E - E'$). As $q^2 \to 0$, $\sigma_s \to 0$ and $\sigma_t(q^2, \nu) \to \sigma_\gamma(\nu)$, where $\sigma_\gamma(\nu)$ is the photoabsorbtion cross-section for real photons of energy ν.

Alternatively, $\mathrm{d}^2\sigma/\mathrm{d}\Omega\,\mathrm{d}E'$ can be written in terms of the « structure functions » W_1 and W_2:

$$(5.2) \qquad \frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \frac{e^4}{4E^2}\,\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\,[W_2 + 2W_1\,\mathrm{tg}^2(\theta/2)]\,.$$

W_1 and W_2 are related to σ_t and σ_s by

$$(5.3) \qquad \begin{cases} W_1 = K\sigma_t\,, \\[2mm] W_2 = K\,\dfrac{|q^2|}{|q^2| + \nu^2}\,(\sigma_t + \sigma_s)\,, \\[2mm] K = \dfrac{W^2 - M^2}{8\pi^2 M\alpha} \end{cases}$$

and to the experimental cross-section by

$$(5.4) \qquad \begin{cases} W_1 = \left[\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right]_{\mathrm{exp}}\left(\dfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{-1}_{\mathrm{Mott}}\left((1+R)\,\dfrac{|q^2|}{|q^2| + \nu^2} + 2\,\mathrm{tg}^2(\theta/2)\right)^{-1}, \\[4mm] W_2 = \left[\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right]_{\mathrm{exp}}\left(\dfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{-1}_{\mathrm{Mott}}\left(1 + 2\left(\dfrac{1}{1+R}\right)\dfrac{|q^2| + \nu^2}{|q^2|}\,\mathrm{tg}^2(\theta/2)\right)^{-1}, \end{cases} \qquad R = \frac{\sigma_s}{\sigma_t}.$$

For small values of $\mathrm{tg}^2(\theta/2)$, W_2 depends much less critically than W_1 on R.

The experimental findings, which are of extreme interest and have stimulated a quantity of theoretical speculations, can be summarized in the following points:

a) R, although rather poorly measured (see Fig. 21), appears to be quite small and is compatible with being constant over the full q^2 and ν range explored. The average value is $R = 0.18 \pm 0.10$. This fact is often inter-
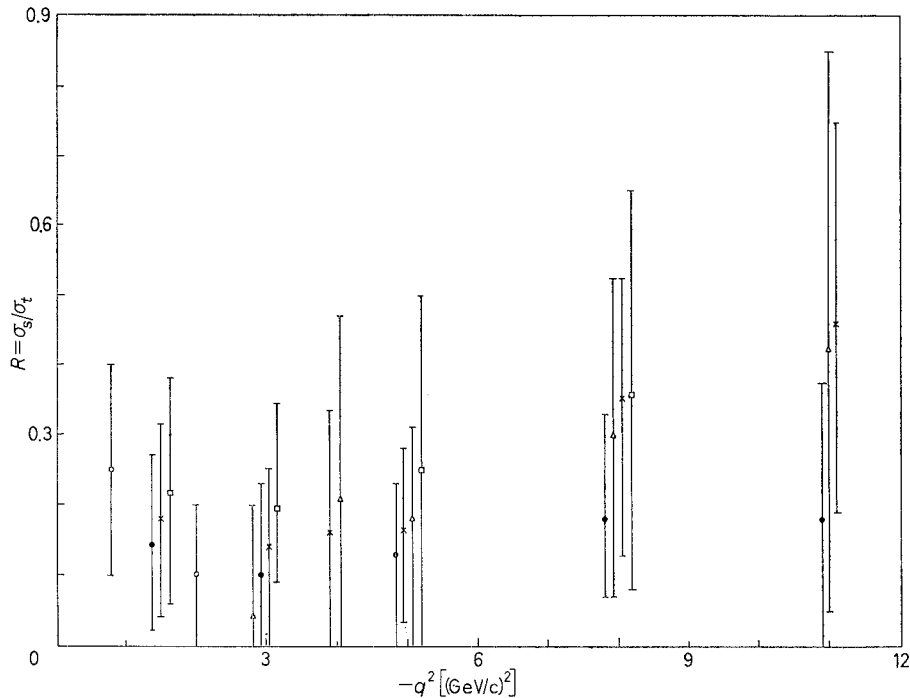
Fig. 21. - The ratio σ_s/σ_t (σ_s and σ_t are defined as in (5.1)) plotted as a function of $-q^2$, for different values of W. W is the invariant mass of the unobserved hadronic final state in GeV: o $W = 1.9 \div 3.1$, ref. [104]; • 2.0, ref. [102]; ▵ 2.5, ref. [102]; × 3.0, ref. [102]; □ 3.3÷3.5, ref. [102].

preted as an indication that the contribution to the total cross-section from the scattering on virtual scalar (or pseudoscalar) mesons is small. Actually in the infinite-momentum limit when all the particles involved in the reaction travel along the beam direction (a situation which is *not* necessarily satisfied in the experiment) helicity cannot be conserved if a longitudinal photon is absorbed by a boson.

b) When W is well above the resonance region, $d^2\sigma/d\Omega dE'$ has, at fixed W, a rather weak dependence on q^2. This is shown in Fig. 22, where

$$\Gamma = \frac{d^2\sigma}{d\Omega\,dE'} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}$$

is plotted as a function of $-q^2$ for different values of W. With increasing W, Γ becomes flatter. This is one of the facts which has suggested the idea of the proton built up of pointlike constituents [105] (partons) which, due to the above observation *a*), should be mostly fermions.
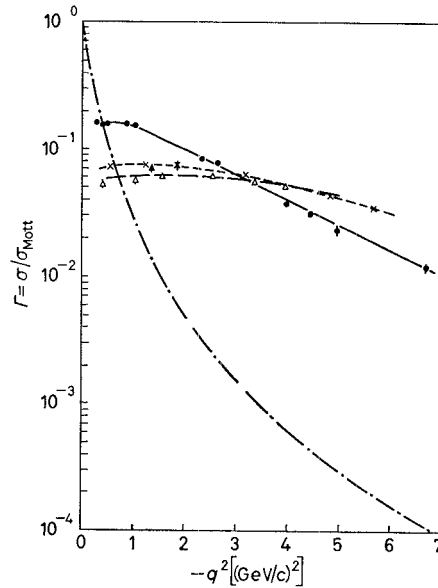
Fig. 22. – The ratio of the experimental cross-section to σ_{Mott} in deep inelastic electron-proton scattering is plotted as a function of $-q^2$ for different values of W. For comparison, the same ratio is also shown for elastic electron-proton scattering: ● ———— $W=2$ GeV; × – – – $W=3$ GeV, △ – – – – $W=3.5$ GeV, —·—·— elastic scattering.

c) νW_2, determined in the hypothesis that R is constant and equal to 0.18 ± 0.10 (W_2 however depends rather weakly on R, so that this hypothesis is not very drastic), appears to depend on the ratio $\omega = 2M\nu/q^2$ rather than on q^2 and ν separately. The experimental situation is presented in Fig. 23, where νW_2 is shown as a function of $-q^2$ for various values of ω. At $\omega=4$, νW_2 is clearly independent of q^2 within the errors (see also Fig. 24). At different values of ω, the data suggest that a plateau is being reached at higher values of $|q^2|$. This behaviour was first suggested by BJORKEN [106] («scaling law») to occur in the so-called Bjorken limit $|q^2| \to \infty$, $\nu \to \infty$ (ω constant), and the fact that this limit appears to be reached at relatively small values of $|q^2|$ and ν was considered surprising. Other variables (e.g. $\omega' = \omega + M^2/q^2 = 1 + W^2/q^2$) in terms of which the scaling behaviour is reached even earlier have also been proposed [107].

From an intuitive point of view, the scaling law of the structure functions can be understood as follows. Consider a function of the type (one-dimensional light-cone singularity)

(5.5) $$f(x)\delta(x-t),$$

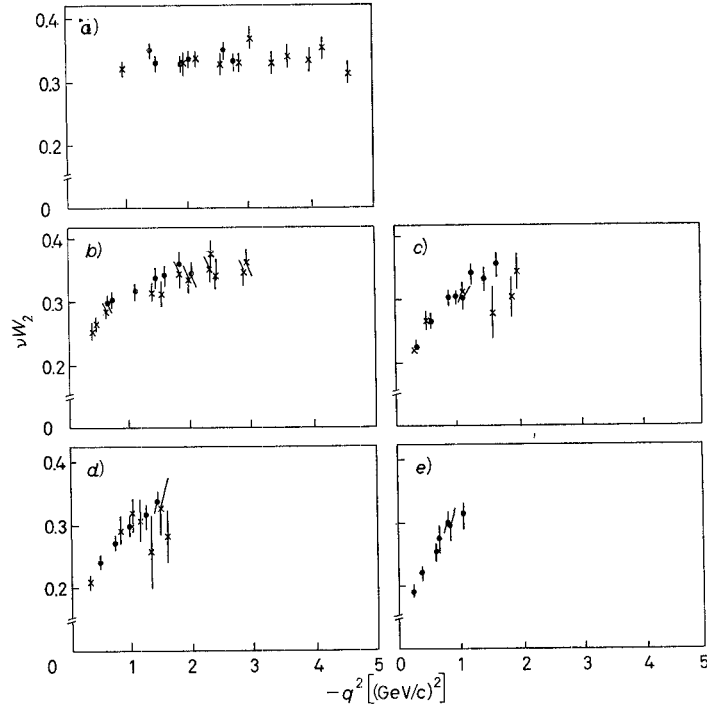where x is the direction of the incoming virtual photon of momentum $q_\mu \simeq$

Fig. 23. – The structure function νW_2 is plotted as a function of $-q^2$ for different values of $\omega = 2M\nu/q^2$. $R = 0.18$ was assumed, see ref. [102]: $\times\ = 10°$, $\bullet\ = 6°$; a) $4 \leqslant \omega \leqslant 6$, b) $8 \leqslant \omega \leqslant 12$, c) $12 \leqslant \omega \leqslant 18$, d) $16 \leqslant \omega \leqslant 24$, e) $24 \leqslant \omega \leqslant 36$.

