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MASSIVE LEPTON PAIR PRODUCTION AND THE SIZE OF THE PHOTON

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An intuitive picture for understanding scaling in hadronic as well as non hadronic processes is proposed which makes use of shrinkage of the size of the particle with its mass. An explicit model in agreement with experiment is also discussed.

In a recent work scaling laws for e^+e^- annihilation into hadrons and for electroproduction were shown [1] to emerge in a natural manner from the energy-momentum sum rules. In the present paper we extend this analysis to massive (large Q^2) photon production in hadron collisions. We propose a physical picture which does not stipulate a cut-off in the transverse momentum q_T but rather in a transverse scaling variable $x_T = q_T^2/Q^2$, there by restoring symmetry between the longitudinal and transverse directions. The intuitive picture which we propose and discuss later, suggests that cross sections are determined by some transverse size characteristic of the process, thus providing a unified framework for understanding the scaling properties of hadronic as well as non hadronic processes. Thus, by virtue of the cut-off in transverse momentum hadronic cross sections should approach constants while cross sections for e^+e^- annihilation into hadrons, virtual photon scattering off nucleons and massive photon production should all fall off as $1/Q^2$, because of the decreasing of the transverse size of the photon.

Our main conclusions are:

(i) Scaling for the cross section of the process $pp \to \mu^+\mu^- + X$

$$\frac{d\sigma^\gamma}{dQ^2} \xrightarrow[\tau \equiv Q^2/s \text{ fixed}]{Q^2 \to \infty} \frac{1}{Q^4} F(\tau) . \qquad (1)$$

This property is also shared by the parton model [2] but not by the bremsstrahlung model [3] or the light cone approach [4].

(ii) The average transverse momentum of the photon grows as $\langle q_T^2\rangle \sim Q^2$ for large Q^2. This is to be contrasted with the parton model where $\langle q_T^2\rangle$ is bounded and rather small, independentely of Q^2. This growth of $\langle q_T^2\rangle$ seems to be present in the multiperipheral model [5] and is also found in high energy QED calculations [6]. It is amusing to note that in the bremsstrahlung model scaling is violated by an overall multiplying factor $\langle t_{av}\rangle/s$, and may thus be restored, at least formally, if $\langle t_{av}\rangle \sim Q^2$. The growth of $\langle q_T^2\rangle$ with Q^2 can be easily tested experimentally by observing the large angle production of the pair. We also argue that similar effects must be present, in the appropriate kinematical regions, for the transverse momenta of hadrons strongly correlated with a photon in processes such as $\gamma(Q^2) + p \to c + X$ and $p + p \to \gamma(Q^2) + c_1 + \ldots + c_n + X$.

(iii) We assume that the dependence of the distribution function on the Feynman variable x_F is similar to that found in other meson production processes (e.g. π, k, ...) i.e. a sharp ($\sim$ gaussian) fall-off in x_F peaking around $x_F = 0$. This sharp fall-off in x_F is again to be compared with the parton model where the x_F distribution is proportional to the product of parton and antiparton contributions to the electromagnetic structure function of the proton which, if assumed similar, lead to a rather slow decrease with x_F. It is our claim that the main reasons for the sharp

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increase with energy in the integrated $\mu^+\mu^-$ cross section of the BNL-Columbia experiment [7] is due to this sharp decreasing with x_F.

(iv) We also present an explicit realization of this scheme in an extended VMD model which extends some earlier work [8] which showed scaling for e^+e^- annihilation and for deep inelastic scattering.

