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SHIELDING FOR ULTRARELATIVISTIC MUONS*

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The survival probability $P_E(x)$ of a muon penetrating through a thickness x of material is calculated with the Monte Carlo method. Considering standard earth as shielding material, the values of $P_E(x)$ and range distribution $R_E(x)$ are given for $E = 100$, 200 and 500 GeV.

1. Introduction

In designing shielding around very high energy accelerators, radio protection from muons in hadronic cascades or from muon beams however they may be produced, is of primary importance.

The difficulties encountered in calculating the thickness necessary to reduce muons to safe levels are represented:

a) by defective knowledge of the energy loss of high-energy muons for certain processes;

b) by the fact that energy losses are subject to fluctuations. For this reason, the muon maximum range may differ from the mean range used in calculation. This difficulty arises when the flow levels must be reduced by factors between 10^{10} and 10^{12}.

The purpose of this paper is to calculate, with the Monte Carlo method, the survival probability[1]) $P_E(x)$ of a muon of energy E that must penetrate through a thickness x of material.

2. Muon energy loss

The main processes through which muons lose energy are:

1) collision with excitation and ionization of atoms;
2) pair production;
3) bremsstrahlung;
4) nuclear interactions.

2.1. Collision processes

For these processes we use the results given by Rossi[2]), modified for particles of spin $\frac{1}{2}$,

$$-\frac{dE}{dx} = \frac{2Cm_e}{\beta^2}\left[\ln\frac{2m_e\beta^2 E}{(1-\beta^2)\,I^2(Z)} - 2\beta^2 + 0.25\right], \quad (1)$$

where $I(Z)$ is the average ionization potential of the atom of atomic number Z [for this we can put $I(Z) = I_H Z$ where $I_H = 13.5$ eV is the energy corresponding to Rydberg's frequency] and $C = \pi N(Z/A)r_e^2 = 0.15\,Z/A\ \mathrm{g^{-1}\,cm^2}$; the meaning of the other symbols is well-known.

This formula neglects the density effect, which is within 1% and which regards small energy transfers.

The fluctuation problem was faced using the solution given by Symon[3]).

2.2. Pair production

Most solutions regarding energy loss by pair production refer to the extremes "no shielding" and "complete shielding" of the atomic electrons.

The intermediate case was only recently resolved by Kelner et al.[4]) for relativistic muons through the use of the model of Thomas for the shielding effect. The diagrams of Feynman used are the usual ones which, in the theory of the lowest disturbance order, correspond to the pair producing process. The differential probability for the pair production in a case of this kind is given by:

$$d\sigma_{pp} = \frac{16}{\pi}\,(Z\alpha r_e)^2\,F(E,v)\,\frac{dv}{v}, \quad (2)$$

where v is the fraction of energy lost by the muon in the collision. The values of the function $F(E,v)$ for different values of v and E are tabulated in the above-mentioned work by Kelner et al.[4]).

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From eq. (2) the same authors derive the following expression for muon energy loss through the pair producing process

$$-\frac{1}{E}\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{16}{\pi}(Z\alpha r_e)^2\, n\chi(E), \tag{3}$$

where n is the number of atoms per unit of volume and $\chi(E)$ the integral of $F(E,v)$ extended to all possible energies transferred in the process in question.

2.3. Bremsstrahlung

At present, there are two good estimates of this contribution.

The first is that of Rozental[5]), who calculates the radiative cross section $\sigma_b(E,v)$ for muons of energy $> 10^2$ GeV on a stationary nucleus with corrections owing to the influence of the medium, to recoil, to the interactions of muons with electrons and to radiative corrections.

The second estimate we use is that of Erlykin[6]), which takes into account the finite dimension of the nucleus and the incompleteness of Born's approximation. The differential cross sections in the field of the nucleus is

$$\frac{\mathrm{d}\sigma_b}{\mathrm{d}v} = \alpha Z^2 r_\mu^2 \frac{R(E,v)}{v}, \tag{4}$$

where $R(E,v)$ is a function including all correcting terms[6]) and r_μ is the muon electromagnetic radius.

