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G. Pancheri-Srivastava: REGGEIZATION OF THE PHOTON IN QUANTUM ELECTRODYNAMICS. -

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ABSTRACT

Under certain assumptions it is shown that in massless QED the photon may reggeize. Our technique involves a comparison of the infrared spectrum with that of the di-triple Regge expansion of inclusive processes.

(x) - Scholar of the Radcliffe Institute, 1972-1973

Many years ago, Gell-Mann et al.[1, 2, 3] showed that in spinor-massive neutral vector meson scattering the spinor particle reggeizes (i.e. even though it is introduced initially as an elementary particle, it lies on a Regge trajectory when one considers higher orders). Mandelstam[4] also considered the problem of Reggeization of the vector particle and answered in the negative - at least up to 3-particle unitarity terms. However, the problem of the "true" (i.e. massless photon) QED has remained an open one. We use an approach different from the above authors to show that in true QED the photon may indeed reggeize.

Our basic idea is this: in QED any measurable reaction between charged particles is necessarily an inclusive one because of soft photon radiation. Thus, the question of Reggeization must involve a consideration of inclusive amplitudes. Since the so called "triple" and "di-triple" Regge limits of inclusive spectra are now available[5, 6] as well as the infrared corrected cross section in the appropriated exponential form[7], such a question can be answered as follows.

Let us consider the following inclusive reaction in QED

(1) $$e(p_1) + e(p_2) \longrightarrow e(p_3) + e(p_4) + X$$

where X stands for "anything" (made up of the photons or pairs produced by photons). Process (1), in the limit in which the undetected X four-momentum is much smaller than that of any of the detected ones, is really ee elastic scattering accompanied by the usual soft photon emission. Hence, in the above limit the cross section for process (1) is given by ee elastic cross-section corrected for the infrared radiation. Let us call such a cross-section $d\sigma^{(QED)}$. On the other hand, at very large energies (and fixed momentum transfers) a 2-particle inclusive reaction enters into the so-called di-triple Regge region[6] and the relative cross-section has a limit (the di--triple Regge limit) which is an explicit function of the Regge trajectories exchanged. Call such a limiting cross-section $d\sigma^{Regge)}$. Thus, it is expected that a comparison between $d\sigma^{(QED)}$ (if it reggeizes) and $d\sigma^{(Regge)}$ for process (1) will allow us to determine the photon trajectory in QED. In order to do so, we first obtain an expression for $d\sigma^{(QED)}$. This expression in the high energy, fixed momentum transfer limit explicitly shows a factorization into an energy dependent part (which is raised to a power which is a function of the momentum transfer) times an energy independent function (dependent on the momentum transfer). This is to be compared with the di-triple Regge limit.

The infrared problem has been widely studied but for our purpose it is most convenient to refer to a work done by the author together with E. Etim and B. Touschek[17], where the infrared corrections to all orders in e^2 were determined by the use of the Bloch-Nordsieck theorem. To wit, the cross section for the process[(x)]

$$e^-(p_1) + e^-(p_2) \longrightarrow e^-(p_3) + e^-(p_4)$$

can be written, in case of no momentum resolution, as

$$\frac{d\sigma}{dt} = N^{-1} \left(\frac{\Delta E}{E}\right)^{\tilde{\beta}} \frac{d\tilde{\sigma}}{dt} \tag{2}$$

where N^{-1} is a normalization factor which depends on $\tilde{\beta}$, ΔE the energy resolution, E the C.M. energy and t the momentum transfer. Since both $d\tilde{\sigma}/dt$ as well as $\tilde{\beta}$ require some explanation, we shall first analyze $\tilde{\beta}$, leaving aside for the moment $d\tilde{\sigma}/dt$. $\tilde{\beta}$ is an invariant function of the electron momenta and is defined as

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(x) - In ref. (7) the process examined in detail is $e^+e^- \longrightarrow \mu^+\mu^-$ but the extension to $ee \to ee$ is straight forward.

(3) $$\tilde{\beta} = -\frac{e^2}{(2\pi)^2}\int d^2\hat{n}\,\Big(\sum_{i=1}^{4}\frac{p_{i\mu}\epsilon_i}{(p_i\cdot n)}\Big)\Big(\sum_{j=1}^{4}\frac{p_j^{\mu}\epsilon_j}{(p_j\cdot n)}\Big)$$

where n is such that $n^2 = 1-|\hat{n}|^2 = 0$, and $\mathcal{E}_i = \pm 1$ for the creation or distriuction of a negative particle.