$\simeq (\nu + M/\omega, 0, 0, \nu)$. The Fourier transform of (5.5) is

$$\int f(x)\,\delta(x-t)\exp\left[i\nu t - i\left(\nu + \frac{M}{\omega}\right)x\right]\mathrm{d}x\mathrm{d}t = \int f(x)\exp\left[-i\,\frac{M}{\omega}\,x\right]\mathrm{d}x = F(\omega)\ .$$

Therefore a function of the type (5.5) has a scaling Fourier transform. BJORKEN first pointed out that the e.m. current commutators, whose Fourier
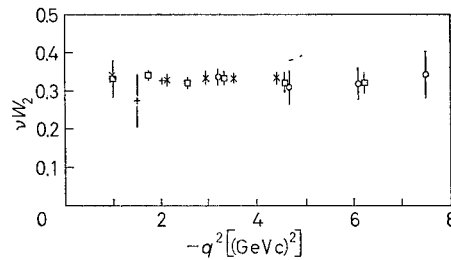

Fig. 24. – The structure function νW_2 is plotted as a function of $-q^2$ for $\omega = 4$, see ref. [102]. $R = \sigma_s/\sigma_t = 0.18$ was assumed: $+$ 6°, $\times$ 10°, □ 18°, ○ 26°.

transforms are closely connected with the structure functions, must be dominated, in the infinite-momentum (Bjorken) limit by singularities on the light-cone $(x_\mu x^\mu = 0)$. The experimental result on the scaling law was therefore surprising mainly because scaling is reached at relatively low energy.

The main trend of the present theoretical work on scaling [108] is essentially along two lines: a) techniques to evaluate the light-cone singularities of the e.m. current commutators and b) parton models, in which the proton structure assumes quite naturally the form of objects moving in the infinite-momentum limit with the velocity of light (the simplest form of the charge distribution of the proton being $\sum Q_i \delta(x_i - t)$ with Q_i the parton charges).

Both these approaches have a nonnegligible predictive power: in the first case the transformation properties of the currents under different groups (Poincaré group, SU_2, SU_3 etc.) can be used to correlate cross-sections and to predict sum rules; in the second case many predictions can be made once the partons are identified, e.g., with the quarks.

Experimentally, some data on the comparison [102] of deep inelastic scattering on protons and neutrons are also available.

Finally, the results of ref. [36] (see Sect. 1) allow one to conclude that deep-inelastic-scattering data of muons on proton are consistent with the electron data.
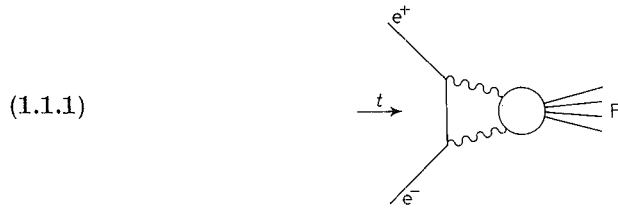
II. Timelike region.

The investigation of the e.m. structure of hadrons in the timelike region has only recently begun with the operation of e^+e^- storage rings and has been pursued up to now only in few and rather small laboratories, essentially at Orsay, Novosibirsk and Frascati.

The experimental information is very important for an understanding of the physics of elementary particles, but is still quite meagre. This is due not only to the fact that this kind of physics has a short history and a rather limited geography, but also to the fact that hadrons are produced in e^+e^- interactions with very small cross-sections (in the range of nanobarns—10^{-33} cm²—when the total energy is above 1 GeV). In addition, in storage rings beams of $\sim 10^{11}$ particles are sent one against another, to be compared with the use of beams against targets in the conventional machines. The fact that the beam-beam impact occurs $\sim 10^7$ times per second does not compensate the difference in intensity. For this reason, very small beam dimensions are generally used in storage rings to obtain a high target density, which, while making the machine operation quite delicate and difficult, brings the counting rate just to the limit of experimental feasibility. Counting rates of a few events per day, or even per week, are not unusual in typical experiments.

1. – Validity of the one-photon–exchange, pointlike leptons and $1/q^2$ photon propagator hypotheses in the timelike region.

1˙1. *One-photon exchange*. – In the timelike region the smallness of the usual two-photon exchange contribution from the graph

(1.1.1)

has not been experimentally tested, and we have therefore to trust the calculations based on the absence of strong enhancement mechanisms. It is worth noticing that in the timelike region the number of exchanged virtual photons is directly related to the charge conjugation eigenvalue C of the final state. If the charge of the produced particles is not recognized (as was the case for all the experiments performed up to now with e^+e^- storage rings), then the interference between even and odd states of C cancels, so that the two-photon-exchange process can contribute only to an α^4 order.

There is however an additional two-photon contribution which is expected to be quite important in e^+e^- interactions, namely

(1.1.2)

(1.1.3)
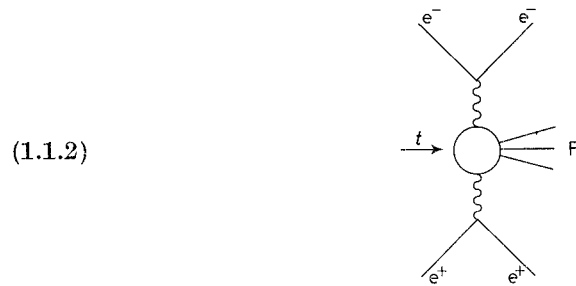
The additional α^2 factor appearing, *e.g.*, in diagram (1.1.3) is in fact expected to be at least partially compensated by the fact that this process can involve much lower momentum transfers than the usual one-photon–annihilation graph.

The contribution from the above diagrams might give rise to an important background in the e^+e^- experiments. However, they deserve also some interest by themselves since they can provide useful information on the coupling of hadrons with a two-photon system when F of diagram (1.1.2) is of hadronic nature (F could be, for instance, an η or an η' particle). This will be particularly true with higher-energy storage rings, since the cross-section is expected to
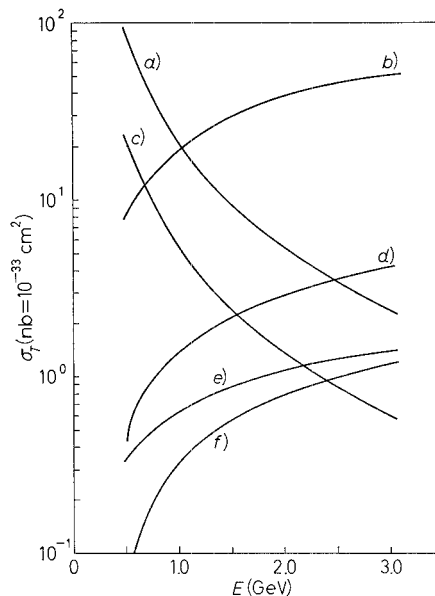

Fig. 25. – The cross-sections for different e^+e^- induced processes via two-photon interactions as calculated by BRODSKY *et al.*, ref. [109]. We see that as the energy E of the electron and positron beams is above ~ 1.5 GeV, the total cross-sections for two-photon interaction processes are expected to overtake the cross-sections for the usual $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \pi^+\pi^-$ annihilation processes: *a*) $e^+e^- \to \mu^+\mu^-$, *b*) $ee \to$ $\to ee\mu^+\mu^-$ (equivalent photon), *c*) $e^+e^- \to \pi^+\pi^-$ (pointlike), *d*) $ee \to ee\pi^+\pi^-$ (equivalent photon), *e*) $ee \to ee\pi^0$ (exact), *f*) $ee \to ee\eta$ (exact).

increase logarithmically with increasing energy, while the usual annihilation cross-sections are expected to decrease with increasing energy E (as $1/E^2$ for production of pointlike particle pairs, see Fig. 25). Fortunately, the processes (1.1.2) can be separated since in the final state the incident electron and positron survive. This separation is often possible also without detecting the scattered electrons, since the angular and energy distribution for these two-photon reactions is quite peculiar. For instance in the reaction $e^+e^- \to e^+e^-e^+e^-$ two

leptons are expected to be emitted in a narrow cone along the beam directions, while the other two, when emitted at large angles, should have a $\Delta\phi$ distribution strongly peaked around $\Delta\phi = 0$ [109] ($\Delta\phi$ is the angle between the planes defined by the beam axis and the two emitted particles).

