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On the Possibility of Measuring Meson and Baryon Electromagnetic-Structure Functions Using NAL and ISR (*).

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Using the high-energy machine at SLAC, it has been possible to measure the proton (and neutron, albeit less accurately) electromagnetic-structure functions (1), which has excited a great deal of theoretical interest due to their scaling properties (2). It is the purpose of this note to suggest the possibility of using the very high-energy « strong » machines (*e.g.* NAL) to measure the $\pi^{\pm}$, $K^{\pm}$ electromagnetic (e.m.) form factors and the inelastic-structure functions. Usually it is not possible to « see » the e.m. interactions in a hadronic process since they are overwhelmed by the strong interactions. But a strong process suffers from an exponential transverse momentum cut-off (roughly of the type $\exp[-3p_{\perp}^2]$). The e.m. vertex of a hadron has only a ($\sim$ power law) damping in the four-momentum transfer. Thus, if we consider a reasonably large $p_{\perp}$ (say, 3 GeV) and as small $-t(\geqslant p_{\perp}^2)$ as is kinematically allowed, one easily see that the e.m. cross-section is much larger than the strong one. One needs a very high-energy machine (say, $E_L \geqslant 200$ GeV) to keep-t very close to $p_{\perp}^2$ and yet remain in the interesting scaling ragion (*i.e.* $M_x^2 \gg m^2$), thus, keeping the cross-section not too low to measure.

Consider as a prototype the following inclusive reaction (Fig. 1):

$$\text{projectile } A(k) + \text{proton } (p) \to \text{proton } (p') + \text{anything}$$

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(1) E. D. BLOOM, G. BUSCHHORN, R. L. COTTRELL, D. H. COWARD, H. DE STABLER, J. DREES, C. L. JORDAN, G. MILLER, L. MO, H. PIEL, R. E. TAYLOR, M. BREIDENBACH, W. R. DITZLER, J. I. FRIEDMAN, G. C. HARTMANN, H. W. KENDALL and J. S. PAUCHER: in *Recent results in inelastic electron scattering*, SLAC Report, SLAC-PUB-796 (1970). Report presented to the *XV International Conference on High-Energy Physics* (Kiev, 1970).
(2) J. D. BJORKEN: *Phys. Rev.*, **179**, 1547 (1969).

in the one-photon–exchange approximation. Here A is used generally for any projectile (*e.g.* π, K or protons).

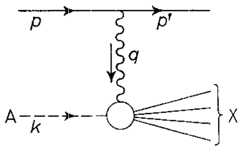

Fig. 1.

This inclusive differential cross-section can be written as

$$ {}^{(\gamma)}f_{\mathrm{AD}}^{(\mathrm{D})} \equiv \frac{\mathrm{d}^3\sigma}{\mathrm{d}^3\boldsymbol{p}/E_{\mathrm{D}}'} = \frac{\alpha^2 m_{\mathrm{A}}}{\sqrt{(k\cdot p)^2 - m_{\mathrm{A}}^2 m^2}}\, \frac{1}{Q^4}\, J_{\mu\nu}^{(\mathrm{D})}\, T_{(\mathrm{A})}^{\mu\nu}\,, \tag{1}$$

where

$$ J_{\mu\nu}^{(\mathrm{D})} = \frac{1}{2}\sum_{\mathrm{spin}} \langle p'|j_\mu|p\rangle\langle p'|j_\nu|p\rangle^* \tag{2a}$$

and

$$ T_{(\mathrm{A})}^{\mu\nu} = -(g^{\mu\nu} - q^\mu q^\nu/q^2)\, W_1^{(\mathrm{A})} + \frac{1}{m_{\mathrm{A}}^2}\, W_2^{(\mathrm{A})}\left(k^\mu - \frac{(k\cdot q)}{q^2}\, q^\mu\right)\left(k^\nu - \frac{(k\cdot q)}{q^2}\, q^\nu\right), \tag{2b}$$

$$ s = (k+p)^2\,, \qquad q^2 = (p-p')^2 \equiv -Q^2 < 0\,. $$

Defining the usual « scaling functions »

