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CONFORMAL INVARIANCE AND BJORKEN SCALING

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Invoking conformal invariance of the electromagnetic vertices $\gamma\pi\pi$ and γNN we calculate the structure functions of the pion and the nucleon and show that they scale in the Bjorken limit only if the dimensions of the corresponding fields are canonical.

Ever since Bjorken's [1] proposed scaling of electroproduction structure functions was experimentally confirmed [2] there has been an ambitious quest for an understanding of this phenomenon by appeal to conformal symmetry arguments [3]. An application of such arguments in the classification of the basis of operators in a general Wilson expansion [4] of the product of two local operators by Ferrara, Gatto and Grillo [5] indicates that the essential condition for scaling, namely correlated dimensions, is not guaranteed by conformal invariance although it is compatible with it**. Recently Migdal [6] has introduced an interesting connection between the covariant quark model and asymptotic conformal invariance by means of which one can calculate the pion and nucleon form factors as powers in momentum transfer, the power exponent being determined by the difference between the anomalous and canonical dimensions.

Apart from its specific predictions, the importance of such a calculation. from the point of view of conformal invariance, lies in its possible extension to compute structure functions permitting thus to investigate the question of whether conformal invariance implies Bjorken scaling or not. This investigation is carried out in this letter. The central idea in such an application is that there is additional information, beyond current conservation and asymptotic conformal invariance, on the 4-point function (c.f. eq. (2)) coming from a knowledge of the set of operators of lowest dimensions appearing in the expansion of the product $J_\mu(x)A(y)$ of the electromagnetic current $J_\mu(x)$ and the particle field operator $A(y)$. The set of these operators is formed by all derivatives of the field $A(y)$. To make use of this observation it will be necessary to allow for certain interchanges of limits, a manupulation we cannot at present justify mathematically. With this extra leverage we show that conformal invariance implies Bjorken scaling only if field dimensions are cannonical or more to the point only for a free field theory. We show in addition that predictions obtainable from such a theory are identical to those of the parton model. This latter fact is traceable to similar assumptions made in both approaches, namely that hadron-quark-quark vertex functions are constants in the one case and partons are free point-like constituents of the hadron in the other. This assumption is crucial for the success of the Migdal approach because it guarantees the dilational invariance of the hadron propagator functions and the conformal invariance of the electromagnetic vertices $\gamma\pi\pi$ and γNN in the quark model. For instance, from figs. 1a and 1b which represent the pion and nucleon propagators one finds, making use of the graphical rules of the quark model with constant vertex functions

$$\langle 0|T(\phi(x_1)\,\phi(x_2))|0\rangle \sim ((x_1-x_2)^2)^{-(l_B+1)}\,\mathrm{Tr}\,[\Gamma_5(\hat{x}_1-\hat{x}_2)\,\Gamma_5(\hat{x}_2-\hat{x}_1)] = C_B((x_1-x_2)^2-\mathrm{i}\epsilon)^{-l_B} \tag{1.1}$$

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** Correlated dimensions of tensor fields have been proven under certain assumptions [c.f. 11]. What has not been demonstrated is that the second rank tensor field to lowest dimension in the Wilson expansion of the product of two currents $J_\mu(x)\,J_\nu(0)$ has dimension $l = 4$.

$$\langle 0|T(\psi(x_1)\,\bar\Psi(x_2))|0\rangle \sim (\hat{x}_1-\hat{x}_2)((x_1-x_2)^2)^{-(l_F+\frac{3}{2})}\,\mathrm{Tr}\,[(\hat{x}_1-\hat{x}_2).(\hat{x}_2-\hat{x}_1)] = C_F(\hat{x}_1-\hat{x}_2)((x_1-x_2)^2-\mathrm{i}\epsilon)^{-(l_F+\frac{1}{2})}\,. \tag{1.2}$$

where l_B, l_F are the dimensions of the fields $\phi(x)$ and $\psi(x)$ respectively and c_B, c_F are dimensionless constants; $\hat{x} = \Gamma_\mu x_\mu$ with Γ_μ a Dirac gamma matrix. Eqs. (1.1) and (1.2) are the dilational invariant forms of the pion and nucleon propagators; they can also be obtained by other means [7].

