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G. Parisi : SERIOUS DIFFICULTIES WITH CANONICAL DIMENSIONS. -

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ABSTRACT. -

We present a strong non perturbative argument which implies that the fundamental fields have anomalous dimensions if the interaction is scale invariant. A physical consequence is that Bjorken scaling cannot be present in asymptotically scale invariant field theories.

A very interesting problem is to understand the observed simple properties of deep inelastic e-p scattering[1] in the framework of standard Lagrangian Quantum field theory. We know that Bjorken scaling[2] is connected with the light-cone singularities of the product of two currents[3]: the scaling law implies that they are as singular as in free field theory.

Under general assumptions we have proven[4] that canonical light cone singularities can be present only and only if the fundamental fields have canonical dimensions. In this letter we present a non perturbative argument which implies that in a renormalizable quantum field theory with adimensional coupling constant the fundamental fields must have anomalous dimensions. (The argument is valid for interaction of the type $\lambda\phi^4$, $\bar{\psi}\psi\sigma$, $\bar{\psi}\gamma_5\psi\pi$, some care must be taken in the gluon model, where the Hilbert space of the states has no more positive metric and the dimensions of the fermion field is a function of the gauge).

It is well known that canonical dimensions are possible if and only if there exist a value of the renormalized coupling constant $g_c \neq 0$ which is a simultaneous zero of the two Callan[5] Symanzik[3] functions $\beta(g)$ and $\gamma(g)$. We prove that this is impossible, the only simultaneous zero being located at $g = 0$.

We consider the C. S. eq. for $g = g_c$; all the anomalous term are absent and it reads as:

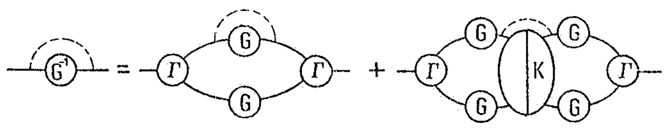
(1) $$\left[K_i \frac{d}{dK_i} - 6 + 2N\right] \Gamma^N (K_1, \ldots, K_{N-1}) = \eta(g_c)\, m^2 \frac{d}{d m_o^2}\, \Gamma^N (K\ , \ldots K_{N-1})$$

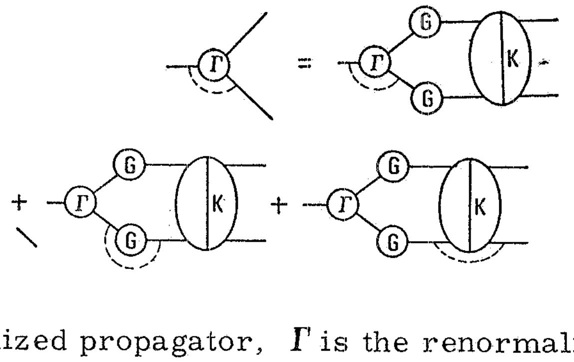
Standard arguments based on the Weimberg theorem[7] suggest that the r. h. s. can be asymptotically neglected in the deep euclidean region: the propagator will be $G(K^2) = 1/K^2 + m^2 0(1/K^4)$; also for the general N point function in the deep euclidean region the preleading terms will be a factor $1/K^2$ smaller than the leading ones. Because of the Federbush Johnson Pohlmeyer[8] theorem the theory is asymptotically free and only the disconnected part of the leading terms of high order Green functions will be different from zero.

As a consequence the dressed vertex will go to zero at infinity a power of K^2 faster that in perturbation theory. We stress that this is true only at $g = g_c$; for different values of the coupling constant the speed of the approach to the asymptotic region is ruled by the form of the zero of function $\beta(g)$[6, 9].

Looking to the renormalized equation of motion we find that this smooth behaviour of the vertex may be present only in a free theory. For sake of diagrammatical semplicity we consider the very unphysical case of a $\lambda\phi^3$ in six dimensions: the whole argument can be extended to the other cases only drowing more complicated diagrams.

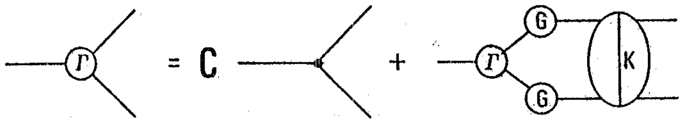
The Dyson equations can be written as[10]:

(2) 

(3) 

G is the renormalized propagator, Γ is the renormalized vertex and K is the Bethe Salpheter kernel, which can be written as the sum of all two particle irreducible skeleton graphs. The dushed lines denote differentiation respect to the momenta connecting the two points. The recorsive solutions of these equations with the condition $\Gamma(0, 0, 0) = g$ yields standard perturbation theory.

If we have no trouble on the convergence of the integrals eq. (3) can be rewritten as

(4) 

'C is an opportune constant that should not be identified with the physical coupling constant. We assume that all these integrals are convergent: they are only logarithmical divergent in perturbation theory and now the effective interaction is more soft that the bare one.

If we send all the momenta to infinity we find that the left hand side go to zero, while the left hand side is proportional to C. C must be zero and a bootstrap type condition must be satisfied for $g = g_c$. However, doing some manipulations which are justified by the convergence of the integrals, it is an easy task(11) to show that the self energy becomes a constant if we let the bootstrap condition eq. (4) into eq. (2). The propagator is just the free one, the theory is free(8) and g_c must be equal to zero. Canonical dimensional are present only in free theories.

The really delicate point of the whole argument is the use of the Weimberg theorem outside perturbation theory.

The conclusions of this letter are that, if some very strange patalogies are not present, it is impossible that fields have canonical dimension in presence of a scale invariant interaction. In this type of theories Bjorken scaling cannot be valid, however each of the integrals

$$\int_1^\infty F_{1,2}(\omega, q^2)\, \omega^{-N}\, d\omega$$

still scales in a not trivial way: they(12) must be asymptotically proportional to

$$C_N \left(\frac{q^2}{M^2}\right)^{\sigma_N} .$$

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