The contribution of atomic electrons to the cross sections is described by means of four interaction diagrams. Two of them are effective only up to energies of a few hundred GeV and the corresponding processes are treated as Compton virtual scattering in the muon field.

It follows that

$$\frac{\mathrm{d}\sigma_b}{\mathrm{d}v} = \left[\frac{\mathrm{d}n(E,v)}{\mathrm{d}v}\right]\sigma_{\mathrm{tot}}(E,v), \tag{5}$$

where $\mathrm{d}n/\mathrm{d}v$ is the spectrum of the virtual photons associated with the muon and σ_{tot} considers both the case of the free and the recoil electron.

The other two diagrams correspond to processes that are fairly similar to those in the field of the nucleus.

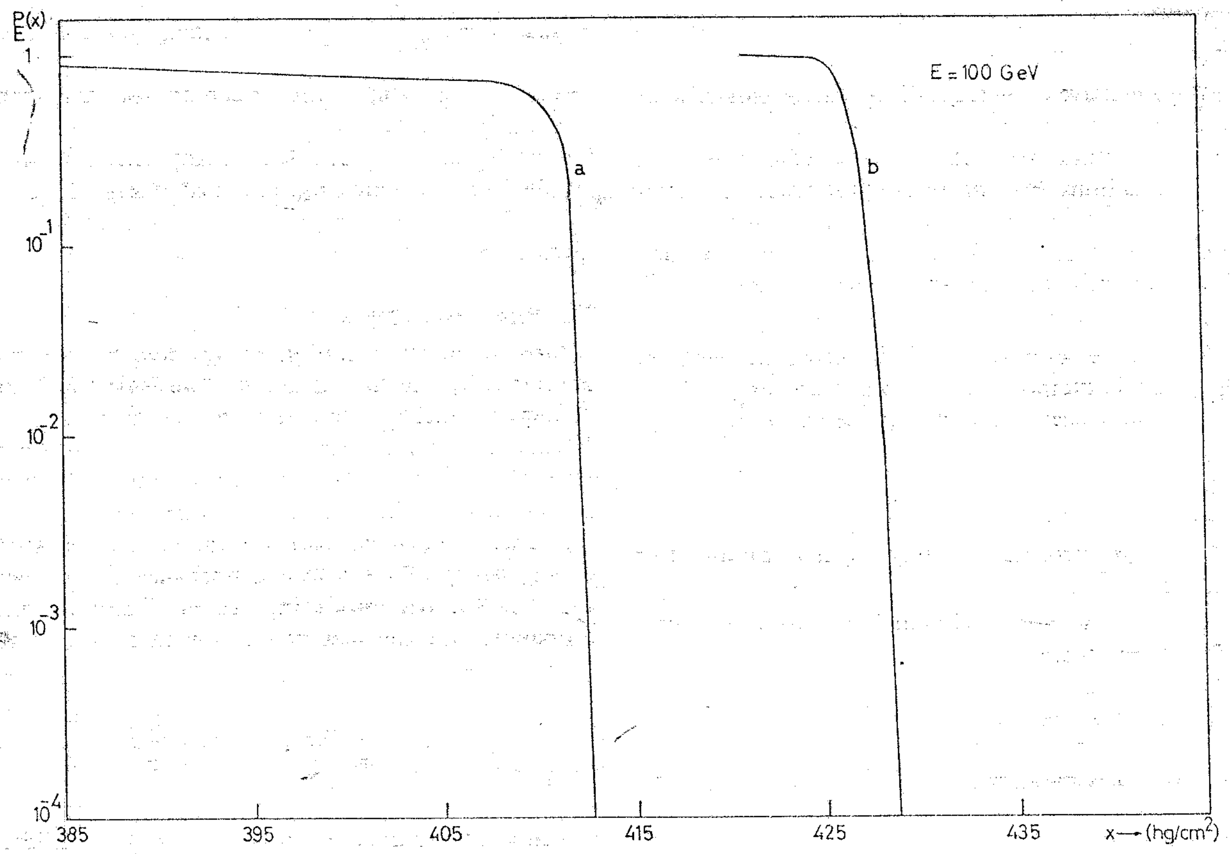

Fig. 1. Survival probability $P_E(x)$ of a muon that has penetrated through a thickness x at $E = 100$ GeV. The curve (a) refers to $P_E(x)$ taking into account all the processes of energy loss, and the curve (b) refers to $P_E(x)$ for ionization only.