To calculate the electroproduction structure functions defined by

$$W_{\mu\nu}(q^2, p\cdot q) = \frac{1}{N}\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{d}^4z\,\exp\{-\mathrm{i}p(x-y)+\mathrm{i}qz\}\,D_p(x)\,\sum\langle 0|T(A(x)J_\mu(-\tfrac{1}{2}z))|n\rangle\langle n|T(J_\nu(+\tfrac{1}{2}z)\bar{A}(y))|0\rangle\,\bar{D}_p(y) \tag{2}$$

from the above techniques one needs the Wilson type operator expansion of the time ordered products of the electromagnetic current and the hadron field $A(x)$ ($A(x)\equiv\phi(x)$ for the pion and $\psi(x)$ for the nucleon). In eq. (2) N is a normalisation factor equal to $1/2M$ for the pion and 1 for the nucleon; $D(x)$ and $\bar{D}(y)$ are differential operators which in the conformal symmetry limit are given by

$$D_P(x) = (\Box_x^2)^{-(l_B-2)}\,,\qquad \bar{D}_P(y) = (\Box_y^2)^{-(l_B-2)}\,, \tag{3.1}$$

for the pion and

$$D_P(x) = \bar{u}(p)\,\Gamma_\mu\,\frac{\partial}{\partial x_\mu}\,(\Box_x^2)^{-(l_F-\frac{3}{2})}\,,\qquad \bar{D}_P(y) = -\frac{\partial}{\partial y_\mu}\,\Gamma_\mu\,u(p)\,(\Box_y^2)^{-(l_F-\frac{3}{2})}\,, \tag{3.2}$$

for the nucleon where $u(p)$ is a spinor wave function. Note that in the dilational invariance limit eqs. (3.1) and (3.2) give the correct momentum space inverse propagators of the pion and nucleon respectively. For simplicity we shall henceforth treat only the case of the pion in detail and only quote the corresponding results for the nucleon structure functions.

The Wilson expansion of the product $J_\mu(y)\,\phi(z)$ is obtained from a comparison of the vertex $\gamma\pi\pi$ given in configuration space by

$$\langle 0|T(\phi(x)J_\mu(y)\phi(z))|0\rangle \sim (x-y)^{-4}(y-z)^{-4}(z-x)^{-2(l_B-1)}\,\mathrm{Tr}\,[\Gamma_5(\hat{x}-\hat{y})\,\Gamma_\mu(\hat{y}-\hat{z})\,\Gamma_5(\hat{z}-\hat{x})]$$
$$= K_B\,[(x-y)^{-2}\,\frac{\overleftrightarrow{\partial}}{\partial y_\mu}\,(y-z)^{-2}]\,(z-x)^{-2(l_B-1)} \tag{4}$$

with eq. (1.1) and making use of the generalised Feynman integral [8]

$$(x^2)^{-a}\,((x-y)^2)^{-b} = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^1 \mathrm{d}\lambda\,\lambda^{b-1}(1-\lambda)^{a-1}((x-\lambda y)^2)^{-(a+b)}\,. \tag{5}$$

The result is

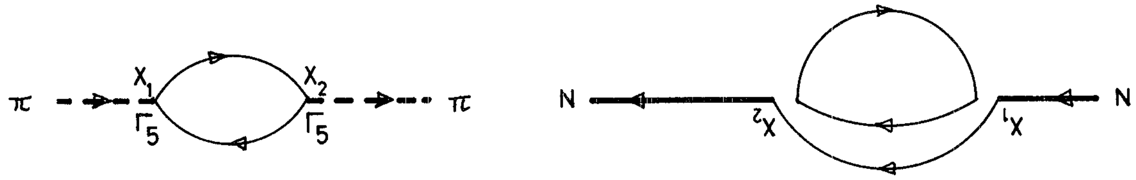

Fig. 1. (a) The pion propagator in the quark model. Internal lines stand for quark propagators. (b) The nucleon propagator in the quark model. Internal lines stand for quark propagators.

$$J_\mu(y)\,\phi(z) = (\partial_\mu\partial_\nu - g_{\mu\nu}\Box^2)\frac{(y-z)_\nu}{(y-z)^2}\int_0^1 \mathrm{d}\lambda\, f(\lambda)\,\phi(z+\lambda(y-z)) \tag{6}$$

where we have incorporated manifest current conservation and replaced the weight function $(1-\lambda)^{l_B-2}$ coming from eq. (5) by $f(\lambda)$. This replacement is not just a trivial generalisation because it is necessary to ensure the convergence of the integrals defining the structure functions (c.f. eq. (12)).