$$ F_1^{(\mathrm{A})} \equiv \frac{W_1^{(\mathrm{A})}}{(2m_{\mathrm{A}})}\,, \tag{3a}$$

$$ F_2^{(\mathrm{A})} \equiv \frac{(k\cdot q)}{m_{\mathrm{A}}}\, W_2^{(\mathrm{A})} \tag{3b}$$

with the Bjorken scaling variable $\omega = 2(k\cdot q)/Q^2$, we find to leading order in s

$$ {}^{(\gamma)}f_{\mathrm{AD}}^{(\mathrm{D})} \xrightarrow[Q^2,\ \omega\ \text{fixed}]{s\ \text{large}} \frac{\alpha^2}{(Q^2)^2}\cdot\frac{4s}{Q^2+4m^2}\left[\frac{F_2^{(\mathrm{A})}(\omega)}{\omega}\right]\left[G_{\mathrm{M}}^2(Q^2) + \frac{4m^2}{Q^2}\, G_E^2(Q^2)\right], \tag{4}$$

where G_E and G_{M} are the proton form factors ([3]).

Now, for what ranges of the parameters do we expect expression (4) to be much larger than the strong one (2)? Let us assume, to obtain an *order of magnitude estimate*, that the hadronic part goes as ([4])

$$ {}^{(\mathrm{h})}f_{\mathrm{pp}}^{(\mathrm{p})} \approx (10\ \mathrm{mb/(GeV)^2})\exp[-3p_\perp^2] \tag{5}$$

([3]) The normalization is the usual one so that approximately $G_E \simeq G_{\mathrm{M}}/(1+k)(1/[1+Q^2/0.7\ (\mathrm{GeV})^2]^2)$ where $k \simeq 1.79$ is the proton anomalous magnetic moment.

([4]) L. G. RATNER, R. J. ELLIS, G. VANNINI, B. A. BABCOCK, A. D. KRISCH and J. B. ROBERTS: *Phys. Rev. Lett.*, **27**, 68 (1971).

For $p_\perp = 3$ GeV, we obtain then

$$(6) \qquad {}^{(h)}f^{(p)}_{pp}(p_\perp = 3\ \text{GeV}) \leqslant 10^{-38}\ \text{cm}^2/(\text{GeV})^2\ .$$

This is an *over-estimate*, since the absolute value chosen is *larger* than the value extrapolated from the data of ref. ([4]), also the transverse momentum cut-off chosen is (probably) *smaller* than the expected one (for instance, the indications are that the cut-off is more like 3.5 $(\text{GeV})^{-2}$ ([4]), while dual models provide a value $\sim 4\ (\text{GeV})^{-2}$ ([5])).

Now, when is expression (4) large consistent with $p_\perp = 3\ \text{GeV}^2$? Due to the dampling in Q^2, present in the photon propagator as well as in the proton form factors, we wish to keep Q^2 small. For small angles, in the c.m. frame, we have at high energy

$$\theta^* \simeq \frac{Q^2}{p^* p_\perp}\ ,$$

where $p_\perp \simeq p'^* \theta^*$ and P^* is the c.m. incident momentum. This implies that $Q^2 > p_\perp^2$. Defining the Feynman scaling variable x, we may write

$$x = \frac{p'^* \cos\theta^*}{P^*} \simeq 1 - \frac{M_x^2}{s} - \frac{3m^2 - 2q^2}{s}\ ,$$

where M_x is the « missing mass », $M_x^2 = (p + k - p')^2$. Of course we wish to remain in the Bjorken scaling region where $\omega = (M_x^2 - m_A^2)/Q^2 + 1$ is not near 1 otherwise the function $F_2(\omega)$ vanishes (at least quadratically or even higher ([1])). For definiteness, consider the following values of the parameters which satisfy all the above constraints for NAL energies:

$$\text{I} \begin{cases} E_L = 200\ \text{GeV}\ , & p_\perp = 3\ \text{GeV}\ , \quad Q^2 = 10\ (\text{GeV})^2\ , \\ x = 0.87\ , & M_x^2 = 28.06\ , \quad \omega = 3.72\ . \end{cases}$$