Experimentally, the $\Delta\phi$ distribution for the reaction $e^+e^- \to e^+e^-e^+e^-$ has been investigated at Novosibirsk [110]. The results are shown in Fig. 26, and

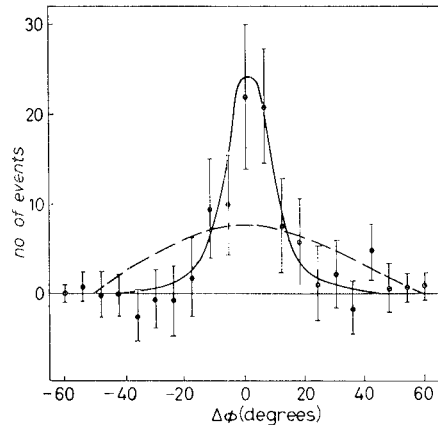Fig. 26. – Distribution of events from reaction $e^+e^- \to e^+e^-e^+e^-$ as a function of the angle $\Delta\phi$ defined as the angle between the two planes which contain each of the two large-angle emitted electrons and the beams axis. The data were collected at three different values of the total energy ($2E = 1020, 1180, 1340$ MeV). The full line is the theoretical calculation of BAIER and FADIN [109]. The dashed line corresponds to a uniform $\Delta\phi$ distribution weighted with the detection efficiency of the apparatus.

are compared with the theoretical calculation of BAIER and FADIN [109]. A measurement of this reaction has also been performed in Frascati with Adone, where also a few candidates from the reaction $e^+e^- \to e^+e^-\mu^+\mu^-$ have been observed. In this experiment counters have been placed near the machine vacuum chamber in order to detect the electrons (and/or positrons) emitted near the beam directions, using as spectrometers the magnets of the machine itself. Some additional information on the kinematics of the reaction can thus be obtained. In Fig. 27 the distribution of the 29 observed events as a function of β, the centre-of-mass velocity of the leptons emitted at large angles, is shown. The sign of β is defined as negative when β has the same orientation as the single detected electron along the beam direction (no event in which both the small-angle leptons were detected was observed). A comparison with theory is also given in Fig. 27. Although the yield of events with $\beta < 0$ depends critically on the lower experimental cut in the energy of the detected particles, it appears that there is a large contribution of events with a kinematical feature not foreseen by the standard theoretical calculations [109],
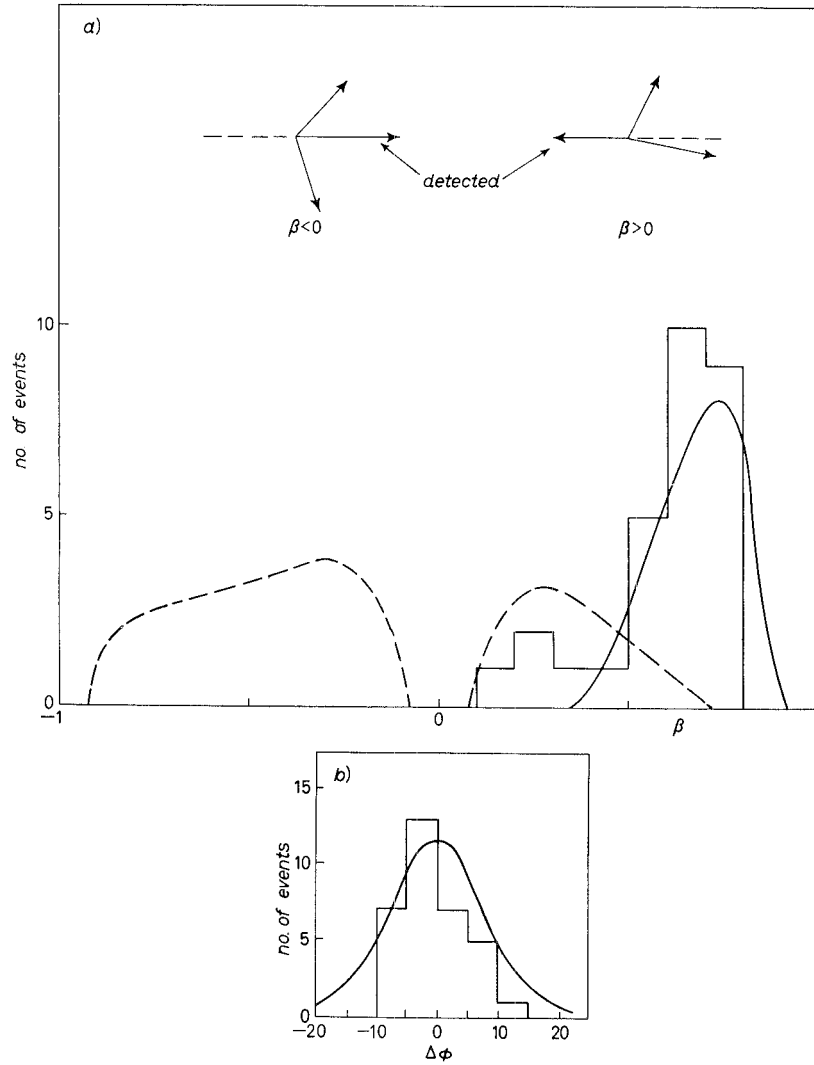
Fig. 27. – The results of the Frascati « γγ-group » on the reaction e⁺e⁻→ e⁺e⁻e⁺e⁻ (ref. [111]): a) Distribution of the events as a function of β, the c.m. velocity of the pair of leptons emitted at a large angle. The convention used for the sign of β is specified in the upper part of the drawing. The results are compared with the theoretical calculations: ———— PARISI [112], ——— — Brodsky et al. [109]. b) Distribution of the events as a function of $\Delta\phi$, compared with the calculation of BAIER and FADIN [109].

whose approximations are based on the hypothesis of the dominance of the kinematical configuration

(1.1.4)

$$e^- \quad\quad\quad e^- \text{ forward}$$
$$e^+ \text{ wide angle}$$
$$e^- \text{ wide angle}$$
$$e^+ \quad\quad\quad e^+ \text{ forward}$$

PARISI [112] has evaluated the contribution from the kinematical configuration

(1.1.5)

$$e^- \quad\quad\quad e^- \text{ forward}$$
$$e^+ \text{ forward}$$
$$e^- \text{ wide angle}$$
$$e^+ \quad\quad\quad e^+ \text{ wide angle}$$

which appears to account for most of the observed events.

Preliminary data from a second experiment [113], in which a lower-energy cut is set on the observed wide-angle electrons, show a contribution of events from the configuration (1.1.4) but also a nonnegligible contribution from configuration (1.1.5). Better theoretical calculations would therefore be welcome. Also the possibility of some higher-energy storage rings to be operated both with e^+e^- and with e^-e^- appears as a convenient facility to study, in the e^-e^- mode of operation, the two-photon interactions without contamination from the annihilation channels.

In any case, there is good experimental evidence that the data reported in the next Sections as hadronic events from e^+e^- interactions receive, if at all, only a small contamination from the two-photon interaction graph (1.1.2) (see Sect. **5**).

1˙2. *Pointlike leptons and* $1/q^2$ *photon propagator.* – Experimental tests of these hypotheses can be obtained by measuring the $\mu^+\mu^-$ or e^+e^- pair production cross-section in e^+e^- interactions. However, as we already mentioned in in Sect. **1** (Part I), the cross-section for the reaction $e^+e^- \rightarrow e^+e^-$ is dominated by the scattering rather than by the annihilation graph. Tests of the pointlike electrons in the timelike region will be possible only by measuring the cross-section for the process $e^+e^- \rightarrow e^+e^-$ with recognition of the charge of the produced electron pair, in which case the separation of the annihilation contribution will be possible. The possibility that the electron has a complex form factor F_e, even if $|F_e|^2 = 1$, can also be tested in this reaction [114].

At present, only the annihilation process $e^+e^- \rightarrow \mu^+\mu^-$ has been experimentally investigated. A comparison of the electron and muon form factors

in the timelike region is then available only at $q^2 \simeq 0.5$ (GeV/c)^2 through the measurement of the $\rho \to \mu^+\mu^-$ and $\rho \to e^+e^-$ decay rates: the result is [115]

$$\frac{\Gamma(\rho \to e^+e^-)}{\Gamma(\rho \to \mu^+\mu^-)} = 0.97 \pm 0.17 = \frac{|F_e|^2}{|F_\mu|^2}.$$

The experimental situation on the reaction $e^+e^- \to \mu^+\mu^-$ is presented in Fig. 28. The results [116-118] are expressed in terms of the ratio

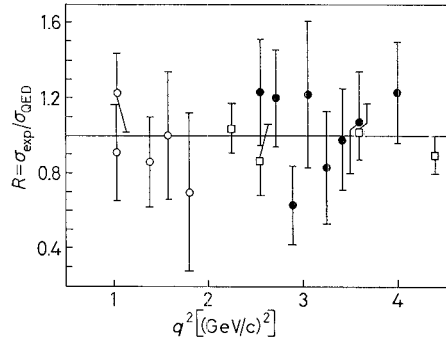$$R = \frac{\sigma_{exp}}{\sigma_{QED}} = |F_e(s)|^2 \cdot |F_\mu(s)|^2 |M(s)|^2.$$

Fig. 28. – Results on the reaction $e^+e^- \to \mu^+\mu^-$, expressed in terms of the ratio $R = \sigma_{exp}/\sigma_{QED}$. The machine luminosity is monitored in this case with the wide-angle e^+e^- elastic scattering: o Novosibirsk ref. [116], □ Frascati ($\mu\pi$) ref. [117], ● Frascati (BCF) ref. [118].

The agreement is good within the large experimental errors (typically $(15 \div 20)$ %) up to momentum transfers as high as

$$s = E_+ + E_- = q^2 = 4.4 \text{ (GeV/c)}^2.$$

This kind of data is often parametrized in terms of a cut-off parameter Λ^2 $(R = (1 - q^2/\Lambda^2)^2)$ by assigning the form $1 - q^2/\Lambda^2$ to either F_e or F_μ or to the photon propagator modification M. This parametrization is arbitrary, and sometimes gives rise to serious theoretical difficulties (when assigned, for instance, to M or to a lepton propagator modification [23]). However it is usually justified with the need of comparing different experiments. This attitude is misleading in our opinion, since it invites one to consider a rough experiment at high energy equivalent to a good-precision low-energy experiment: in fact in the cut-off philosophy deviations from QED are expected to increase with increasing energy. This might very well be wrong. Actually, a break-down of QED *is* expected due to vacuum polarization effects originated by hadrons

coupled to the virtual photon: this kind of break-down is not expected to be more important at higher energy. This is demonstrated experimentally in Fig. 29 where we show the results of another experiment on the reaction $e^+e^- \to \mu^+\mu^-$ [119] performed at Orsay at a lower energy than the experiments quoted in Fig. 28. The energy region explored is around the ϕ mass, and a vacuum polarization effect shows up, although at the limit of the experimental errors.
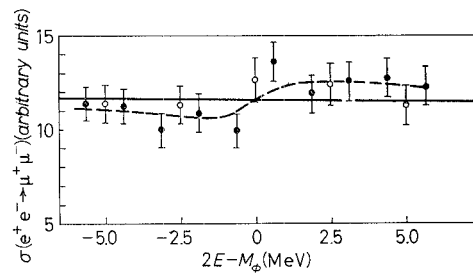

Fig. 29. – Results of the Orsay group showing the vacuum polarization effect in the reaction $e^+e^+ \to \mu^+\mu^-$. Open circles refer to events collected at five fixed energies E. Black circles refer to events collected with a continuous cycling of the beam's energy in a range of total c.m. energy $2E$ of ± 6 MeV around the ϕ-meson mass. The full line corresponds to no polarization effect; the dashed line represents a best fit to the data with vacuum polarization effect included: $B = \Gamma_{\phi \to e^+e^-} \cdot \Gamma_{\phi \to \mu^+\mu^-} / \Gamma_\phi$: $- - - B = = 2.6 \cdot 10^{-4}$, $\chi^2 = 7.1$; $\underline{\quad\quad} B = 0$, $\chi^2 = 15$.