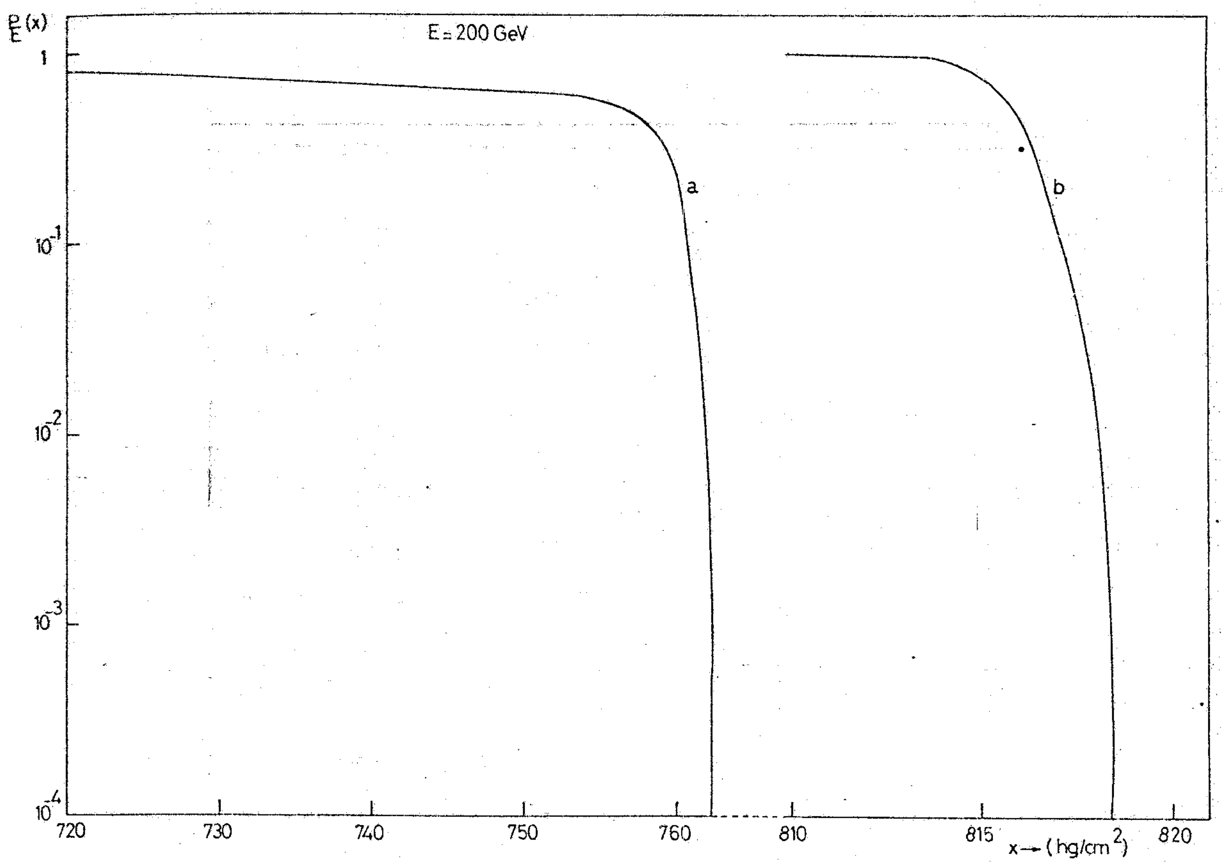

Fig. 2. As fig. 1, for $E = 200$ GeV.

