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CRITICAL INDICES FOR THE SPHERICAL MODEL FROM CONFORMAL COVARIANT SELF CONSISTENCY CONDITIONS *

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Critical indices for the spherical model are computed in arbitrary dimension space. Well-known exact results are recovered in three-dimensional space. The key tools used are conformal covariant self consistency conditions.

Recently it has been suggested that changing the dimension of the space may be an useful tool to compute critical indices [1]. One finds that the critical indices for the spherical model in a $4-\epsilon$ dimensional space are $\nu = \frac{1}{2} + \frac{1}{4}\epsilon + \frac{1}{8}\epsilon^2 + O(\epsilon^3)$, $\eta = 0 + O(\epsilon^4)$.

In this letter we extend this result to arbitrary dimension space using conformal covariant self consistency conditions [2, 3]. The equivalence between our and Wilson's approach was recently proved by Mack [4]. The critical indices turn out to be: $\nu = 1/(2-\epsilon)$, $\eta = 0$. This expression is equal Wilson's for small $\epsilon = 4-d$ and yields the correct values $\eta = 0$, $\nu = 1$ for $\epsilon = 1$ [5].

The self consistency conditions are written in fig. 1: G is the "one particle" propagator, D is the "phonon" propagator, V is the phonon-particle vertex, K_1 and K_2 are the two different Bethe–Salpeter kernels and the dotted lines mean momentum differentiation [2, 6]. These self consistency condition are derived following ref. [1] using the Landau–Ginsburg functional for the free energy as a starting point and introducing an auxiliary "phonon" field [7]. We need the solution of the equations in the limit $N \to \infty$. The factor N plays the role of a multiplicity factor.

The form of the propagator and vertices at the critical temperature follows from conformal invariance requirements:

$$G(x) = 1/|x|^{d+\eta-2}, \qquad D(x) = 1/|x|^{2d-2/\nu},$$
$$V(x,y,z) = \frac{g\,\Gamma^{-1}(2+\nu^{-1}-\eta-d)}{|x-z|^{1/\nu}\,|z-y|^{1/\nu}\,|x-y|^{d+2-\eta-1/\nu}}. \tag{1}$$

The normalization of the coupling constant is chosen in such a way that the Fourier transform of the vertex is always finite, for any value of the indices. The kernels K_1 and K_2 have a standard diagrammatical expansion in terms of G, D and V.

Letting (1) in the self consistency equations we find a system of three equations in three unknowns whose solution yields the value of the critical indices. We look now for solution of the type $1/\nu = 2-\epsilon+\Delta\nu$, where η and $\Delta\nu$ are small parameters.

In this limit we find that the three self-consistency equations are respectively:

$$\frac{g^2}{\Delta\nu-\eta} + \ldots = C_1;$$
$$\frac{g^4}{\Delta\nu-\eta} + \ldots = C_2\eta; \tag{2}$$
$$\frac{Ng^4}{\Delta\nu-\eta} + \ldots = C_3,$$

where C_1, C_2, C_3 are some constants dependent on the dimension of the space. These equations are obtained taking only the simplest diagrams in the self consistency conditions; a more careful analysis shows that neglected terms do not change our conclusions [7].

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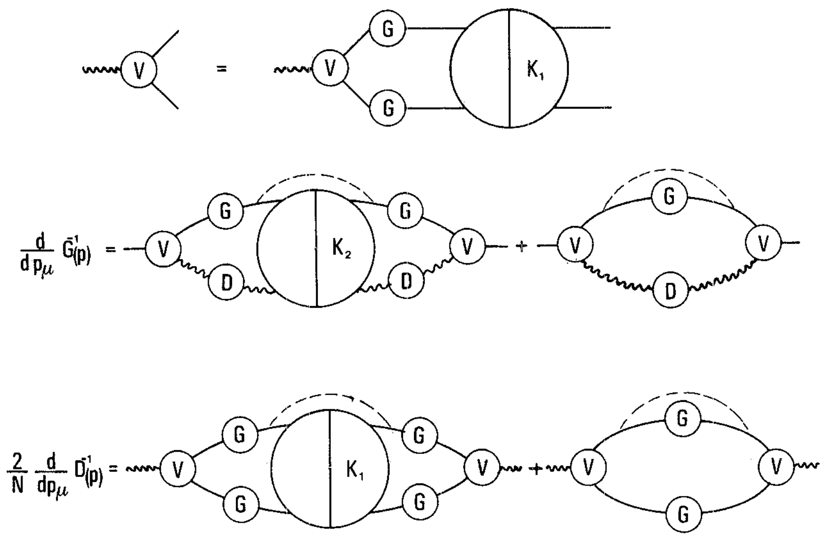

Fig. 1. Self consistency conditions for the vertex, the "one particle" and the "phonon" propagators.

In the $N \to \infty$ limit we find that the solution is $\nu = 1/(2-\epsilon')$, $\eta = 0$. Any ambiguity in the solution of the equations is removed by the condition that in the 4-dimensional limit we recover the classical indices.

The simplicity of our result is due to the compensation of the singularities of the denominator with the large N limit.

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