The relevant point is the comparison between experiment and theory; and the pertinent parameter, at whatever energy, is the precision of the experiment rather than the cut-off parameter Λ.

2. – Proton form factors.

A first measurement of the cross-section for the reaction

$$(2.1) \qquad\qquad\qquad e^+e^- \to p\bar{p}$$

has been recently performed at Frascati by the Naples group [120]. 21 ± 5 events from the reaction (2.1) have been observed at $q^2 = 4.4$ (GeV/c)^2. The separation of background from reaction (2.1) is achieved by means of energy E and dE/dx measurements in thick scintillation counters, time-of-flight determination and collinearity of the observed tracks as measured in optical spark chambers. The sample of events is then quite clean in spite of the very low counting rate (~ 1 event/(2 ÷ 4) days). 14 of the events show the antiproton annihilation star, as expected on the basis of the known detection efficiency of the apparatus.

To determine from the observed number of events the total cross-section, extrapolation over the full solid angle of the counting rate is needed. (The

experimental apparatus covers ~ 0.6 of the total 4π solid angle, although the detection efficiency is equal to 1 only around $90°$.) For this purpose, the angular distribution of the events must be known.

The equivalent of the Rosenbluth formula in the timelike region is [121]

$$(2.2) \quad \begin{cases} \dfrac{d\sigma}{d\Omega} = \dfrac{\pi}{8}\alpha^2\lambdabar^2\beta\left(|G_M|^2(1+\cos^2\theta) - \dfrac{1}{\tau}|G_E|^2\sin^2\theta\right) \\ \left(\sigma \simeq 10^{-32}\left(|G_M|^2 - \dfrac{1}{2\tau}|G_E|^2\right)\mathrm{cm}^2\right), \end{cases} \qquad \tau = \dfrac{-q^2}{4M^2};$$

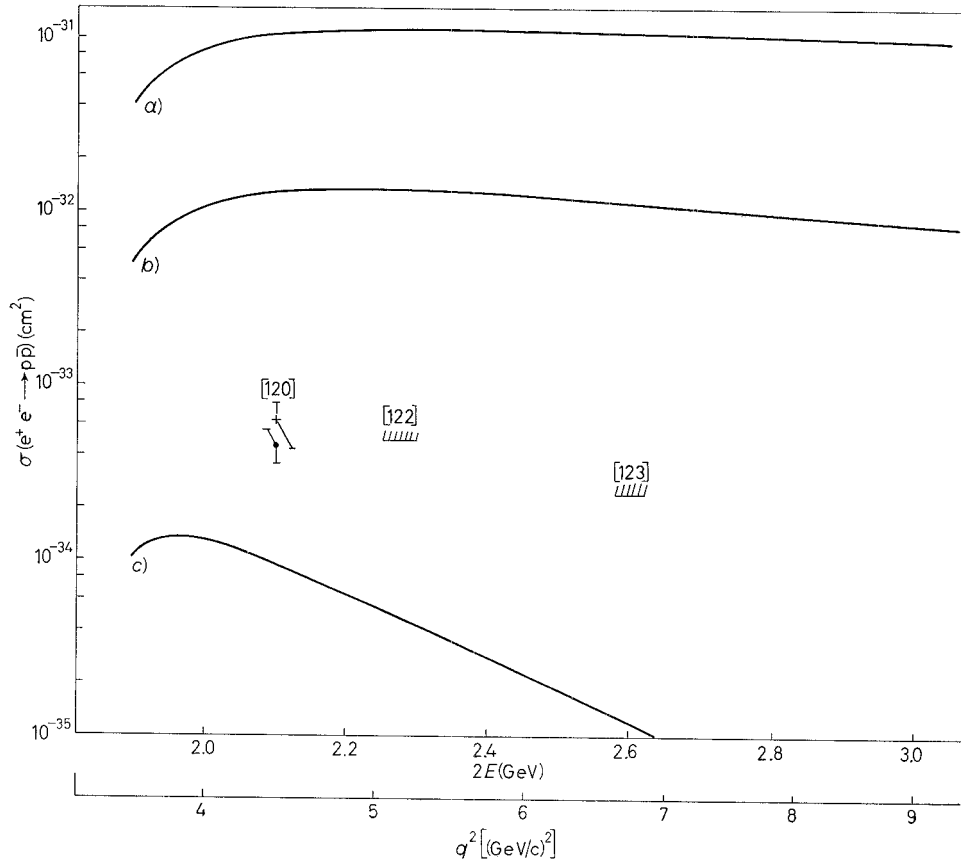$\beta, \theta = $ velocity and angle of emission of the proton.

Fig. 30. – The result of the Naples group (ref. [120]) on the reaction $e^+e^- \to p\bar{p}$. The black circle $\bullet$ represent the cross-section as determined by using all the detected events; the cross $+$ represents the cross-section as determined by using only the events in which the antiproton annihilation star is detected. Previously available upper limits from the reaction $p\bar{p} \to e^+e^-$ are also shown (ref. [122, 123]). The full lines represent theoretical predictions for some particular values of the form factors: a) $F_1 = 1$, $F_2 = 1.79$; b) $G_E = G_M = 1$; c) $G_E = G_M = 1/(1 + q^2/0.71)^2$.

The angular distribution depends on $|G_M|^2/|G_E|^2$ and some hypothesis is needed, unless the experiment covers a wide enough θ region so that $|G_E|^2$ and $|G_M|^2$ can be separately determined; this is not the case for the Naples experiment. However this experiment has been performed near threshold where one would expect the angular distribution to be isotropic, i.e. $|G_E|^2 = |G_M|^2$.

In fact, unless $G_E = G_M$ at threshold $(\tau = -q^2/4M^2 = -1)$, F_{1p} and F_{2p} (see Subsect. 2·2, Part I) become infinite, producing an unwanted divergence in the e.m. current of the proton.

With the hypothesis of an isotropic production distribution the authors obtain a value of $\sigma_{e^+e^- \to p\bar{p}} = (4.6 \pm 1) \cdot 10^{-34} \text{ cm}^2 \ (\sigma_{e^+e^- \to p\bar{p}} = (6.2 \pm 1.8) \cdot 10^{-34} \text{ cm}^2$ using only the events with the detected annihilation star). In Fig. 30 these values are compared with previously available upper limits from the reaction $\bar{p} + p \to e^+ + e^-$ [122, 123] and with calculations for a few particular choices for G_E and G_M.

In the above hypothesis $|G_E| = |G_M|$, the measured value of the cross-section corresponds to $|G_E| = |G_M| = 0.19 \pm 0.03$.

This experiment is obviously only a first approach to a new field of investigation. New storage ring experiments permitting the determination of $|G_E|$ and $|G_M|$ separately (and also the form factors of unstable baryons) are planned both at Frascati and Stanford.

3. – The pion form factor.

The pion form factor is simply related in the timelike region to the cross-section for the reaction

$$(3.1) \qquad\qquad e^+e^- \to \pi^+\pi^-$$

by the relations [121]

$$(3.2) \qquad \begin{cases} \dfrac{d\sigma_{\pi^+\pi^-}}{d\cos\theta} = \dfrac{\pi\alpha^2}{4} \dfrac{\beta_\pi^3}{q^2}|F_\pi(q^2)|^2 \sin^2\theta\,, \\[3mm] \sigma_{\pi^+\pi^-} = \left(\dfrac{\pi\alpha^2}{3}\right)\dfrac{\beta_\pi^3}{q^2}|F_\pi(q^2)|^2. \end{cases}$$

A measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ therefore permits one to measure $|F_\pi(q^2)|^2$. Notice that a $\pi^+\pi^-$ system with $l = 1$ (as it must be in the one-photon-exchange hypothesis) must have $T = 1$ so that $F_\pi(q^2)$ is related to the coupling of isovector photons with hadrons. Actually, F_π coincides exactly with F_V below threshold for 4π production and in practice below $q^2 = 1 \ (\text{GeV}/c)^2$. Phase-space factors are expected to depress strongly the $e^+e^- \to 4\pi$ channels below $q^2 = 1 \ (\text{GeV}/c)^2$, as confirmed by the first experimental measurements (see Sect. **5**).

Measurements of $\sigma(e^+e^- \to \pi^+\pi^-)$ at $q^2 \lesssim 1$ (GeV/c)^2 have been quite extensively performed during the last ~ 5 years at Novosibirsk [124] and Orsay [125]. The phenomenology is dominated by the production of the vector meson ρ, the foreseen interference term with the ω contribution (via the electromagnetic decay mode $\omega \to \pi^+\pi^-$) having also been observed. The results are summarized in Fig. 31, showing $|F_\pi|^2$ as a function of q^2.