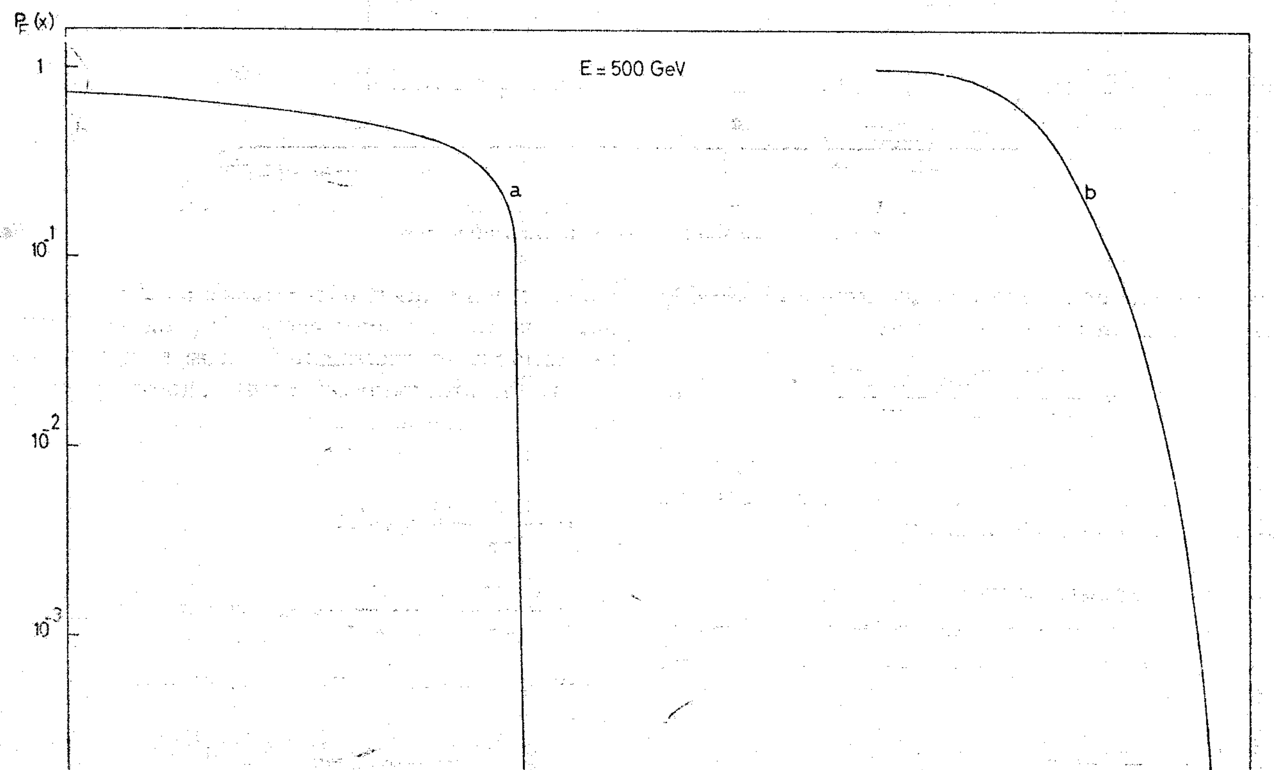

Fig. 3. As fig. 1, for $E = 500$ GeV.

