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G. Parisi: HOW TO MEASURE THE DIMENSION OF THE  
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ABSTRACT. -

We show that the anomalous dimension of the fundamental field is connected to the anomalous dimensions of the high spin bilinear operators. The dimensions of these operators can be determined looking to the violations of Bjorken scaling law in deep inelastic electron proton scattering. The structure function at field  $\omega$  near to one has a power dependence from  $q^2$ , the exponent being the anomalous dimension of the "parton" field.

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## 1. INTRODUCTION. -

In these last years many speculations have been done on the possibility that the short distance behaviour of an interacting theory is different from that of a free theory<sup>(1)</sup>: the dimensions of nearly all the operators are different from the canonical one the Green functions of the renormalized fields are more singular at short distances than in free field theory; canonical commutation relations are therefore destroyed.

This very interesting conjecture must be tested looking to off mass shell quantities, and this can be done only using currents coupled to leptons. One can measure only the dimensions of those operators which contribute to the leading term of the Wilson expansion of two currents near the light cone; the field is not one of these operators; it seems therefore impossible to measure the dimension  $d_\phi$  of the "parton" field.

The aim of this paper is to show that the situation is not so sad:  $d_\phi$  cannot be measured directly, but it can be connected to the observable dimensions of other operators.

In deep inelastic electron proton scattering we define in the standard way the function

$$F_2(\omega, q^2) = \nu W_2(\nu, q^2) \quad (\omega = -\frac{2\nu}{q})$$

If anomalous dimensions are present, Bjorken scaling is no more valid, but each of the following integrals has asymptotically a simple power behaviour<sup>(3)</sup>:

$$(1) \quad \int_1^\infty d\omega \omega^{-N} F_2(\omega, q^2) \xrightarrow{q^2 \rightarrow -\infty} C_N \left( \frac{M^2}{-q} \right)^{A_N/2}$$

$A_N$  is the anomalous dimension of the spin  $N$  operator and can be determined from a careful analysis of the experimental data.

We prove that in a  $\lambda \phi^4$  theory in four dimensions

$$(2) \quad A_\infty \equiv \lim_{N \rightarrow \infty} A_N = 2(d_\phi - 1) \equiv 2A_\phi$$

and

$$(3) \quad A_\infty - A_N = G N^{-H} \quad G > 0 \quad H > 0$$

We note that the dimension  $d_N$  of the operator

$$(4) \quad O_{\mu_1, \mu_N} = \phi^+ \overset{\leftrightarrow}{O}_{\mu_1} \dots \overset{\leftrightarrow}{O}_{\mu_N} \phi - \text{Traces}$$

is equal to  $2+N+A_N$ ; the physical interpretation of formula (2) is that the dimension becomes additive in the case of high spin operators.

Additional interest in this fact comes from the observation that for high  $N$  the integrals (1) are dominated by the region at  $\omega \sim 1$ . This implies that<sup>(4)</sup>

$$(5) \quad F_2(\omega, q^2) \rightarrow f(\omega) \left( \frac{M^2}{-q} \right)^{A_\phi}$$

for fixed  $\omega$  such that  $1 \gg (\omega - 1) \neq 0$ .

The anomalous dimension of the fundamental field is thus simply connected to the breakdown of Bjorken scaling at  $\omega$  near to one.

The difficulties to extend the proof to more realistic situations like the  $\sigma$  model are due to the more complicated structure of the Dyson equations in presence of more than one fundamental field.

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In the second section of this paper we discuss the scaling properties of the zero mass limit and the Dyson equation for the vertex of the  $O_{\mu_1, \mu_N}$  operators with two fields  $\phi$ .

In the third section we study the properties of the Bethe Salpeter Kernel and we present a detailed proof of formulae (2, 3).

In the last section we discuss our results and some possible extensions.

## 2. THE DYSON EQUATION. -

The best way to investigate the dimension spectrum of operators is to study the solution of the Dyson equation for the vertex of these operators. To simplify the computations we restrict ourselves to a zero mass scale invariant theory. This restriction yields no problem because the dimensions of the operators are mass independent<sup>(5)</sup> and it has been shown by K. Symanzik<sup>(6)</sup> that the Gell-Mann Low limit of a renormalizable quantum field theory exists and it is scale invariant, provided that a certain function  $\beta(g)$  has a zero.