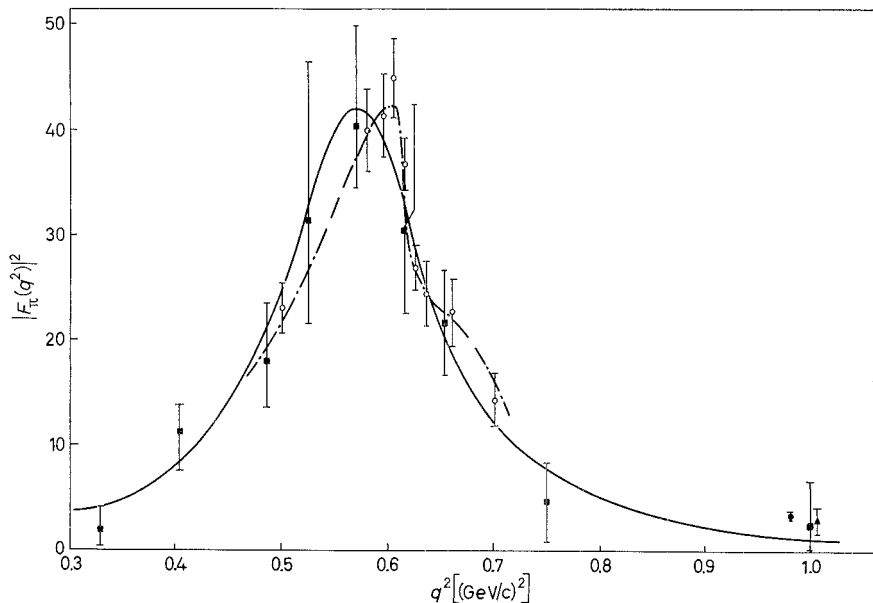Fig. 31. – $|F_\pi(q^2)|^2$ as determined by the Novosibirsk and Orsay groups through measurements of the reaction $e^+e^- \to \pi^+\pi^-$ at $q^2 \lesssim 1$ (GeV/c)^2. The full line is the Breit-Wigner best fit to the Novosibirsk points. The dash-dotted line is the best fit to the Orsay points using a Gounaris-Sakurai formula and taking into account the ω contribution via the decay channel $\omega \to \pi^+\pi^-$. ——— ■ Novosibirsk 1, ref. [124]; —·—·— ○ Orsay 1, ref. [125]; ▲ Novosibirsk 2, ref. [124]; ● Orsay 2, ref. [125].

The full line is the Breit-Wigner best fit to the Novosibirsk points; the dashed line is the best fit to the Orsay points including the $\omega \to \pi^+\pi^-$ contribution. On the top of the ρ peak the cross-section is ~ 1.5 μb. In terms of ρ parameters, the results can be summarized as follows.

	Orsay	Novosibirsk
m_ρ (MeV)	775.4 ± 7.3	754 ± 9
Γ_ρ (MeV)	149 ± 23	105 ± 20
$\Gamma_{\rho \to e^+e^-}/\Gamma_{\rho \, total}$	$(4.0 \pm 0.5) \cdot 10^{-5}$	$(5 \pm 1) \cdot 10^{-5}$
$\Gamma_{\rho \to e^+e^-}$ (keV)	6.1 ± 0.7	5.2 ± 0.5

The difference in the parameters obtained at Novosibirsk appears to be essentially due to the fact that the ω contribution is not taken into account. Actually, the best fit to the Orsay points (which provides the additional information $(\Gamma_{\omega\to2\pi}/\Gamma_{\omega\to\text{total}})^{\frac{1}{2}} = 0.2 \pm 0.05$, the phase angle between the ω and ρ amplitudes being $\phi_{\omega\rho} = (87.05 \pm 15.04)°$) appears to fit perfectly also the Novosibirsk points.

The fact that the cross-section for $\pi^+\pi^-$ production can be accounted for by the ρ and ω contributions alone (actually the ρ, ω and ϕ account for all the hadronic production below 1 GeV) was a strong support in favour of the « vector dominance » hypothesis [126]. The data above 1 GeV, however, give
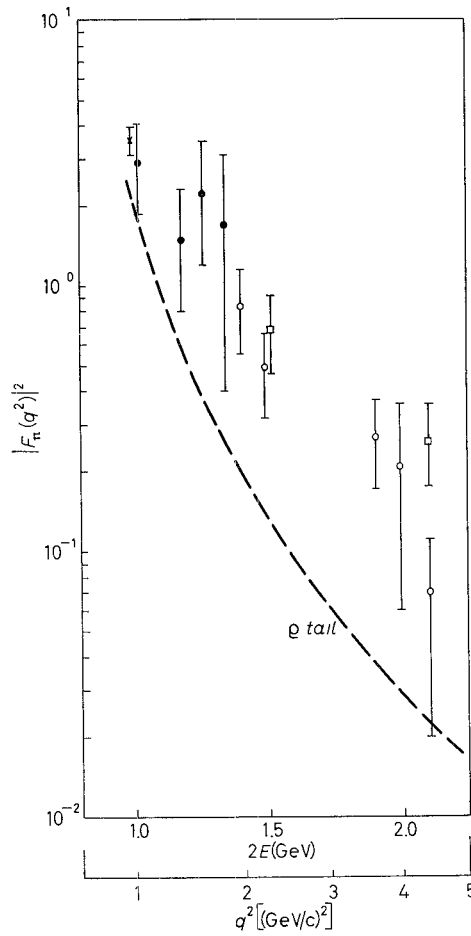

Fig. 32. – $|F_\pi(q^2)|^2$ as determined by measurements of the reaction $e^+e^- \to \pi^+\pi^-$ at $q^2 \geqslant 1$ (GeV/c)^2. The expected contribution from the ρ tail is shown. Corrections for a possible contamination of kaons in the sample of pions are not applied: × Orsay ref. [144]; ● Novosibirsk, ref. [129]; o Frascati (BCF), ref. [142]; □ Frascati ($\mu\pi$), ref. [143].

evidence in favour of a nonnegligible contribution from other mechanisms (*).

Above 1 GeV, the results are presented in Fig. 32. We see that for $1 < q^2 < 4$ (GeV/c)², $|F_\pi|^2$ is larger than the expected contribution of the ρ tail. In this energy region, the corresponding cross-sections are of a few nanobarn

$$\left(|F_\pi|^2 = 1 \text{ would correspond to a cross-section } \sigma(e^+e^- \to \pi^+\pi^-) \simeq \frac{20 \cdot 10^{-33} \text{ cm}^2}{q^2 \text{ (GeV/c)}^2}\right).$$

A separation of the π's from the K's has not been achieved in the Frascati points, so that the interpretation of the data of Fig. 32 as $|F_\pi|^2$ requires the hypothesis that the contribution to the counting rate from the channel $e^+e^- \to K^+K^-$ is negligible.

4. – The kaon form factor.

The kaon form factor is related to the cross-section for the process

(4.1)
$$e^+e^- \to K^+K^-$$

by a relation of exactly the same form as eq. (3.2). However, also isoscalar photons can couple to a K^+K^- pair (rather than only isovector photons as in $\pi^+\pi^-$ production).

Around the ϕ mass, $|F_K|^2$ is dominated by the process $e^+e^- \to \phi \to K^+K^-$. The production of ϕ-mesons in e^+e^- interactions has been investigated at Orsay [127] and Novosibirsk [128]. The results can be summarized in the following Table.

(*) However, the simple ρ, ω and ϕ vector dominance model was already in some trouble due to the behaviour of the isovector form factors F_V of the nucleons as a function of the four-momentum squared $t = -q^2$:

$$F_V \propto \frac{1}{t^2}.$$

In fact, using the relation

$$F_V(t) \propto \int \frac{\text{Im} F_V(s)}{t-s} \, ds$$

and since $1/(t-s) = 1/t + (1/t)(s/(t-s))$, we have

$$F_V(t) \propto \frac{1}{t} \int \text{Im} F_V(s) \, ds + \frac{1}{t} \int \frac{s \, \text{Im} F_V(s) \, ds}{t-s}.$$

$F_V(t) \simeq 1/t^2$ requires $\int \text{Im} F_V(s) \, ds = 0$. This however cannot be satisfied if only the ρ contributes to $F_V(s)$ since the ρ contribution has a large imaginary part with definite sign.

TABLE II.

	Orsay	Novosibirsk
$\sigma_{K^+K^-}$ $(q^2 = M_\phi^2)$	$(2.41 \ \pm 0.13) \cdot 10^{-30}$ cm^2	$(2.13 \ \pm 0.17) \cdot 10^{-30}$ cm^2
$\sigma_{K^0\bar{K}^0}$ $(q^2 = M_\phi^2)$	$(1.47 \ \pm 0.21) \cdot 10^{-30}$ cm^2	$(1.01 \ \pm 0.15) \cdot 10^{-30}$ cm^2
$\sigma_{\pi^+\pi^-\pi^0}$ $(q^2 = M_\phi^2)$	$(1.01 \ \pm 0.21) \cdot 10^{-30}$ cm^2	$(0.81 \ \pm 0.21) \cdot 10^{-30}$ cm^2
$\sigma_{\text{all modes}}$ $(q^2 = M_\phi^2)$	$(4.99 \ \pm 0.40) \cdot 10^{-30}$ cm^2	$(3.96 \ \pm 0.35) \cdot 10^{-30}$ cm^2
Γ_ϕ	$(4.09 \ \pm 0.29)$ MeV	$(4.67 \ \pm 0.42)$ MeV
$\Gamma_{\phi \to e^+e^-}/\Gamma_{\phi \to \text{all}}$	$(3.52 \ \pm 0.28) \cdot 10^{-4}$	$(2.81 \ \pm 0.25) \cdot 10^{-4}$
$\Gamma_{\phi \to K^+K^-}/\Gamma_{\phi \to \text{all}}$	0.483 ± 0.043	0.540 ± 0.034
$\Gamma_{\phi \to K^0\bar{K}^0}/\Gamma_{\phi \to \text{all}}$	0.295 ± 0.040	0.257 ± 0.030
$\Gamma_{\phi \to \pi^+\pi^-\pi^0}/\Gamma_{\phi \to \text{all}}$	0.202 ± 0.035	0.203 ± 0.042
$\Gamma_{\phi \to e^+e^-}$	$(1.44 \ \pm 0.12)$ keV	$(1.31 \ \pm 0.12)$ keV

Above the ϕ mass, only four events have been observed at Novosibirsk at three different values of q^2.
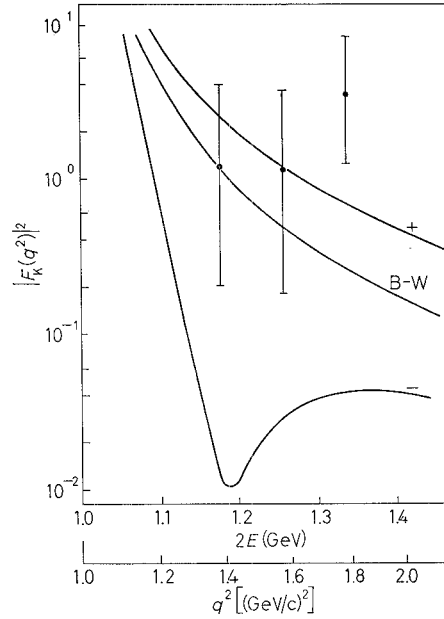

Fig. 33. – $|F_K(q^2)|^2$ as determined by the Novosibirsk group (4 events in total). The meaning of the curves is explained in the text.