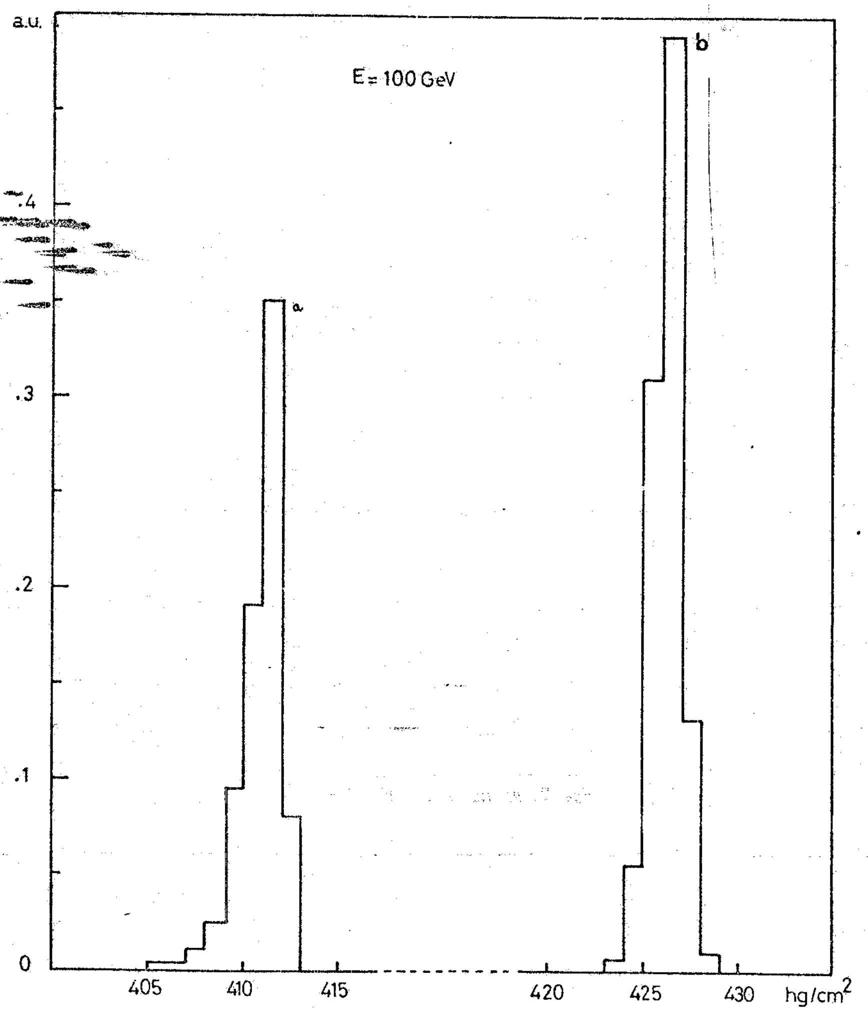

Fig. 4. Range distribution $R_E(x)$ as a function of x for $E = 100$ GeV. Curve (a) refers to $R_E(x)$, taking into account all processes of energy loss, curve (b) to $R_E(x)$ for ionization only.

The contribution of the atomic electrons is obtained by replacing Z^2 with $(Z+\varepsilon_r)Z$, where

$$\varepsilon_r = \frac{\ln[1440(m_\mu/m_e)\,Z^{-\frac{2}{3}}]}{\ln[191(m_\mu/m_e)\,Z^{-\frac{1}{3}}] - F}, \tag{6}$$

F being the sum of the two corrections owing to the incompleteness of Born's approximation and to the finite dimensions of the nucleus.

2.4. Nuclear interactions

The general expression for the inelastic differential cross section for nuclear interaction processes is given by[7])

$$\frac{d^2\sigma}{dq^2\,d\nu} = \frac{4\pi\alpha^2}{q^4}\,\frac{1}{E^2} \times$$

$$\times\,[\tfrac{1}{2}q^2\,W_1(q^2,\nu) + (E^2 - E\nu - \tfrac{1}{4}q^2)\,W_2(q^2,\nu)], \tag{7}$$

where q^2 is the square of the transferred 4-momentum and ν the virtual photon energy; W_1 and W_2 are the two form factors referring to the cross sections σ_T and σ_L for the absorption of virtual photons polarized transversally and longitudinally respectively.

W_1 and W_2 are given by

$$W_1 = \frac{\nu - q^2/2M}{4\pi^2\alpha}\,\sigma_T(q^2,\nu),$$

$$W_2 = \frac{\nu - q^2/2M}{4\pi^2\alpha}\,\frac{q^2}{q^2+\nu^2}\,[\sigma_T(q^2,\nu) + \sigma_L(q^2,\nu)]. \tag{8}$$

For small values of $q^2\,(q^2 < m_\rho^2)$ as in our case, we have

$$\sigma_T = \sigma_{\gamma N}(\nu)\left(\frac{m_\rho^2}{m_\rho^2+q^2}\right)^2,$$

