Laboratori Nazionali di Frascati

V. Montelatici and G. Tomassetti : MEASUREMENTS OF USEFUL
PARAMETERS WHICH INDICATE THE DIFFUSION PROCESSES
IN THE DYNAMIC POLARIZATION

# Measurements of Useful Parameters which Indicate the Diffusion Processes in the Dynamic Polarization.

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The behaviour of the dynamic polarization can be studied analitically by means of a differential-equation system, in which the polarization $p(t)$ and $P(t)$ of the two groups of spins $(I)$ and $(S)$ respectively, are present.

We refer to the protons $(I)$ of the crystallization water and to neodimium ions $(S)$ of the crystal:

$$La_2Mg_3(NO_3)_{12} \times 24H_2O + 1\% \ Nd \ .$$
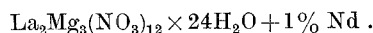
We shall infer some possible properties of the spin diffusion through measurements of the spin heat bath relaxation time of the protons, of the stationary value of the protons polarization, and of the time constant to build up this value. The differential equations, which describe the time evolution of the dynamic polarization, have the stationary solution for system $(I)$ given by

$$(1) \qquad \frac{p_s}{p_0} = \frac{T_I(N/n)\beta W}{1 + (1 + f) T_I(N/n)\beta W} \simeq \frac{W T_s(1 - P_0^2)^{-1}}{1 + W T_s(1 - P_0^2)^{-1}} \ ,$$

where $W$ is proportional to the pumping power, $P_0 = \mathrm{tgh}\,(h\nu_s/2KT)$, $p_0 = \mathrm{tgh}\,(h\nu_I/2KT)$ are the values of the polarization at thermal equilibrium with the heat bath of the spin $(S)$ and $(I)$ respectively, $f = (T_s/T_I)(n/N)$ is the ratio of the spin heat bath relaxation time multiplied by the ratio between the proton number and the ion number, $T_I = T_s(n/N)[\beta(1 - P_0^2)]^{-1}$, where $\beta$ is a coefficient due to the magnetic dipolar interaction between the spin $(S)$ and $(I)$.

The time constant with which $p(t)$ approaches the value $p_s$ is given by the approximate relation

$$(2) \qquad \tau^{-1} = T_I^{-1}[1 + W T_s(1 - P_0^2)^{-1}]$$

as one finds from the solution of the differential equations, note (*). Thus, by meas-

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(*) The differential equation system in ref. [1]

$$\dot{p}(t) = -(p - p_0)w\beta\frac{N}{n}\left(\frac{1}{P_0} - P_0\right) - \beta W(\nu_s)\frac{N}{n}(p \mp P) \ , \qquad \dot{P}(t) = -(P - P_0)\frac{w}{P_0} - \beta W(\nu_s)(P \mp p) \ ,$$

$w/P_0 = T_s^{-1}$, $\beta W(\nu_s)$ is the induced transition probability at frequencies $\nu_\pm = \nu_s \pm \nu_I$.

[1] V. Montelatici and G. Baldacchini: *Nuovo Cimento*, **68** B, 253 (1970).

uring the value of the quantities: $\tau$, $T_I$, $p_s/p_0$, one can verify the relation:

$$(3) \qquad \frac{\tau^{-1} - T_I^{-1}}{T_I^{-1}} = \frac{(p_s/p_0)(P_0/p_0)^{-1}}{1 - (p_s/p_0)(P_0/p_0)^{-1}} \,,$$

as a function of the relative power, after the magnetic field and the temperatures are fixed.