For this case

$${}^{(\gamma)}f^{(p)}_{pp} \approx F_2^{(p)}(\omega = 3.72) \times \left(\frac{10^{-36}\ \text{cm}^2}{\text{GeV}^2}\right)\ .$$

From the SLAC-MIT experiment ([1]), we know that $F_2^{(p)} \simeq 0.3$ for $\omega \simeq 3.7$. We expect that F_2 for the π- or K-meson is also of the same order of magnitude in analogy with the hadronic total cross-section.

Thus we find that even with a rather conservative $p_\perp$ cut-off, the *one-photon–exchange graph dominates over the strong one and more importantly the cross-section is large enough to be measurable.* These experiments are in principle possible at ISR where values of s up to $\sim 2500\ (\text{GeV})^2$ are available, thus probing the proton structure functions in the truely deep inelastic region. The present ISR luminosity ($L \simeq 2 \cdot 10^{28}\ \text{cm}^{-2}\ \text{s}^{-1}$) is however too low to measure cross-sections as small as the one estimated above. Thus we have to wait, even for the proton case, for NAL. There the usual technique of using nuclei instead of proton can also augment the cross-sections roughly by a factor of $A^{\frac{2}{3}}$.

A glance at formula (4) shows that (in general) the e.m. term, bein gproportional to s, does *not* scale in the Feynman sense, while the strong part does (*i.e.* becomes inde

([5]) C. DE TAR, K. KANG, C. I. TAN and J. WEIS: *Phys. Rev. D*, **4**, 425 (1971).

pendent of s for large s). Thus, apart from the transverse-momentum cut-off, it is this factor which helps keep the e.m. contribution large. It is perhaps not without interest to remark, however, that for the kinematic configuration $M_x^2 \gg Q^2 \simeq p_\perp^2/x$, the e.m. part *does* scale in the Feynman sense, provided $F_2(\omega)$ approaches a constant for large ω. That is to say

$$ {}^{(\gamma)}f^{(p)}_{AD} \xrightarrow[s\to\infty]{x,p_\perp \text{ fixed}} \frac{\alpha^2}{(p_\perp^2)^2}\left[\frac{4p_\perp^2}{p_\perp^2+4m^2x}\right]\left[\frac{x^2}{1-x}\right] F_2^{(A)}(\omega=\infty)\left[G_M^2\left(\frac{p_\perp^2}{x}\right)+\frac{4m^2x}{p_\perp^2}\,G_E^2\left(\frac{p_\perp^2}{x}\right)\right] $$

tends to a *finite* limit, provided that $F_2^{(A)}(\omega=\infty)$ is finite. Of course, this result is a simple example of the « triple-Regge » graphs ([6]), which dominate when M_x^2 as well as s/M_x^2 are large. Here one « Regge » pole is the photon with $\alpha_\gamma(Q^2)=1$ and the other is the pomeron (diffractive part) with $\alpha_P(0)=1$, which keeps $F_2^A(\omega=\infty)$ finite.

Meson form factors. If we claim to know the proton structure functions in the scaling region, then by performing a slightly different experiment we can try to obtain the meson e.m. form factors. Consider (Fig. 2)

$$ \text{meson } (k) + \text{proton } (p) \to \text{meson } (k') + \text{anything} $$

in the one-photon-exchange approximation.

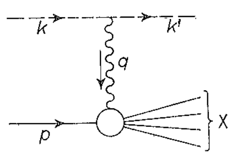

Fig. 2.