These results [129], in terms of $|F_K|^2$, are presented in Fig. 33. The curve B-W represents the tail of the ϕ Breit-Wigner; the other two curves are $|F_K|^2$

as expected in the vector dominance model calculated as follows [129]:

$$(4.2) \qquad F_{\rm K}(q^2) = \frac{g_{\rho {\rm KK}}}{g_\rho} \frac{m_\rho^2}{m_\rho^2 - q^2} + \frac{g_{\omega {\rm KK}}}{g_\omega} \frac{m_\omega^2}{m_\omega^2 - q^2} + \frac{g_{\phi {\rm KK}}}{g_\phi} \frac{m_\phi^2}{m_\phi^2 - q^2} \, .$$

If we put $m_\rho^2 \simeq m_\omega^{2}$,

$$F_{\rm K}(q^2) = \frac{m_\rho^2}{m_\rho^2 - q^2} \left(\frac{g_{\rho {\rm KK}}}{g_\rho} + \frac{g_{\omega {\rm KK}}}{g_\omega} \right) + \frac{g_{\phi {\rm KK}}}{g_\phi} \frac{m_\phi^2}{m_\phi^2 - q^2}$$

and use $F_{\rm K}(0) = 1$, $A = g_{\rho {\rm KK}}/g_\rho + g_{\omega {\rm KK}}/g_\omega$ can be evaluated in terms of $g_{\phi {\rm KK}}/g_\phi$. Then the curves $+$ and $-$ in Fig. 33 represent respectively what is expected if A has the same phase as $g_{\phi {\rm KK}}/g_\phi$, or a 180° difference in phase.

We see that the extremely poor statistics are still insufficient to decide whether $|F_{\rm K}|^2$ is accounted for, in this energy region, by the simple ρ, ω and ϕ vector dominance model.

5. – Multihadron production in e⁺e⁻ interactions.

Up to values of $q^2 \sim 1.1$ (GeV/c)² multihadron production is essentially limited to the production of $\pi^+\pi^-\pi^0$ via the isoscalar vector mesons ω and ϕ. The contribution from the channel $e^+e^- \to \phi \to \pi^+\pi^-\pi^0$ is already summarized in Table II. The ω production was also investigated at Orsay [130]. The results can be summarized as follows:

$$\sigma(e^+e^- \to \pi^+\pi^-\pi^0) \ \text{(at the } \omega \text{ mass)} = (1.76 \pm 0.13) \ \mu b \, ,$$

$$\Gamma_\omega \qquad = 12.2 \ {\rm MeV} \ \text{(from the world average)} \, ,$$

$$\Gamma_{\omega \to e^+ e^-} = (1.00 \pm 0.18) \ {\rm keV} \, .$$

Above $q^2 \sim 1.1$ (GeV/c)², multihadron production has been investigated at Orsay, Novosibirsk and especially at Frascati.

The first experimental values for multiparticle production cross-sections were presented at the Kiev Conference by the Frascati « boson » [131] and « $\mu\pi$ » [132] groups; the values of the cross-section ($\geqslant 30 \cdot 10^{-33}$ cm²) were at least one order of magnitude larger than expected on the basis of an extrapolation from the lower energy range data.

On the basis of pulse-height analysis, shower recognition and investigation of the interaction properties in the spark chamber plates, it was soon possible to conclude that the produced particles are hadrons (π or K) with a contamination from e and μ which is at most (5 ÷ 10) % [133-136].

In addition, it was possible to conclude that the production occurs essentially

via the annihilation channel, with at most a small contribution (a few percent) from graphs of the type (1.1.2). In fact none of the observed multihadron events were detected in coincidence with a small-angle electron.

In addition, the distribution of the events as a function of the noncoplanarity angle $\Delta\phi$ appears to be flat [137] (Fig. 34) with no appreciable contribution
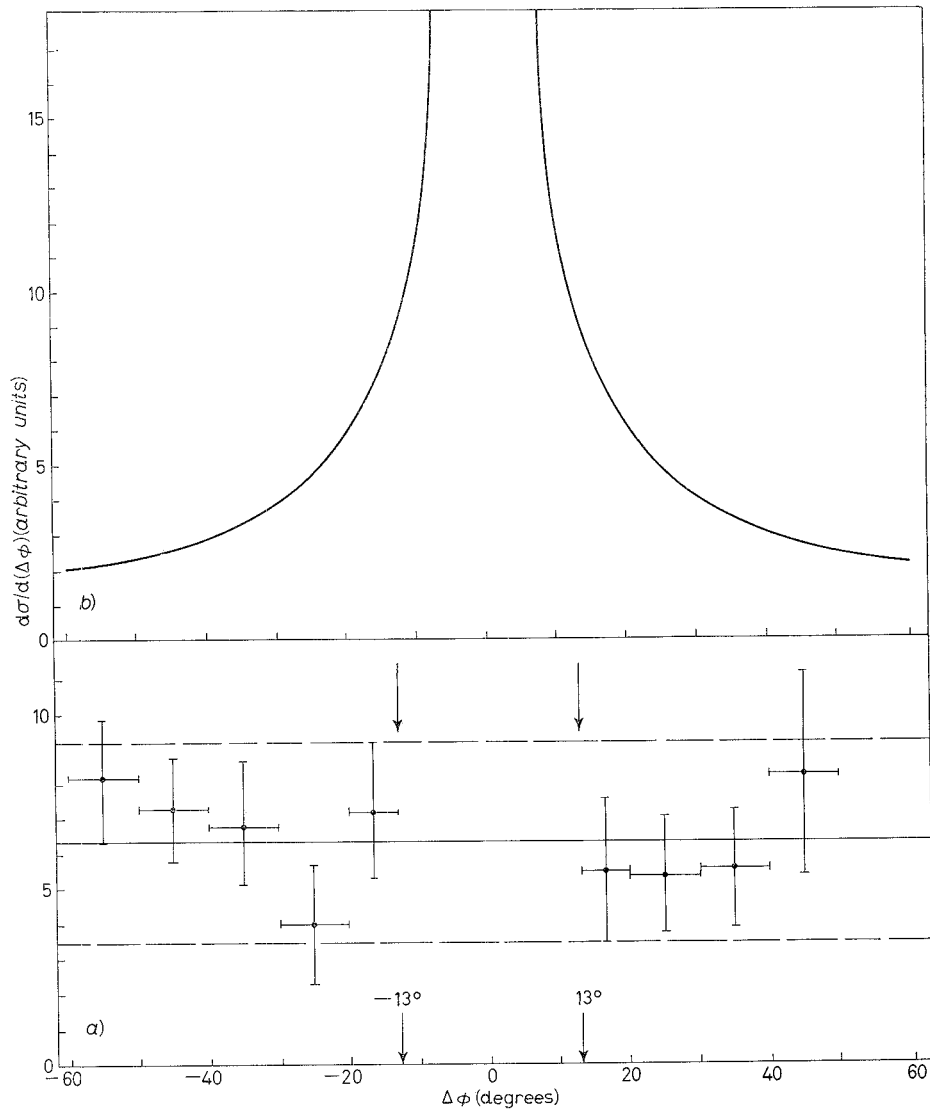