Consider the single particle distribution function

$$f^c_{ab}(q, q^2 = Q^2, s) \equiv \frac{1}{\sigma^{tot}_{ab}(s)} \frac{d\sigma^c_{ab}}{d^3q/q_0} \qquad (2)$$

for the process $a(p_1) + b(p_2) \to c(q) + X$. The various kinematical variables used are $s = (p_1 + p_2)^2$, $t = (p_1 - q)^2$, $u = (p_2 - q)^2$, $M^2_X = (p_1 + p_2 - q)^2$ along with the Bjorken and Feynman scaling variables

$$x^B_j = 2p_j q/Q^2 , \quad j = 1, 2 \qquad (3)$$

$$x_F = \frac{2q^x_{\parallel}}{\sqrt{s}} = \frac{Q^2}{s} \frac{x^B_1 - x^B_2}{\sqrt{1 - 4m^2/s}}$$

and

$$x_T = \frac{q^2_T}{Q^2} \underset{x^B_j \text{ fixed}}{\overset{Q^2,\, s \text{ large}}{\sim}} \frac{Q^2}{s}\left(x^B_1 x^B_2 - \frac{s}{Q^2}\right),$$

where for simplicity $m_1 = m_2 = m$.

The Feynman scaling of $f^c_{ab}(q, Q^2, s)$ follows from the energy–momentum sum rule

$$(p_1 + p_2)_\mu = \sum_c \int (d^3q/q_\mu f^c_{ab}(q, s) , \qquad (4)$$

where c runs over the stable hadrons, the photon (in general off mass shell of mass $\sqrt{Q^2}$) and the weakly interacting particles, under the assumption that the asymptotic form of $f^c_{ab}(q, s)$ is uniform in x_F and that the q_T integrals exist.

To discuss the large Q^2 behaviour of f^γ_{ab} one needs some information on the average transverse momentum $\langle q_T \rangle$. This is usually settled by admitting a cut-off in q_T. From eq. (3) one sees that in Bjorken limit i.e. Q^2 large, τ and x^B_j fixed, $q^2_T \sim Q^2$. If unlike the parton model we do not insist that the cut-off in q_T is independent of Q^2, we are naturally led to a cut-off in the $x_T = q^2_T/Q^2$ variable. To be precise, this implies that the product $x^B_1 x^B_2$ is not necessarily constrained to be s/Q^2, for large Q^2.

Therefore, in the above limit

$$f^\gamma_{ab}(Q^2, s, x^B_j, x_F, q_T) \xrightarrow[\tau,\, x^B_j,\, x_F \text{ fixed}]{Q^2 \text{ large}}$$

$$\left(\frac{1}{Q^2}\right)^\alpha G^\gamma_{ab}(\tau, x^B_j, x_F, x_T) , \qquad (5)$$

where α is as yet an unknown constant. This uniformity assumption in x_T, x_F, etc. implies that the integrated cross section

$$\sigma^\gamma_{ab}(Q^2, s) \underset{\tau \text{ fixed}}{\overset{Q^2 \text{ large}}{\sim}} \pi \left(\frac{1}{Q^2}\right)^{\alpha - 1} \sigma^{tot}_{ab}(s)$$

$$\times \int \frac{dx_T\, dx_F}{\sqrt{x^2_F + 4\tau(1 + x_T)}} G^\gamma_{ab}(\tau, x^B_j, x_F, x_T) , \qquad (6)$$

where the integral on the r.h.s. is supposed to exist. One notices immediately that the average multiplicity $\langle n^\gamma_{ab}(Q^2, s)\rangle$ defined as $\sigma^\gamma_{ab}(Q^2, s)/\sigma^{tot}_{ab}(s)$ grows as $(1/Q^2)^{\alpha-1} \ln (s/Q^2)$ which provides for the usual ln s growth when Q^2 is fixed ($\tau \to 0$).

To determine the parameter α we resort to the following intuitive picture which provides a unified description of the hadronic as well as non-hadronic processes. This picture consists in the statement that cross sections are determined by some transverse size characteristic to the problem. Thus all the hadronic total cross sections approach a constant due to the existence of a transverse momentum cut-off. On the other hand we expect that e^+e^- total cross section into hadrons, the deep inelastic cross section and the process under consideration fall off as $1/Q^2$, since the transverse size of the photon decreases as $1/\sqrt{Q^2}$. This implies the choice $\alpha = 2$ which is our statement of scaling. Thus as in the parton model we have the scaling for the integrated cross section but f^γ_{ab} behaves differently.