We are interested to operators which gives the leading contribution on the light cone: they must be symmetric traceless tensors transforming like the  $(n/2, n/2)$  representation of the Lorentz group: they can be defined as in (4).

They must satisfies the following Dyson equation:

(6)

where G is the propagator of the field  $\phi$ , V is the  $\phi\phi O_{\mu_1, \mu_N}$

vertex, and  $K$  is the Bethe Salpeter Kernel which is the sum of all two particle irreducible graphs. The inhomogeneous term cannot be present because it transform itself under scale transformations in a different way from the other term<sup>(7, 8)</sup>.

If we introduce the Fourier transform of the functions into the Dyson equation can be rewritten as

$$(7) \quad V_{\mu_1, \mu_N}(p+K, -p+K) = \int d^4q V_{\mu_1, \mu_N}(q+K, -q+K) G(q+K) G(-q+K) K(q+K, -q+K; p+K, -p+K)$$

The scale properties of the functions involved in the Dyson equation are:

$$(8) \quad G(\lambda p) = \lambda^{-4+2d} G(p) = K(\lambda p_1, \lambda p_2; \lambda p_3, \lambda p_4) = \lambda^{4-4d} K(p_1, p_2; p_3, p_4)$$

$$V_{\mu_1, \mu_N}(\lambda p_1, \lambda p_2) = \lambda^{-2d+d_N} V_{\mu_1, \mu_N}(p_1, p_2)$$

This equation must be satisfied at arbitrary values of the external momenta; we study it when  $K$  is equal to zero.

The existence of the Fourier transform of the vertex in this particular situation, i. e. the absence of infrared divergences, can be proved when the dimension of  $O_{\mu_1 \mu_N}$  is greater than 2 using the Araky-Haag-Ruelle<sup>(9)</sup> theorem on cluster decomposition in zero mass theories.

From eq. (8)

$$(9) \quad V_{\mu_1, \mu_N}(p, -p) \alpha(p^2)^{-d+\frac{1}{2}(d_N-N)} \left[ p_{\mu_1} \dots p_{\mu_N} - \text{Traces} \right]$$

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The constant of proportionality is of no interest, eq. (8) being homogeneous in the vertex.

We introduce an arbitrary vector  $t_\mu$

$$(10) \quad V_{\mu_1, \mu_N}(p, -p) t^{\mu_1} \dots t^{\mu_N} = C_N(p^2) (p^2)^{-d_\phi + \frac{1}{2}(d_N - N)}$$

where  $C_N$  is the Chebyshev polynomial of order  $N$ .

The Dyson equation at zero momentum transfer can therefore be rewritten as:

$$(11) \quad (p^2)^{-\delta} C_N(p^2) = \int d^4 q \bar{K}(p, -p; q, -q) (q^2)^{-\delta} C_N(q^2)$$

where

$$(12) \quad \delta = d_\phi - \frac{1}{2}(d_N - N) \equiv A_\phi - \frac{1}{2} A_N = \bar{K}(p, -p; q, -q) = K(p, -p; q, -q) G^2(q) \\ - \bar{K}(\lambda p, -\lambda p; \lambda q, -\lambda q) = \lambda^{-4} \bar{K}(p, -p; q, -q)$$

The possible values for the dimension of a spin  $N$  operator are obtained improving the validity of (12).

In the next section we shall prove that irrespectively of the detailed form of  $K$ , equation (11) admits solutions in which  $A_N$  goes to  $2A_\phi$  when  $N$  go to infinity.

### 3. - THE SOLUTIONS OF THE DYSON EQUATION.-

The key ingredient we need to prove our statements, is the fact that  $\bar{K}(0, 0; q, -q)$  is free of infrared divergences also in a zero mass field theory. This fact was firstly recognized by Baker Johnson

and Wiley in their fundamental paper on zero mass electrodynamics. Their proof can be extended without difficulties also to the case of  $\lambda \phi^4$  theory. This result united with the scaling properties of the function  $\bar{K}$  tell us that

$$\bar{K}(p, -p; q, -q) \xrightarrow[q^2 \rightarrow \infty]{} (q^2)^{-2}$$

i. e., the short distances singularity of the kernel are logarithmic in the  $x$  space.

