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ROTATION OF THE VERTICAL POLARIZATION OF A CIRCULATING ELECTRON BEAM INTO THE HORIZONTAL PLANE BY EXCITATION OF AN IMPERFECTION RESONANCE

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It is shown, theoretically, that the vertical polarization of a circulating beam in a synchrotron can be turned into the horizontal plane at an energy corresponding to an imperfection resonance of the magnetic moment motion. The evolution in time of the polarization vector is investigated under the circumstance when radiation loss causes the electrons to approach the resonance energy while spiraling into a suitable forcing perturbation.

Particular attention is given to maintaining the spin motion essentially unchanged before the particles leave the phase stable position. It is expected that the spilled beam will have, at the resonance radius and at each azimuthal location, a known direction of polarization. The possibility of using such a method on the Frascati 1 GeV Constant Gradient Electron Synchrotron is considered, and shown to be worth trying.

1. Introduction

Theoretical considerations^{1,2,7)} suggest the possibility of accelerating, in an electron synchrotron, electron beams whose polarization is aligned along the main magnetic field without significant polarization loss. In order to obtain circularly polarized photons, which are of great experimental interest, is it necessary to produce longitudinally polarized electrons.

The most straight-forward way in which to rotate by 90° the direction of the polarization vector is to vertically deflect³⁾ the beam, once extracted. The corresponding deflection angle is given by

$$\delta = \frac{90^\circ}{1 + \gamma G}, \quad (1)$$

where γ is the ratio of the total energy E of the electron to its rest energy m_0c^2 , and $G = \frac{1}{2}g - 1 \approx 1.16 \times 10^{-3}$ is the anomalous part of the magnetic moment (g is the gyromagnetic ratio of the electron).

It follows that $\delta \approx 6^\circ$ for 6 GeV electrons (DESY) and $\approx 28^\circ$ for 1 GeV electrons (Frascati synchrotron). Hence practical difficulties become greater as the final energy decreases.

To overcome the need for vertical deflection we suggest a method for bringing the spin vectors of the circulating beam into the horizontal plane. This is accomplished, at the end of acceleration, by artificially producing a selected resonance between the proper frequency of the magnetic moment of the particle and the periodic perturbation due to a (small) localized magnetic field, deliberately introduced in a region of small azimuthal width. In the laboratory reference

frame the perturbing field is a stationary one, and hence the field as seen by the circulating particle as an impulsive periodic field.

2. General considerations

In a reference frame, Σ , which is attached to the particle and which has one of its coordinate axes pointing in the direction of motion, the electronic spin motion in the field of a synchrotron is described in first-order approximation by the equation^{1,2,4-7)}

$$\begin{aligned} \dot{\mathbf{S}} = & \omega_p \mathbf{S} \times \mathbf{k} + \omega_\perp \frac{B_r}{B_0} \mathbf{S} \times \mathbf{n} + \omega_\parallel \frac{B_\theta}{B_0} \mathbf{S} \times \mathbf{w} \\ & + \frac{\omega_p}{\omega_c} \frac{z}{RA} \mathbf{S} \times \mathbf{w}, \end{aligned} \quad (2)$$

where:

\mathbf{S} is the polarization vector of the particle; the dot denotes differentiation with respect to the (lab) time; \mathbf{w} , \mathbf{n} , \mathbf{k} are the space unit vectors (\mathbf{w} is the space unit vector pointing in the direction of motion);

$\mathbf{B}_0 = k\mathbf{B}_0$ is the magnetic field at the equilibrium orbit; \mathbf{B}_r and \mathbf{B}_θ denote the periodic magnetic fields seen by the particle;

z is the vertical displacement of the particle from the equilibrium orbit;

RA is the mean radius (R is the bending radius); ω_c is the angular velocity of revolution;

$$\left. \begin{aligned} \omega_p = \omega_c \gamma G \text{ is the angular velocity of the spin} \\ \text{precession about the direction of the} \\ \text{guiding field } B_0; \\ \omega_\perp = (1 + \gamma G)\omega_c; \\ \omega_\parallel = \frac{1}{2}g\omega_c. \end{aligned} \right\} \quad (3)$$