In the "quasi-elastic" approximation, i.e. $p_1+p_2-p_3-p_4 \simeq 0$, eq. (3) can be rewritten as

(4) $$\tilde{\beta} = \frac{2e^2}{\pi}\left\{-I_{12}+I_{13}+I_{14}-2\right\}$$

where

$$I_{ij} = 2(p_i\cdot p_j)\int_0^1 \frac{dy}{2y(1-y)\left[(p_i\cdot p_j)-m^2\right]+m^2}$$

We notice here that the elastic approximation corresponds to a photon momentum much smaller than that of any of the interacting electrons. Defining

$$s = (p_1+p_2)^2, \quad t = (p_1-p_3)^2, \quad u = (p_1-p_4)^2$$

we can take the high energy, fixed t limit of eq. (4). We get

$$I_{12} \xrightarrow[s\to+\infty]{} -2\ln\frac{m^2}{S}\;;\qquad I_{14} \xrightarrow[u\to-\infty]{} -2\ln\left|\frac{m^2}{\mu}\right|$$

so that

$$I_{12}-I_{14} \xrightarrow[\substack{\text{large } s\\ \text{fixed } t}]{} 0$$

and we can write

(5) $$\tilde{\beta}(s,t,u) \xrightarrow[\text{fixed } t]{\text{large } s} \beta(t) = \frac{2e^2}{\pi}\left[(2m^2-t)\int_0^1 \frac{dy}{m^2-ty(1-y)} - 2\right]$$

The integral which appears in (5) is an analytic function of t with a cut from $t = 4\,m^2$ to ∞ and is thus regular in s-channel physical region.

We now turn to the meaning of $d\bar{\sigma}$. Following the procedure of ref. (7), we expand eq. (2) in powers of $\tilde{\beta}$ (i.e. in powers of e^2) to obtain, in first approximation,

$$\frac{d\sigma}{dt} \approx (1 - \tilde{\beta} \ln \frac{E}{\Delta E}) \frac{d\bar{\sigma}}{dt}$$

The lowest order radiative corrections obtained from perturbation theory can always be written in the form

$$\frac{d\sigma}{dt} \approx \left[1 - \tilde{\beta} \ \ln \frac{E}{\Delta E} + \lambda\right] \frac{d\sigma_o}{dt}$$

where $d\sigma_o/dt$ is the Møller cross-section. The term λ, which may depend upon the energy and the momentum transfer but not on ΔE, represents what may be called the genuine (as opposed to the infrared) radiative corrections. It has been calculated by many authors[8, 9]. An approximate definition of $d\bar{\sigma}/dt$ in terms of $d\sigma_o/dt$ of perturbation theory is then

(6) $$\frac{d\bar{\sigma}}{dt} \approx (1+\lambda) \frac{d\sigma_o}{dt}$$

This equation tells us that $d\sigma/dt$ in the large s, fixed t limit is only a function of t, at least up to order e^6. In fact we can see from ref. (9) for instance that both λ as well as $d\sigma_o/dt$ have no energy dependence in such limit. From eq. (6) we then write

$$\frac{d\bar{\sigma}}{dt} \xrightarrow[\text{fixed } t]{\text{large } s} \bar{f}(t)$$

We leave to future investigations the correctness of the above limit at all orders of perturbation theory, but we feel that at least up to order e^6 we can write

$$\frac{d\sigma}{dt} \xrightarrow[\text{fixed } t]{\text{large } s} \left(\frac{\Delta E}{\sqrt{s}}\right)^{\beta(t)} f(t) \tag{7}$$

The cross section in (9) corresponds to an experiment in which only the energy of the outgoing electrons is measured and the missing energy has been integrated up to its maximum value ΔE. Notice that eq. (9) is valid only if the undetected four momentum $\Delta P = (\Delta E, \Delta \vec{P})$ is much smaller than any of the electron momenta so that at very high energies this allows ΔE to be relatively large and still the process to be almost elastic. If we then define the five independent invariants for the inclusive process (1) as

$$s_1 = (p_1+p_2-p_3)^2,\ s_2 = (p_1+p_2-p_4)^2,\ t_1 = (p_2-p_4)^2,\ t_2 = (p_1-p_3)^2,$$
$$M_x^2 = (p_1+p_2-p_3-p_4)^2$$

we see that limit (7) corresponds to $t_1 \simeq t_2 = t$ fixed, $M_x^2 \ll s$, s_1 and s_2. In other words, eq. (17) is valid in a subset of the di-triple Regge region of process (1).