This inclusive differential cross-section can again be written as

$$ {}^{(\gamma)}f^{M}_{MD} = \frac{d^3\sigma}{d^3\boldsymbol{k}'/E'_K} = \frac{\alpha^2 m}{\sqrt{(k\cdot p)^2 - m^2\mu_M^2}}\,\frac{1}{Q^4}\,J^{(M)}_{\mu\nu}\,T^{\mu\nu}_{(p)}\,, $$

where the mesonic part is

$$ J^{(M)}_{\mu\nu} = \langle k'|j_\mu|k\rangle\langle k'|j_\nu|k\rangle^* $$

and $T^{\mu\nu}_{(p)}$ is as defined before in eq. (2*b*) with $k \leftrightarrow p$ and $m_A \leftrightarrow m$. (Here μ_M denotes the meson mass.) To leading order in s we obtain, for $Q^2 = -(k-k')^2$ and $\omega = 2(p\cdot q)/Q^2$ fixed,

$$ {}^{(\gamma)}f^{M}_{MD} \xrightarrow[Q^2,\omega \text{ fixed}]{s \text{ large}} \frac{\alpha^2}{(Q^2)^2}\,F_M^2(Q^2)\left(\frac{4s}{Q^2}\right)\frac{F_2^{(p)}(\omega)}{\omega}\,. $$

As expected this is very similar to eq. (4) with G_M for the proton replaced by the e.m. form factor F_M for the meson. Now if, as is generally believed, the meson form factors

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([6]) C. De Tar, C. E. Jones, F. E. Low, J. Yang and J. Weis: *Phys. Rev. Lett.*, **26**, 675 (1971).

do fall less fast than the proton one (7), ${}^{(\gamma)}f^{M}_{Mp}$ is even larger than in the case considered before and thus overwhelms even more the hadronic piece ${}^{(p)}f^{M}_{Mp}$. For example, for the kinematics set-up I, considered before, we can obtain an absolute value in the following way. Consider for definiteness the case of pions and approximate the isovector form factor by the ρ pole, *i.e.* choose

$$F_\pi(Q^2) = \frac{m_\rho^2}{m_\rho^2 + Q^2}\,.$$

We obtain, for $E_L = 200$ GeV, $p_\perp = 3$ GeV, $Q^2 = 10\ (\text{GeV})^2$, $F_2^{(p)}(\omega = 3.7) \simeq 0.3$

$${}^{(\gamma)}f^{\pi}_{\pi p} \simeq 1\cdot 10^{-35}\ \text{cm}^2/(\text{GeV})^2\,,$$

which is certainly large enough to be a measurable cross-section. Again using nuclei, the cross-sections can be enhanced by the usual $A^{\frac{2}{3}}$-factor.

Thus it seems to us that the method is feasible to determine π and K e.m. form factors using NAL for large Q^2. The « contamination » from purely strong interactions should be truely negligible for these large transverse momenta as evidenced by our earlier expression (6). It should be obvious that the value $p_\perp = 3$ GeV was chosen for $\exp[-3p_\perp^2]$. If, in fact, the cut-off is even stronger, one can select a lower value for $p_\perp$, thus resulting in larger e.m. cross-sections (for $Q^2 \simeq p_\perp^2$).

As a final important point, we wish to remark that in view of the estimates provided above, the interpretation of the large energy, large $p_\perp$ data in *purely dadronic processes* (*e.g.* $p + p \to p + X$), becomes highly unclear since the e.m. part of the amplitude also comes into play. Thus, in all those processes where one-photon exchange can occur, the $p_\perp$ behaviour as energy increases should show a discrepancy.

After the completion of this work, we become aware of some related attempts by BERMAN, BJORKEN and KOGUT (8) and by LOW and TREIMAN (9). The first of these relies heavily on the parton picture and an abundance of details about almost all leptonic, semi-leptonic and hadronic inclusive processes can be found here. The second work deals with the e.m. contribution to i) elastic hadronic scattering at large transverse momenta and ii) inclusive processes of the type $a + b \to X_a + X_b$.

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It is a pleasure to thank Prof. S. DRELL for helpful remarks.

(7) See, for instance, S. VITALE: in *Proceedings of the Informal Meeting on Electromagnetic Interactions* (Frascati, Roma, 1972).

(8) S. M. BERMAN, J. D. BJORKEN and J. B. KOGUT: SLAC-PUB-944 (August 1971).

(9) F. E. LOW and S. B. TREIMAN: NAL Preprint THY 20.