$$\sigma_L = 0, \tag{9}$$

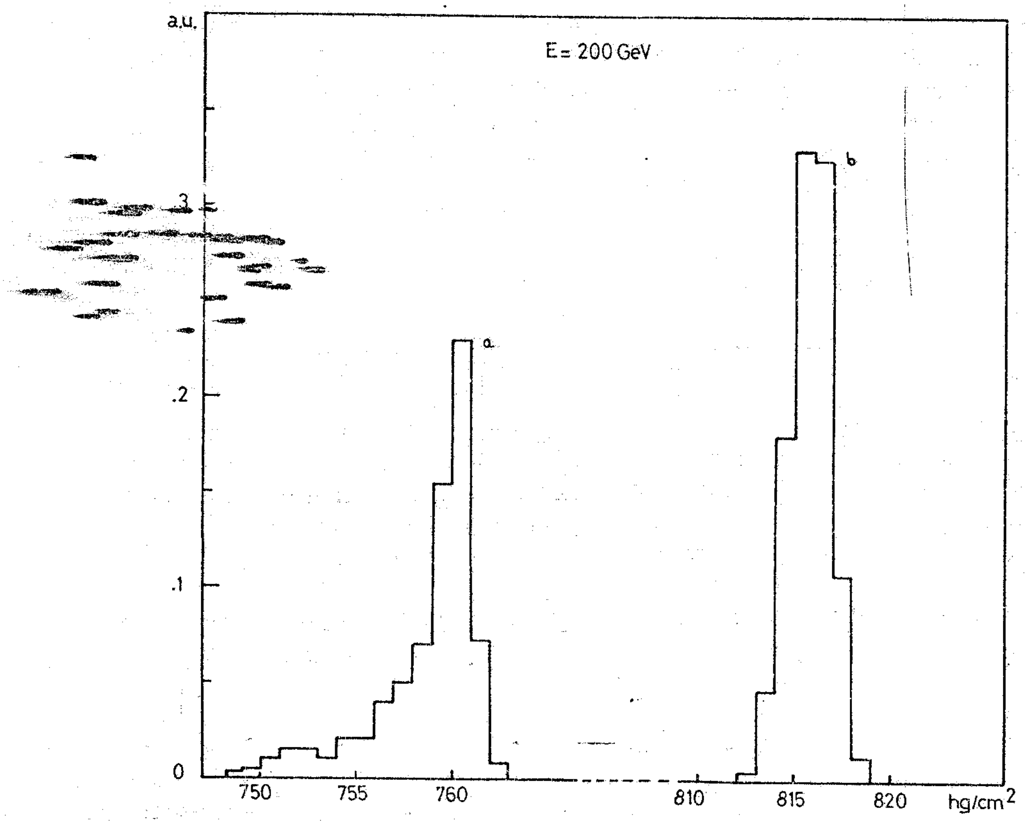

Fig. 5. As fig. 4, for $E = 200$ GeV.