We now present a model which has been succesful in obtaining scaling results in e^+e^- annihilation as well as deep inelastic ep scattering [8]. This model visualizes the production of an infinite number of vector mesons which then decay into the $\mu^+\mu^-$ pair. Under the above hypothesis $\sigma_{\mu\bar{\mu}}(Q^2, s)$ is given by

$$\sigma_{\mu\bar{\mu}}(Q^2, s) = \sum_n \frac{\Gamma_n^{\text{strong}}\,\Gamma_{n\to\mu\bar{\mu}}}{(Q^2 - m_n^2)^2 + m_n^2\Gamma_n^2}, \tag{7}$$

where

$$\Gamma_n^{\text{strong}} \equiv \sigma^{\text{strong}}(m_n^2, s)\, m_n^2 \Gamma_n \tag{8}$$

and

$$\Gamma_{n\to\mu\bar{\mu}} = \frac{\alpha^2}{3}\left(\frac{4\pi}{f_n^2}\right)\left(1 - \frac{4\mu^2}{m_n^2}\right)^{\frac{1}{2}}\left(1 + \frac{2\mu^2}{m_n^2}\right) m_n . \tag{9}$$

Scaling in e^+e^- into hadrons was obtained [8] under the replacement

$$\sum_n \frac{m_n^2}{f_n^2}\, \frac{m_n\Gamma_n}{(Q^2 - m_n^2)^2 + m_n^2\Gamma_n^2} \to \frac{\pi}{3f_\rho^2} . \tag{10}$$

Using this result, (8), (9) and (7) we obtain

$$\sigma_{\mu\bar{\mu}}(Q^2, s) = \tfrac{4}{3}\pi\alpha^2\left(1 - \frac{4\mu^2}{Q^2}\right)^{\frac{1}{2}}$$

$$\times\left(1 + \frac{2\mu^2}{Q^2}\right)\frac{\pi}{3f_\sigma^2}\,\sigma^{\text{strong}}(Q^2, s) . \tag{11}$$

Thus if $\sigma^{\text{strong}}(Q^2, s)$ scales as $1/Q^2 \cdot G(\tau)$ then also $\sigma_{\mu\bar{\mu}}(Q^2, s)$ scales in the same way.

We now discuss the large Q^2 behaviour of $\sigma^{\text{strong}}(Q^2, s)$ which is defined as

$$\sigma^{\text{strong}}(Q^2, s) = \sigma_{\text{ab}}^{\text{tot}}(s)\int(\mathrm{d}^3q/q_0) f_{\text{ab}}^{(Q^2)} . \tag{12}$$

From the kinematic relations (3) it follows that even for Q^2 large but s/Q^2, t/Q^2 and u/Q^2 also large we are in the so-called pionization region. In the Mueller–Regge language this corresponds to the diagram of fig. 1 which consists of 2 Reggeon exchanges. This diagram gives:

$$f_{\text{ab}}^{(Q^2)} \sim \beta_{\text{QQR}}^2(0)\, h(q_{\text{T}}^2, Q^2) , \tag{13}$$

where $\beta_{\text{QQR}}(0)$ is the coupling of the mass Q "photon" to the Reggeon R, which was obtained previously from

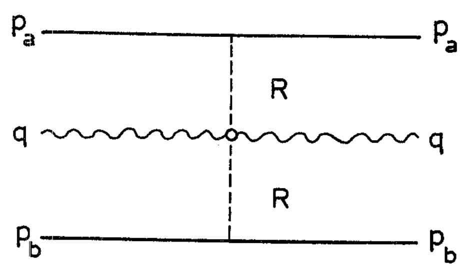

Fig. 1.

the analysis of the deep inelastic scattering in the same EVMD model [8] and was found to go as $1/Q^2$. Taking our cue from the previous analysis it seems reasonable to demand that this diagram continues to dominate even if $Q^2 \sim s$ (so long as we are not too near "thresholds" i.e. $x_j^{\text{B}} \nrightarrow 1$). Thus scaling for $\sigma^{\text{strong}}(Q^2, s)$ follows from (12) and (13) only if $h(q_{\text{T}}^2, Q^2)$ for large Q^2 becomes a function of $x_{\text{T}} = q_{\text{T}}^2/Q^2$. This reinforces our earlier claim that for large variable mass $\sqrt{Q^2}$, the damping is not in the q_{T}^2 variable but in x_{T}. Very near the kinematic boundary (t, u small) we aspect exchange type models to apply. An example is provided by the bremsstrahlung model of Berman, Levy and Neff [3] – possibly modified (as discussed earlier) to satisfy scaling.