We can use this information to write a Deser Gilbert Sudershan representation for spacelike  $p$  of the form <sup>(12, 13)</sup>:

$$(13) \quad \bar{K}(p, -q; q, -q) = \int_0^\infty da \int_{-1}^1 db \frac{\bar{\sigma}(a, b, p^2)}{[a + K^2 + p^2 + 2bpK]^2}$$

The limit of integrations are fixed as usually by the spectrum condition.

We insert this representation in the Dyson equation and we make a Wick rotation on the internal integration to avoid any possible convergence problem.

It is now clear that if  $\bar{K}$  would be a singular function, at fixed  $A_N$  the r.h.s. would go exponentially to zero, so that the Dyson equation could not ever being satisfied unless we chose values of  $A_\phi$  such that the  $Q^2$  integral becomes singular: this happens just for  $A_N = 2 A_\phi$ . This would imply that in the large  $N$  limit  $A_N$  go to  $A_\phi$  in exponentially rapid way. The real situation is different:  $\bar{K}$  is not a completely regular function, but its singularity can be controlled using the D.G.S. representation; the final result will be that a  $A_N$  to to  $A_\phi$  with a speed that it is proportional to some inverse poweer of  $N$ , the power being fixed by the treshhold behaviour of the



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Kernel.

To proceed further we exchange the D.G.S. representation with the  $q$  integral, The  $q$  integral can be performed using the following formula:

$$(14) \quad \int \frac{d^4 q (q^2)^{-\delta} C_N(tq)}{[p^2 + q^2 + 2bpq + a]^2} = \pi^2 (a+p^2)^{-\delta} b^N B(N+2-\delta, \delta)$$

$${}_2F_1(\delta, N+2-\delta; N; \frac{bp^2}{a+p}) C_N(pt)$$

(we use now the euclidean metric).

This formula reduce itself in the large  $N$  limit to

$$(15) \quad \pi^2 \Gamma(\delta) N^{-\delta} b^N [p^2(1-b)+a]^{-\delta}$$

The self consistency conditions deduced from the Dyson equation can be written for large  $N$  in the following way

$$(16) \quad \pi^2 \Gamma(\delta) N^{-\delta} \int_0^\infty da \int_{-1}^1 db \bar{\sigma}(a, b, p^2) b^N [1-b+a/p^2]^{-\delta} = 1$$

The r. s. d. of (16) must go to zero at fixed  $\delta$  when  $N$  goes to infinity; both the factors  $N^{-\delta}$  and  $b^N$  go to zero. A  $\delta(b-1)$  singularity cannot be present without producing a too singular behaviour of the Kernel at treshold.

The conclusion is that for large  $N$  and small  $\delta$  the r. s. d. can be approximated by  $GN^{-H}/\delta$  where  $H$  and  $G$  are positive numbers connected to the treshold singularity of the Kernel. Solving for  $\delta$  and recalling definition (12) we find the result:

$$(17) \quad 2 A_{\phi} - A_N \xrightarrow{N \rightarrow \infty} G N^{-H}$$

#### 4. - CONCLUSIONS. -

We have just found that the anomalous dimensions of the high spin operators are connected to the anomalous dimension of the parton field,  $A_N = A_{\phi} - G N^{-H}$ .

Unfortunately present SLAC experiment do not pose relevant restrictions on the value of the index  $A_{\infty}$ : one should know the behaviour of the structure functions at  $\omega$  near to one but outside the resonance region and this implies an enormous value of  $q^2$ : future experiments will hopefully yield more stringent limit on  $A_{\infty}$ .

A by product of our analysis is a new proof of two different known result: Bjorken scaling ( $A_N = 0 \forall N$ ) can be valid only in a asymptotic free theory ( $A_{\phi} = 0$ ); if Bjorken scaling is not valid all the integrals in <sup>(1)</sup> must scale with a different power.

As a final remark we note that our results can be readily extended both to a  $\lambda \phi^3$  theory in  $6 + \epsilon$  dimensions and to a  $\lambda \phi^4$  theory in  $4 - \epsilon$  dimensions. They agree with the explicit calculations of Mack <sup>(17)</sup> and Wilson <sup>(18)</sup>.

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