Substituting from eqs. (6) and (3.1) into (2) yields

$$W_{\mu\nu}(q^2, p\cdot q) = (q_\mu q_\alpha - q^2 g_{\mu\alpha})\,(q_\nu q_\beta - q^2 g_{\nu\beta})\int_0^1 \mathrm{d}\lambda\,\mathrm{d}\tau f(\lambda) f(\tau)\,(q_1^2)^{-l_B+2}\,(q_2^2)^{-l_B+2}$$

$$\times\frac{1}{2M}\int \mathrm{d}^4\bar{x}\,\mathrm{d}^4\bar{y}\,\mathrm{d}^4\bar{z}\,\exp(\mathrm{i}q_1\bar{x} - \mathrm{i}q_2\bar{y})\,\exp\{-\mathrm{i}(p+q)\bar{z}\}\left(\frac{\bar{x}_\alpha}{\bar{x}^2-\mathrm{i}\epsilon}\;\frac{\bar{y}_\beta}{\bar{y}^2+\mathrm{i}\epsilon}\right)[\mathrm{i}\Delta^{(+)}(\bar{z})] \tag{7}$$

where $\Delta^{(+)}(\bar{z})$ is the two-point Wightmann function defined by

$$\mathrm{i}\Delta^{(+)}(\bar{z}) = \sum_n \langle 0|\phi(x+\lambda\bar{x})|n\rangle\langle n|\phi(y+\tau\bar{y})|0\rangle \tag{8}$$

and the variables $\bar{x}, \bar{y}, \bar{z}, q_1$ and q_2 are given by

$$\bar{x} = -x - \tfrac{1}{2}z\,,\quad \bar{y} = -y + \tfrac{1}{2}z\,,\quad \bar{z} = -(1-\lambda)\bar{x} + (1-\tau)\bar{y} - z\,,\quad q_1 = \lambda(p+q) - q\,,\quad q_2 = \tau(p+q) - q\,. \tag{9}$$

Now the function $\Delta^{(+)}(\bar{z})$ is completely specified by dilational invariance since from eq. (1.1) one gets [7]

$$\Delta^{(+)}(\bar{z}) = C_B(\bar{z}^2 - \mathrm{i}\epsilon\bar{z}_0)^{-l_B}\,. \tag{10}$$

Substituting from here into eq. (7) and carrying out the three four-fold integrations gives the structure function

$$W_2(\nu, Q^2) = (Q^2)^{-2(l_B-1)}\left[\frac{1}{\pi}\sin(\pi l_B)\,((p+q)_+^2)^{l_B-2}\right]H(\omega-1) \tag{11}$$

where $\nu = p\cdot q/M$, $Q^2 = -q^2 > 0$ and $\omega = 2M\nu/Q^2$.

The distribution $((p+q)_+^2)^{l_B-2}$ is analytic everywhere except for a pole at $l_B = 1$ [9] and the function $H(\omega-1)$ is given by

$$H(\omega-1) = 2(2\pi)^5 M\int_0^1 \mathrm{d}\lambda\,\mathrm{d}\tau f(\lambda) f(\tau)\lambda\tau\,[(1-\lambda)(1-\tau)]^{-l_B}\,[(1+\lambda(\omega-1))\,(1+\tau(\omega-1))]^{-l_B}\,. \tag{12}$$

The constant C_B in eqs. (1.1) and (10) has been identified as

$$C_B = -\mathrm{i}(4\pi)^{-2}\,2^{2l_B}\,\Gamma(l_B)/\Gamma(2-l_B) \tag{13}$$

for reasons that will soon be clear. Taking now the Bjorken limit in eq. (11) we find that $\nu W_2(\nu, Q^2)$ scales provided $l_B = 1$, in other words provided the dimension of the pion field remains canonical. This is the result advertised in the introduction. At $l_B = 1$ the distribution $((p+q)_+^2)^{l_B-2}$ has a pole; the corresponding residue is easily calculated from the identity [9]

$$(p_+^2)^\lambda \sin(\pi\lambda) = \tfrac{1}{2}\mathrm{i}\,[\exp(-\mathrm{i}\pi\lambda)\,(p^2+\mathrm{i}\epsilon)^\lambda - \exp(\mathrm{i}\pi\lambda)\,(p^2-\mathrm{i}\epsilon)^\lambda] \tag{14}$$

from which one gets finally

$$W_2(\nu, Q_2) = \delta(Q^2(\omega-1)\times\text{const.} \tag{15}$$

This coincides with the parton result [10]. The reason for this agreement is obviously to be found in the assumed constant form factors of the hadron-quark-quark vertices, which is the analogue of the point - like structure attributed to partons in the parton model [10]. If one introduces the Källén-Lehmann representation of the function $\Delta^{(+)}(\bar{z})$ one finds from eq. (10)

$$\rho((p+q)^2) = \delta((p+q)^2) \tag{16}$$

which is just the Källén-Lehmann spectral function of a non-interacting particle*. The coefficient multiplying the delta function in eq. (16) is unity from the choice of the normalisation of the constant C_B given in eq. (13). The overall picture then is that conformal invariance is consistent with the constraint of Bjorken scaling only for a free field theory. Similar considerations for the nucleon structure functions yield $l_F = 3/2$ as a condition for scale invariance.

* We have been neglecting the pion mass all along.

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