The measurements are made at $T = 1.6\ °K$ and $H = 9100\ G$ with an apparatus described in ref. [1,2]; the angle between the optical axis of the crystal and the magnetic field was $\simeq 90°$. The measurement of the relaxation time has been:

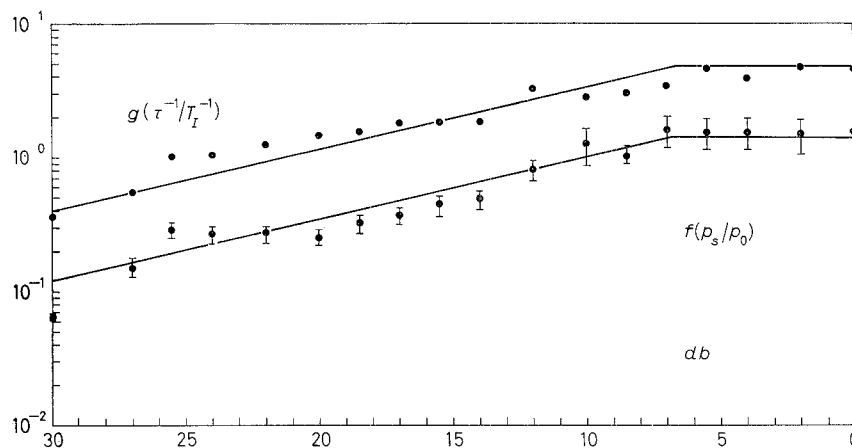$$T_I(9100\ g,\ 1.6\ °K) = 1140\ s \,.$$



Fig. 1. – Experimental points of the functions $g(\tau^{-1}/T_I^{-1})$ and $f(p_s/p_0)$ as a function of the relative power, 0db corresponds to the maximum power $p_0 = 5.94 \cdot 10^{-4}$; $P_0/p_0 = 7.92 \cdot 10^2$ for $H = 9100\ g$, $T = 1.6\ °K$.

In Fig. 1 the values of the functions

$$g\left(\frac{\tau^{-1}}{T_I^{-1}}\right) \equiv (\tau^{-1} - T_I^{-1})\,T_I \qquad \text{and} \qquad f\left(\frac{p_s}{p_0}\right) \equiv \frac{(p_s/p_0)(P_0/p_0)^{-1}}{1 - (p_s/p_0)(P_0/p_0)^{-1}} \,,$$

are reported as a function of the relative power.

The behaviour of the two functions is fairly good within the experimental error (10% on the values of $p_s/p_0$ and on the values of $\tau$ and $T_I$), but there is a discrepancy of a factor of about 3.5 on the numerical values.

We may observe that the expressions (1) and (2) are the same as those obtained from the theory of the solid effect in which one takes into account the diffusion effect (when there are no extraneous relaxations) [3]. This theory foresees a parameter func-

[2] V. MONTELATICI and G. TOMASSETTI: *Lett. Nuovo Cimento*, **3**, 391 (1972).

[3] G. R. KHUTSISHVILI: *Sov. Phys. Usp.*, **8**, 5 (1966); from the Russian edition *Usp. Fiz. Nauk*, **87**, 211 (1965).

tion of the pumping power

$$(4) \qquad\qquad b^* = b(1 + WT_s)^{\frac{1}{4}} \,,$$

where $b$ is the distance from the ion below which the protons are under the direct influence of the ion (*): $b(9100 \text{ G}, 1.6 \text{ °K}) \simeq 1 \text{ Å}$ for the crystal used.

From the value of the functions $g(\tau^{-1}/T^{-1})$ and (4) is deduced the maximum value of $b^*$, which is $\simeq 2b$.

This value implies a « rapid diffusion » process since $b^*$ is smaller than the diffusion barrier $\delta(\simeq 8 \text{ Å})$ as foreseen by the diffusion theory (**). In the case $b^* > \delta$ one has a « limited diffusion » process, where the stationary value (1) is still valid, but the time constant to approach this value is

$$(5) \qquad\qquad \tau^{-1}(b^* > \delta) = \frac{1}{1.6} \frac{\delta^3}{b^3} T_I^{-1}(b < \delta)(1 + WT_s)^{\frac{1}{4}} \,,$$

on the other hand when $b^* < \delta$, the diffusion theory gives

$$(6) \qquad\qquad \tau^{-1}(b^* < \delta) = T_I^{-1}(b < \delta)(1 + WT_s) \,,$$

which is the same as that derived from equation (***) (*₍*₎).