The di-triple Regge limit for process (1) is

$$\frac{d^4\sigma}{dt_1 dt_2 d\left(\frac{M_x^2}{s_1}\right) d\left(\frac{M_x^2}{s_2}\right)} \xrightarrow[\substack{s/M_x^2,\ s_1/M_x^2 \text{ and} \\ s_2/M_x^2 \text{ large}}]{\substack{t_1, t_2 \text{ fixed} \\ M_x^2 \text{ large}}} \frac{1}{s}(M_x^2)^{\alpha_V(0)} \times$$

$$\times \left(\frac{s_1}{M_x^2}\right)^{2\alpha_1(t_1)} \left(\frac{s_2}{M_x^2}\right)^{2\alpha_2(t_2)} \mathfrak{F}(t_1, t_2) \tag{8}$$

where we have assumed that the vacuum singularity is a simple pole and $\alpha_1(t) = \alpha_2(t) = \alpha_\gamma(t)$ is the photon Regge trajectory.

If we further assume $\alpha_V(0) = 1$, eq. (8) becomes

$$\frac{d^4\sigma}{dt_1 dt_2 d(\frac{M_x^2}{s_1}) d(\frac{M_x^2}{s_2})} \longrightarrow (\frac{M_x^2}{s})(\frac{1}{M_x^2})^{2\alpha_\gamma(t_1)} \times \tag{9}$$

$$\times (\frac{s_2}{M_x^2})^{2\alpha_\gamma(t_2)} \mathcal{F}(t_1, t_2) .$$

Let us consider (9) in the particular region where (7) is valid. We write

$$\frac{d^3\sigma}{dtd(\frac{M_x^2}{s_1}) d(\frac{M_x^2}{s_2})} = \int dt_2 \, \delta(t_2 - t) \frac{d^4\sigma}{dtdt_2 d(\frac{M_x^2}{s_1}) d(\frac{M_x^2}{s_2})}$$

so that

$$\frac{d^3\sigma}{d(\frac{M_x^2}{s_1}) d(\frac{M_x^2}{s_2})} \longrightarrow (\frac{s_1 s_2}{M_x^4})^{2\alpha_\gamma(t)-1} \bar{\mathcal{F}}(t) \tag{10}$$

since in the di-triple Regge region $M_x^2/s \simeq M_x^4/s_1 s_2$. As the r.h. side is now only a function of t and M_x^2/s one can write

$$\frac{d^2\sigma}{dt\, d(\frac{M_x^2}{s})} \longrightarrow (\frac{M_x^2}{s})^{1-2\alpha_\gamma(t)} \bar{F}(t)$$

We now integrate both sides up to the maximum value allowed for M_x^2/s which is in fact $(\Delta E)^2/s$ in case of no momentum resolution and given the symmetry of the experiment. We obtain

$$\frac{d\sigma}{dt} \longrightarrow (\frac{\Delta E}{\sqrt{s}})^{4(1-\alpha_\gamma(t))} F(t) \tag{11}$$

The similarity between eqs. (11) and (7) strongly suggests therefore that the photon reggeizes and its trajectory is given by

$$\alpha_\gamma(t) = 1 - \frac{\beta(t)}{4}$$

i.e., by using (5),

$$\alpha_\gamma(t) = 1 - \frac{e^2}{2\pi}\left[(2m^2-t)\int_0^1 \frac{dy}{m^2-ty(1-y)} - 2\right] \tag{12}$$

A consistency check of eq. (12) is of course that $\alpha_\gamma(0) = 1$. This is easily seen by taking the $t = 0$ limit of $\beta(t)$, or a simply considering that at $t = 0$ β, which represents at very large energy the spectrum of soft photons emitted, is zero since no such emission is possible at $t = 0$ by the electron lines.

Of course, to obtain the result of eq. (12) we had to make certain assumptions which have not yet been verified, i.e. i) that the vacuum trajectory is a simple pole and ii) that $\alpha_V(0) = 1$. However the close similarity between the Q.E.D. cross-section (7) and the di-triple Regge limit (11) is so remarkable that we believe the result to be more general than what may seem from on presentation.

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