Fig. 34. − a) Distribution of noncoplanar (multihadron production) events as a function of $\Delta\phi$, the noncoplanarity angle between pairs of observed charged tracks. The distribution is corrected for the detection efficiency. b) $\Delta\phi$ distribution for the reaction $e^+e^- \rightarrow e^+e^-e^+e^-$ as expected according to BAIER and FADIN, see ref. [109].
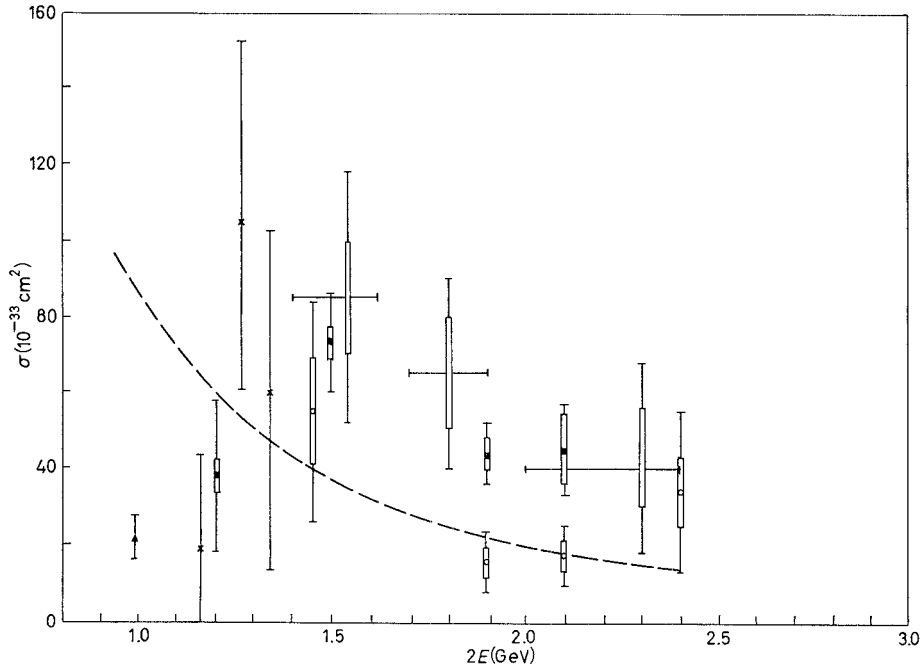
Fig. 35. – The total cross-section $\sigma_{tot}(e^+e^- \to$ more than two hadrons) as a function of the total c.m. energy $2E = E^+ + E^- = \sqrt{q^2}$. For reference, the dashed line shows the calculated total cross-section for production of $\mu^+\mu^-$ pairs: ▲ Orsay, ref. [144]; × Novosibirsk, ref. [145]; □ Frascati (boson), ref. [137]; ■ Frascati ($\mu\pi$), ref. [143]; o Frascati ($\gamma\gamma$), ref. [134].
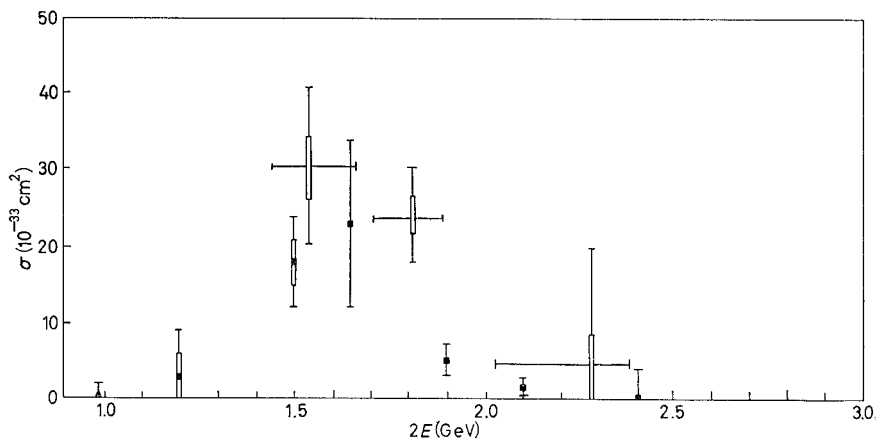

Fig. 36. – The cross-section to produce four charged pions $\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-)$ as a function of the c.m. energy $2E$. In this and in the following Figures the symbols used to indicate the experimental points from different experiments are the same as in Fig. 35: ▲ Orsay, ref. [144]; □ Frascati (boson), ref. [137]; ■ Frascati ($\mu\pi$), ref. [138].
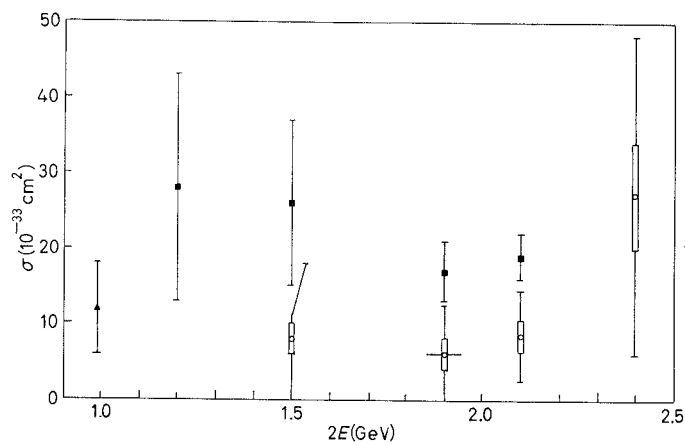
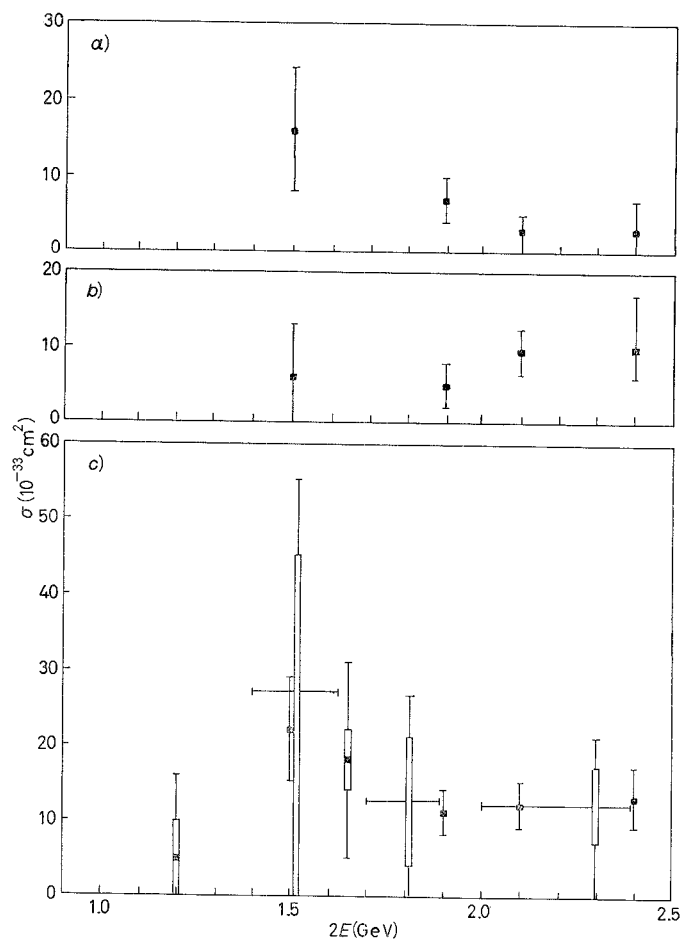Fig. 37. - The cross-section $\sigma(e^+e^- \to \pi^+\pi^-\pi^0\pi^0)$ as a function of the total c.m. energy $2E$.

Fig. 38. - a) The cross-section $\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0)$ as a function of $2E$, b) the cross-section $\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^0)$ as a function of $2E$, c) $\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^0) + \sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0)$ as a function of $2E$.

from the peaked distribution characteristic of the events of the type

$$e^+e^- \to e^+e^-e^+e^-, \qquad e^+e^- \to e^+e^-\mu^+\mu^-, \qquad e^+e^- \to e^+e^-\pi^+\pi^-.$$

Under the assumption that the charged detected hadrons from the reaction

(5.1) $e^+e^- \to$ more than two hadrons

are pions, and that angular distributions are determined by pure phase space, cross-sections have been evaluated for different production channels. The main conclusions would remain unaltered within the errors if the angular and energy distributions are determined by quasi-two-body intermediate states (*e.g.* $e^+e^- \to$ $\to A_2^+\pi^- \to \pi^+\pi^-\pi^+\pi^-$) [134, 136, 137].

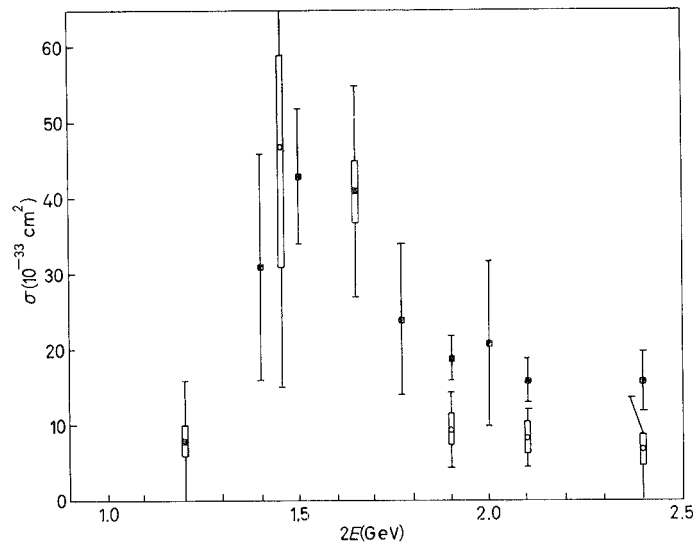The results are shown in Fig. 35-40. The total cross-section appears to be

Fig. 39. – The cross-section $\sigma(e^+e^- \to$ at least four charged pions) as a function of $2E$. The points indicated by the symbol ▨ are taken from ref. [139].

very large, larger than the cross-section for production of a pair of pointlike fermions. After a steep increase between ~ 1.2 and 1.4 GeV, it shows a slow fall-off consistent with a $1/q^2$ dependence. Different channels show different energy behaviours: for instance, while the channel $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ shows a broad bump at a mass of ~ 1.6 GeV (a new vector boson?), the $e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$ shows a slow increase between 1.5 and 2.4 GeV.

Some attempts have been made to interpret the above data in terms of conventional vector dominance [140], or in terms of an extended vector dominance including the contribution of a higher-mass vector meson ρ' [141].
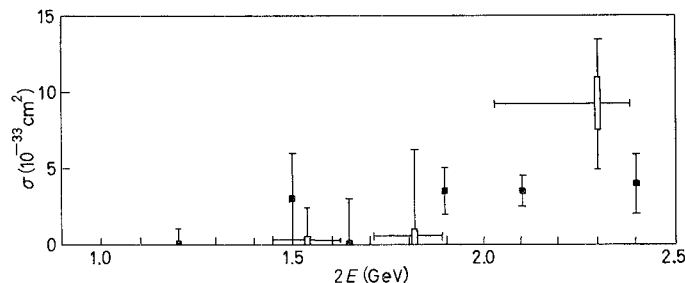
Fig. 40. – The cross-section $\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^+\pi^-)$ as a function of $2E$.