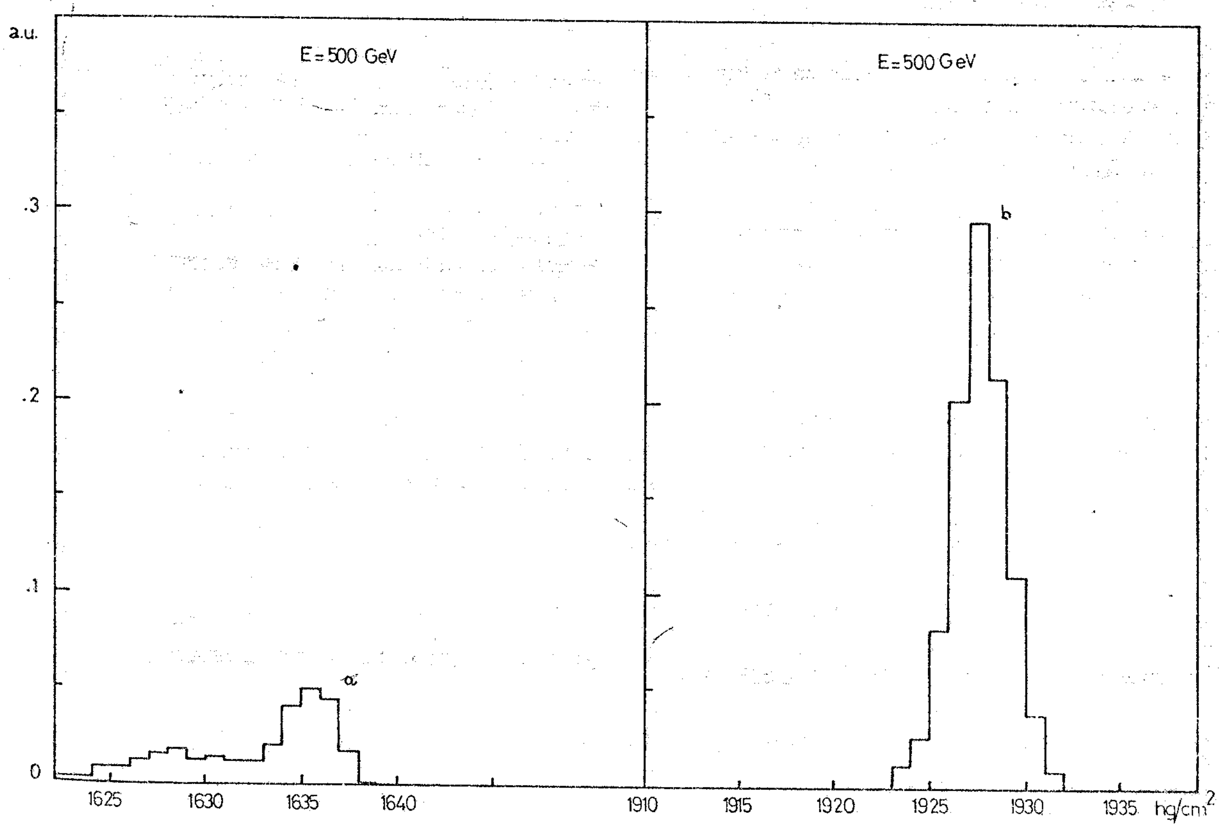

Fig. 6. As fig. 4, for $E = 500$ GeV.

where $\sigma_{\gamma N}$ is the cross section for the absorption of real photons and m_ρ the mass of the meson ρ. The crucial point is the lack of knowledge of $\sigma_{\gamma N}$ for $\nu > 4$ GeV.

Recent data on cross sections obtained with an accuracy of 2% in H and D give the following fit for $\nu > 4$ GeV: for protons $\sigma_{\gamma p} = (94+79\,\nu^{-\frac{1}{2}})$ μb and for neutrons $\sigma_{\gamma n} = (95+47\,\nu^{-\frac{1}{2}})$ μb.

From the results obtained[8]) in C, Cu and Pb we also have the relation between the cross sections of the nucleus and the nucleon

$$\sigma_{\gamma A} = A^{0.9}\,\sigma_{\gamma N}.$$

3. Calculation of $P_E(x)$

As shielding material we considered standard earth ($Z = 11$ and $A = 22$). The thickness is divided into intervals Δx of 100 g cm^{-2}.

With the Monte Carlo method, the fraction v of energy loss in Δx for the four processes is calculated by a muon of energy E, since

$$\int_0^v P(E,v)\,\mathrm{d}v = r, \tag{10}$$

where $P(E,v)$ is the probability of leaving the energy fraction v, and r is a number chosen at random between 0 and 1.

A muon is followed until it dies or succeeds in penetrating through a thickness $n\Delta x = x$ fixed. The values of $P_E(x)$ for $E = 100$, 200 and 500 GeV are shown in figs. 1, 2 and 3. We considered energy losses both in ionization and total.

In figs. 4, 5 and 6 we give the range distribution $R_E(x)$. For $R_E(x)$ we have

$$R_E(x) = \frac{P_E(n\Delta x) - P_E[(n+1)\Delta x]}{\Delta x},$$

which is useful in reduction calculation of muon flow.

4. Conclusions

The conclusions to be drawn from these first results are as follows:

a) Fluctuations are naturally due mostly to discontinuous processes which fortunately, however, produce a notable tail only in the initial part of $R_E(x)$.

b) Fluctuations in muon range are not very important at the energies E of present-day accelerators.

Roe[9]) calculated the fluctuations analytically only for energy losses by ionization and pair production and eliminated all catastrophic interactions in range calculation.

His range estimate, starting from such a restrictive hypothesis, leads to the consideration, in muon flow reduction, of thicknesses greater than those required by our Monte Carlo method.

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