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G. Parisi: EXPERIMENTAL LIMITS ON THE VALUE OF THE ANOMALOUS DIMENSIONS. -

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ABSTRACT -

We show how to use the data on deep inelastic e-p scattering to put bounds on the values of the anomalous dimensions of the operators involved in the Wilson expansion of the product of two currents near the light-cone. Anomalous dimensions of the order of unity are found not to be in contrast with present experimental evidence.

Everybody knows that Bjorken(1) scaling is violated in perturbation theory, because of the appearance of annoying $\log(q^2)$(2). This fact may be completely irrelevant because perturbation theory becomes useless in strong interaction; however there are general theoretical arguments which suggests that Bjorken scaling is still not valid also in the full theory(3).

It remains true that in the large q^2 region the following integrals scale(4, 5)

$$(1)\qquad \int_1^{\infty} F_2(\omega, q^2)\ {}^{-N}\, d\omega \rightarrow C_N \left(\frac{M^2}{q^2}\right)^{\sigma_N/2}$$

but Bjorken scaling can be true only if $\sigma_N = 0\ \forall\ N$.

The physical meaning of σ_N is the anomalous dimension of the operator of spin N which yelds the leading term in the Wilson expansion of two currents near the light-cone(6). The conservation of the energy momentum tensor suggests that $\sigma_2 = 0$; arguments based on positivity imply that $\sigma_{N+1} \geq \sigma_N$.

In this letter we assume that Bjorken scaling is violated e.i. all the σ_N, $N \neq 2$ are different from zero; we try to use the SLAC data on deep inelastic e-P scattering to obtain some bound on the value of σ_N.

In order to define the problem we choose a reasonable but arbitrary parametrization of the anomalous dimensions:

(2) $$\sigma_N = A \left[1 - \frac{12}{(N+1)(N+2)} \right]$$

This type of parametrization is very similar to the results of recent calculations(5)(7). We try now to find out the possible values for A.

In the large q^2 region the old scaling law $F_2(\omega, q^2) = F_2(\omega, K^2)$ is no more valid; there are more complicated relation which allow to compute $F_2(\omega, q^2)$ for all ω and q^2 once we now it as function of at one fixed value $q^2 = K^2$.

The new "scaling law" which may be obtained expanding the r.h.s. of eq. (1) in power of lg (q^2/K^2) and using the faltung Theorem for the Mellin transforms,

(3)
$$F_2(\omega, q^2) = F_2(\omega, K^2) \left[1 - \Delta(\omega, K^2) \frac{A}{2} \lg\left(\frac{q^2}{K^2}\right) + O\left(A^2 \lg^2\left(\frac{q^2}{K^2}\right)\right) \right]$$
$$\Delta(\omega, q^2) = +1 - 12 \int_{1/\omega}^{1} dx (x^2 - x^3) \frac{F_2(\omega x, q^2)}{F_2(\omega, q^2)}$$

If the function $\Delta(\omega, q^2)$ were identically equal to a constant δ, $F(\omega, q^2)$ would be of the form $f(\omega)(M^2/q^2)^{-\delta A}$, the fact that Δ is not constant indicates that the dependence from q^2 in the asymptotic region is more complicated; the physical meaning of this function is clear: $\Delta(\omega, q^2)$ is proportional to the violation in each point of Bjorken scaling law. A typical plot of Δ is shown in fig. 1.

We note that Δ goes to 1 for ω near to 1 and goes to -1 for ω going to infinity and it is much smaller in modulus than 1 in the central region $4 \leq \omega \leq 8$. This form of the function Δ is quite insensitive to changements of the parametrization used, but follows from the

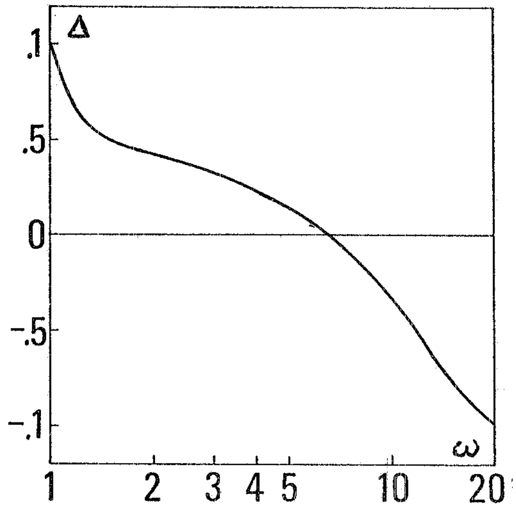

FIG. 1

fact that $\int \Delta(\omega, q^2) F_2(\omega, q^2)\, \omega^{-2} d\omega$ must be zero because $\sigma_2 = 0$.

It is clear that scaling in the central region of ω will be very good nearly indipendently of the amount of violation of scaling. If scaling violations are present the function F should be decreasing with q^2 at small ω and increasing at large ω.

It is interesting to note that this type of behaviour of the function $F_2(\omega, q^2)$ has been experimentally observed[8], however at too low values of q^2 and W to be clearly distinguishible from preasymptotic terms that should die at higher energy.

From the data in the high and low ω region we find that A may be as large as ~ 0.8; If we use also the good "scaling" data at $\omega = 4$ we find $A \lesssim \sim 0.5$. It is amusing that such big violation of the scaling law are not excluded by present data. Their presence may be an explanation of the fact that the "scaling limits" seems to be reached from above at small ω and from below at large ω.

Thanks are due to prof. A. Muller for a useful discussion.

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