The most general attitude is however to consider these large cross-sections as a different manifestation of the same phenomenon which is observed in deep inelastic electron-proton scattering.

6. – Comments and conclusions.

The scattering of charged leptons on hadron targets, as well as the production of hadronic systems in e^+e^- interactions, has been an active field of investigation.

Due to the validity of three experimentally confirmed hypotheses—one-photon–exchange approximation, pointlike leptons and $1/q^2$ photon propagator—the experimental results have a straightforward phenomenlogical interpretation, in terms of a small number of form factors or structure functions closely connected with the electromagnetic structure of the concerned hadrons and of the e.m. current generated by hadrons. The experimental information is quite abundant for nucleons in the spacelike region. On the contrary, the experimental knowledge of the e.m. structure of unstable particles, as well as the data in the timelike region above $q^2 \simeq 1$ $(\text{GeV}/c)^2$, is still scarce or lacking.

In spite of the experimental difficulties—connected with the extremely small cross-sections and, in the timelike region, with the need of using the technique of beam-beam interaction—a general effort to overcome our present lack of experimental knowledge is to be expected—and stimulated—in the near future. Actually, it appears improbable to us that a real understanding of the interactions of the elementary particles will be achieved until their e.m. structure will be known.

From the theoretical point of view, we are still far from being able to predict the behaviour of form factors and structure functions, of connecting them with one another, of understanding the timelike region in terms of the data in the spacelike region; that is to say, as yet we do not have any theory. The experimental information on any particle, in any q^2 region, is therefore extremely useful and will complement the already available phenomenological knowledge

One of the most exciting and elegant ideas in elementary-particle physics was first suggested by the behaviour of e.m. form factors: the idea that a gauge field is generated by each conserved quantum number. The experimental data on hadron production from e^+e^- collisions below 1 GeV have nicely confirmed this idea, suggesting that Nature was applying it in its simplest form—the vector dominance model. New data at higher energy, as well as results in the spacelike region, show that this simple model is not adequate, but do not destroy the validity of the idea. As a compensation for this complication, the new data have suggested another simple idea—the parton models. Also this idea is certainly too simplified, and we already know that the simplest versions are in trouble: in the parton-quark identification, for example, we known that the simple 3-quark structure of the nucleons is not adequate; a sea of virtual quarks is added to the valence quarks, etc.

But we already knew that it is unlikely that the extremely complicated phenomenology of elementary-particle interactions can be completely understood in the frame of a simple model.

However, the close connection between the experimental numbers and fundamental quantities—current commutators, spectral functions, Wigner terms, etc.—tells us that these difficult experiments will certainly give important results since they will provide the foundation on which to build all future theories.

Note added in proofs.

After this paper was submitted to *Rivista del Nuovo Cimento* for publication, new data on e^+e^- interactions have been presented at the *XVI International Conference on*
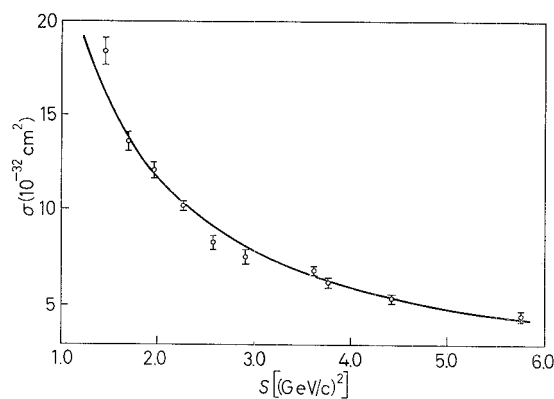

Fig. 41. – Experimental results on $e^+e^- \to e^+e^-$ elastic scattering obtained at Frascati by the BCF group. The results are expressed in term of a yield per unit luminosity as a function of $S = 4E^2$. The full line is a best fit to the experimental data of the form yield $= A/S^n$.
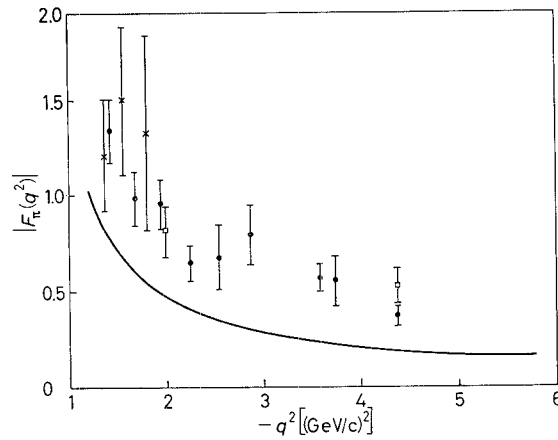
Fig. 42. – The pion form factor $F_\pi(q^2)$ as determined by the measurement of the reaction $e^+e^- \to \pi^+\pi^-$ at $q^2 \geqslant 1$ (GeV/c)^2. Corrections for a possible contamination of kaons are not applied. The full line represents the expected contribution of the ρ tail (Gounaris-Sakurai). × Novosibirsk, ref. [129]; □ Frascati ($\mu\pi$), ref. [146]; ● Frascati (BCF), ref. [146].

High-Energy Physics held at Batavia. We think it convenient to add in proofs these new data.

In Fig. 41 are shown the new results on $e^+e^- \to e^+e^-$ elastic scattering obtained at Frascati by the BCF group. The authors prefer to present their data in terms of yield per unit luminosity Y as a function of $S = 4E^2$. The full line represents a fit
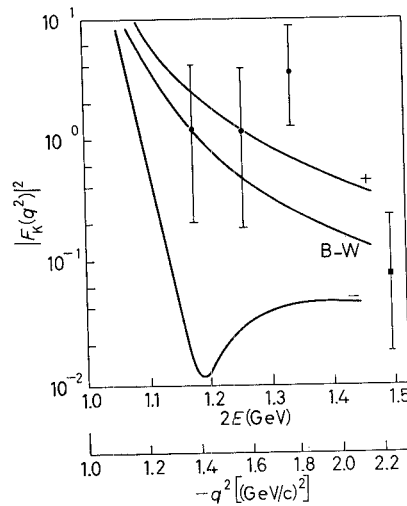

Fig. 43. – The kaon form factor $|F_K(q^2)|^2$ as determined at ● Novosibirsk, ref. [129] and ■ Frascati ($\mu\pi$), ref. [146]. The meaning of the curves is explained in the text, Part II, Sect. 4.

of the form $Y = A/S^n$, where n turns out to be $n = 0.985 \pm 0.040$. Both the absolute value A and the slope n are in agreement with QED predictions within $(4 \div 5)\%$.

In Fig. 42, 43 the present situation on the pion and kaon form factors F_π and F_K is shown. These Figures update Fig. 32, 33.

Figure 44 shows a summary of the results for the total cross-section σ_{tot} for multihadron production in e^+e^- interactions, presented in terms of the ratio $\sigma_{tot}/\sigma_{\mu^+\mu^-}$.
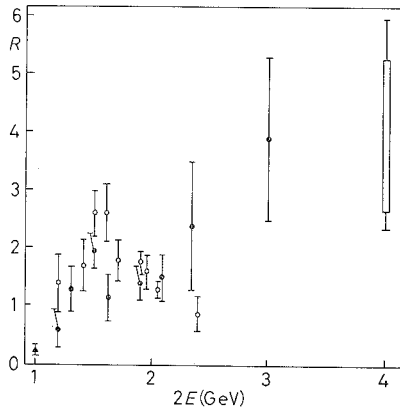

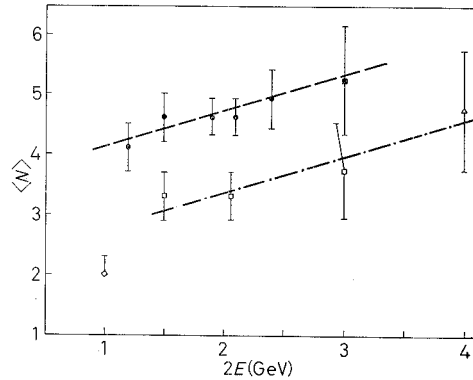Fig. 44.   Fig. 45.

Fig. 44. – Summary of the results on multihadron production in e^+e^- interactions. The results are expressed in terms of the ratio $R = \sigma_{tot}(e^+e^- \to \text{more than two hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ as a function of the total c.m. energy $2E = E^+ + E^- = \sqrt{q^2}$. ▲ Orsay, ref. [144]; ● average of the results of the Frascati (boson), ($\mu\pi$), ($\gamma\gamma$) groups, ref. [137, 143, 134]; ○ Frascati (BCF), ref. [146]; □ CEA, ref. [146].

Fig. 45. – Average values of the multiplicities in multihadron production from e^+e^- interactions as a function of the total c.m. energy $2E$. Black symbols represent total multiplicity, while open symbols represent charged multiplicity. ◇ Orsay, ref. [144]; ● Frascati ($\mu\pi$), ref. [146]; □, ■ Frascati ($\gamma\gamma$), ref. [146]; ▲ CEA, ref. [146]; — — — $\langle N \rangle_{\text{total}}$; — · — · — $\langle N \rangle_{\text{charged}}$.

The open circles (BCF group results) have been analysed under the assumption that the multiplicity distribution in multihadron production is the same as in proton-antiproton annihilation at rest, with phase-space corrections as the energy increases. The black circles are averages of the Frascati groups [137, 143, 134]. The point at $2E = 4$ (GeV) comes from the CEA by-pass experiment.

Finally, in Fig. 45 the average multiplicities in multihadron production are presented.

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