In eq. (2) the first term corresponds to the unperturbed spin motion in a steady field and all the remaining terms are small periodic perturbations. The absolute value of S is a constant of motion, and we may take

$$|S|=1. \tag{4}$$

The analysis ^{1,2,7)} of eq. (2) shows that in an unperturbed synchrotron the only resonance of possible concern is the "intrinsic" depolarization resonance (i.e. due to the focusing structure)

$$\gamma G = Q_z, \tag{5}$$

where Q_z is the number of vertical betatron oscillations per turn. Such a resonance leads to relatively small polarization loss for DESY¹⁾, for the Frascati Synchrotron²⁾ and for ARUS⁷⁾.

During the acceleration cycle, other types of resonance may be excited; these are the so-called "imperfection" resonances. These resonances are found when one of the periodic perturbations in eq. (2) has the form

$$A_n \cos(n\omega_c t + \varphi_n), \quad n = 1, 2, 3, \dots, \tag{6}$$

and occurs at energies

$$\gamma_{\text{res}} G = n, \quad \text{or} \quad E_{\text{res}} = n \frac{m_0 c^2}{G} = n \cdot 440 \text{ MeV}. \tag{7}$$

The corresponding magnetic field perturbations felt by the particles are periodic in azimuth. These are the

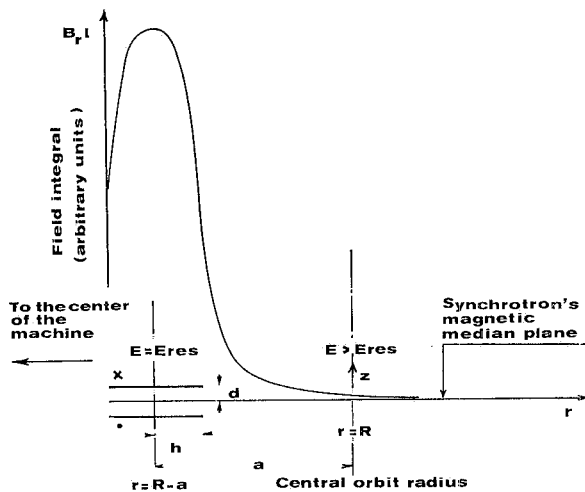


Fig. 1. Shape of the perturbing radial magnetic field (in the median plane). The radial position is denoted by r . The cross section of the perturbing magnet consisting of a pair of parallel equal current strips (as described in section 3) is represented. The signs \times and \bullet represent current flows of opposite directions perpendicular to the plane of the figure. The length l of each strip is much greater than its width $2h$.

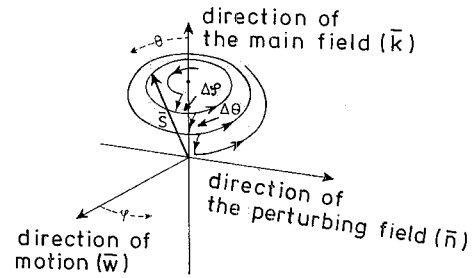


Fig. 2. Sketch of the spin motion under the influence of the perturbing field B_r in the reference frame Σ . The magnitudes of $\Delta\varphi$ and $\Delta\theta$ are exaggerated for the sake of clearness.

resonances which we are interested in, in the present paper.

Let us suppose that the end of the acceleration cycle is at an energy slightly greater than one of the energies (7), i.e. at an integer multiple of $m_0 c^2 / G = 440$ MeV. In these conditions the angular velocity value of the spin precession about the main field B_z is slightly greater than $n\omega_c$.

Suppose now, that the above mentioned local perturbation (a radial field B_r) is produced (fig. 1) by energizing a single non-linear element placed inside the equilibrium orbit in a straight section. B_r has its maximum value at a radius $r = R - a$ corresponding to $E = E_{\text{res}}$. Because of the strong non-linearity introduced, as long as the equilibrium orbit is close to the central orbit, the $(B_r \cdot l)$ field integral is unable to affect the spin motion in a considerable manner.