We now comment on the experimental implications, the details of which shall be presented elsewhere. Presently available data [7] are not precise enough to resolve between various models. However, there are some features of the data regarding which a few remarks are in order. In analogy with the mesonic distribution functions we assume a factorized form,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_{\text{F}}\,\mathrm{d}x_{\text{T}}} = \frac{A}{Q^2} f_1(x_{\text{F}}) f_2(x_{\text{T}}) , \tag{14}$$

where a particular choice (suggested by certain unitarity arguments) for $f_2(x_{\text{T}})$ is made:

$$f_2(x_{\text{T}}) = \frac{1}{2\sqrt{r^2(\tau) - \tau x_{\text{T}}}}$$

$$\times \exp\left\{-\frac{c_1}{\tau}\left[r(\tau) - \sqrt{r^2(\tau) - \tau x_{\text{T}}}\right]\right\} , \tag{15}$$

$r(\tau) = \tfrac{1}{2}\{[1 - (\sqrt{\tau} + 2m/\sqrt{s})^2][1 - (\sqrt{\tau} - 2m/\sqrt{s})^2]\}^{\frac{1}{2}}$

and c_1 is a constant. For $f_1(x_{\text{F}})$ we choose

$$f_1(x_F) = e^{-c_2 x_F^2} . \qquad (16)$$

This already allows us to deduce the following general features about the BNL–Columbia experiment [7]:

(i) $d\sigma/dQ \sim 1/Q^5$ for $\tau \ll 1$. This follows because a factor $1/Q^3$ comes from the scaling part and an extra $1/Q^2$ is obtained from our assumed x_T cut-off and the experimental restriction to very small production angles. This particular result is rather independent of the particular form chosen for $f_2(x_T)$. Our choice, however, meets with the other boundary conditions i.e. for fixed Q^2 and large s it reduces to the usual q_T cut-off as well as satisfying the correct kinematic limits.

(ii) The other and perhaps much puzzling aspect of this experiment is the very sharp rise with s of the integrated cross section which is not explained satisfactorily by any of the other models. In our model taking due account of the experimental cuts (on photon momentum and angle), the strong peaking in $f_1(x_F)$ at $x_F \simeq 0$ (as for other mesons) obtains for us a rise in $\sigma(s)$ as $\sim s^4$ which is consistent with the experiment.

More details as well as our predictions for ISR shall be presented elsewhere.

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References

[1] E. Etim and Y.N. Srivastava, Frascati Report LNF-72/30 (1972) to be published.
[2] S.D. Drell and T.M. Yan, Phys. Rev. Lett. 25 (1970) 316; P.V. Landshoff and J.C. Polkinghorne, Nucl. Phys. B33 (1971) 221; B28 (1971) 240.
[3] S.M. Berman, D.J. Levy and T.L. Neff, Phys. Rev. Lett. 23 (1969) 1363.
[4] G. Altarelli, R.A. Brandt and G. Preparata, Phys. Rev. Lett. 26 (1971) 42;
R.A. Brandt and G. Preparata, to be published;
see also G. Preparata, Lectures given at the Intern. School of Subnuclear Physics, ERICE (1971).
[5] H.D.I. Abarbanel and J.B. Kogut, Institute for Advanced study (Princeton) preprint 1972; to be published.
[6] H. Cheng and T.T. Wu, Phys. Rev. 183 (1969) 1324.
[7] J.H. Christenson et al., Phys. Rev. Lett. 25 (1970) 1523.
[8] A. Bramón, E. Etim and M. Greco, Frascati Report LNF–72/45 (1972), to be published.