Now if we keep in mind that to reach the stationary value $p_s$ at higher temperatures, for example 4 °K, higher pumping power is necessary (since $T_s^{-1}$ is greater, $T_s^{-1}(9100 \text{ g}, 4.2 \text{ °K}) \simeq 8 \cdot 10^4 \text{ s}^{-1}$). Then $b^*$ may become greater than the diffusion barrier, since $b(9100 \text{ g}, 4.2 \text{ °K}) \simeq 3 \text{ Å}$. In this case there should be a different behaviour between the functions $g(\tau^{-1}/T^{-1})$ and $f(p_s/p_0)$, while there must be the same behaviour between the function $f(p_s/p_0)$ and the new function

$$(7) \qquad\qquad g^*\left(\frac{\tau^{-1}}{T_I^{-1}}\right) \equiv \left(\frac{\tau^{-1}}{T_I^{-1}} 1.6 \frac{b^3}{\delta^3}\right)^4 - 1 \,.$$

Thus measurements of $p_s/p_0$, $\tau$ and $T_I$ as a function of the pumping power at various temperatures should give information of the two different diffusion processes, namely the « rapid diffusion » $(b^* < \delta)$ and the « limited diffusion » $(b^* > \delta)$ during the build up of the polarization (*₍*₎). We notice that a sample such as our crystal, in which $b < \delta$, has the possibility to sustain both diffusion processes as the value of the pumpin power is varied. On the contrary one has a rapid diffusion process when the pumping power is switched off. To obtain $b > \delta$ one has to reach higher temperatures (about 10 °K

(*)   The scattering lenght $b = 0.68[(3/10)(\hbar\gamma_s/H)^2 D^{-1} T_s^{-1}]^{\frac{1}{4}}$ cm, $D \simeq 10^{-33}$ cm² s⁻¹ is the diffusion constant; $b(9100 g) \simeq 1.5 \cdot 10^{-1} (T_s^{-1})^{\frac{1}{4}}$Å, where $T_s^{-1} = 3 \cdot 10^{-18} H^5$ ctgh $(\hbar\nu_s/2KT) + 6 \cdot 10^9$ exp $[-47/T]$, ref. (⁴⁵).

(⁴)   P. L. SCOTT and C. D. JEFFRIES: *Phys. Rev.*, **127**, 32 (1962).

(⁵)   T. J. SCHUMUGGE and C. D. JEFFRIES: *Phys. Rev.*, A **138**, 1785 (1965).

(**)   The theory of the spin diffusion is reviewed in ref. (³), we have used the therminology of this article.

(***)   See footnote (*), p. 285.

(*₍*₎)   It can be shown that equations in footnote (*) are valid only under appropriate hypotheses,a nd they give the same results as the theory of diffusion for one case: that for which $b < b^* < \delta$. This discussion will be presented in a later article.

(*₍*₎)   Note that we attribute the terminology « limited diffusion » for $b^* > \delta$ and « rapid diffusion » for $b^* < \delta$. This is done for brevity since these two names are used when the pumping power is switched off, that is during the gradual approach to the thermal equilibrium of the proton polarization.

for our crystal) (*), then

(8) $$\tau^{-1}(b^* > \delta) = T^{-1}(b > \delta)(1 + WT_s)^{\frac{1}{4}},$$

where

(9) $$\frac{T(b > \delta)}{T_s} = \frac{16}{3}\left(\frac{H}{\hbar\gamma_s}\right)^2 R^3 \left\{0.68\left[\frac{3}{10}\left(\frac{\hbar\gamma_s}{H}\right)^2 D^{-1}\right]\right\}^{\frac{3}{4}}(T_s^{-1})^{\frac{3}{4}};$$

one the contrary, when $b < \delta$, one has

(10) $$\frac{T(b < \delta)}{T_s} = \delta^3 R^3 \frac{10}{3}\left(\frac{H}{\hbar\gamma_s}\right)^2,$$

where $R^3 = (V/N)(3/4\pi)$, $V = 1$ is the unite volume of the sample and $N$ is the concentration of ions.

From the measurements of the relaxation times one can ceck (9) and (10) as a function of the temperatures.

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(*)    We suppose $\delta$ independent of the temperature, for more detail see ref. [3,6].
[6]    H. E. RORSCHACH jr.: *Physica*, **30**, 38 (1964).