When the electrons are brought out of synchronism with the rf system (by slowly reducing the rf peak voltage), their equilibrium orbits contract. After an electron is lost from the phase stable position in the rf system, it moves toward the centre of the machine, as a consequence of the energy losses. Thus, the energy E of the spiraling electron approaches the resonance value E_{res} , and simultaneously the perturbing action of $B_r(r) \cdot l$ becomes increasingly more powerful. In this way the magnetic moment of every particle is bent into the horizontal plane, providing the perturbation field integral is sufficiently large, in the course of a few revolutions.

The behaviour of the spiraling spin vector S , in the reference frame Σ , is represented schematically in fig. 2. In the course of one revolution in the machine, the number of spin precessions about k is slightly greater than n , and $\Delta\varphi$ is the corresponding advance of angular precession per revolution. As the particle passes through the perturbed field region, the spin vector rotates about n , and $\Delta\theta$ is the corresponding

angular kick received per revolution by the magnetic moment. At every revolution $\Delta\varphi$ is decreasing and $\Delta\theta$ is increasing.

The system is designed so that the polarization vectors are in the horizontal plane at a radius corresponding to $E=E_{res}$ and $r=R-a$. At this radius, the number of precessions per revolution is just n in the reference frame Σ , and therefore at properly chosen azimuthal positions the polarization vector is aligned with the motion direction.

3. The perturbing magnet

The radial field B_r may be created by means of a pair of parallel equal strips carrying a relatively large current (Fig. 1). The strips must be placed symmetrically with respect to the synchrotron's magnetic median plane.

The expression for the magnetic field, B_r , in the $z=0$ plane is:

$$B_r(\Delta r) = \frac{\mu_0 I}{2\pi h} \Xi, \quad (8)$$

where

$$\Xi = \arctan \frac{\Delta r + a + h}{d} - \arctan \frac{\Delta r + a - h}{d}; \quad (8')$$

and I is the total current in each strip;

$2h$ is the width of each strip;

a is the distance of the axis of the strip pair from the central orbit;

$2d$ is the spacing between the strips;

$\Delta r = r - R$.

The shape of $B_r(\Delta r)$ is outlined in fig. 1.

4. Analysis of the polarization vector motion under the influence of the perturbed magnetic field

To investigate the polarization vector motion under the action of the perturbing field B_r and near the $\gamma G = n$ resonance, it is sufficient to leave in eq. (2) only the first perturbing term. Eq. (2) becomes

$$\dot{\mathbf{S}} = \omega_c \gamma G \mathbf{S} \times \mathbf{k} + \omega_r \mathbf{S} \times \mathbf{n}, \quad (9)$$

where

$$\omega_r = (1 + \gamma G) \omega_c \frac{B_r(\Delta r, t)}{B_0}. \quad (10)$$

Because a lumped perturbation has been considered, $\omega_r \neq 0$ only when a particle passes through the non-linear perturbing field region. For the remainder of this paper the evolution in time of \mathbf{S} will be studied in a reference frame⁶⁾ Σ' which is attached to the particle and which rotates about the main field direction with angular velocity [see eq. (7)]:

$$\Omega = n\omega_c = \gamma_{res} G \omega_c. \quad (11)$$

Eq. (9), when transformed to this new rotating frame, becomes

$$\dot{\mathbf{S}} = \Delta\omega \mathbf{S} \times \mathbf{k} + \omega_r \mathbf{S} \times \mathbf{j}. \quad (12)$$

Here,

$$\Delta\omega = (\gamma G - n)\omega_c, \quad (12')$$

and \mathbf{j} is a unit space vector.

The stability conditions of the spin motion for the particles circulating close to the central orbit (before they slip out of phase stability) are investigated first. We will assume $\Delta\omega \approx \text{constant}$. One sees readily that the spin flip-producing resonance is driven essentially by the zero-th harmonic Fourier component of ω_r , namely

$$\langle \omega_r \rangle = \frac{1}{T} \int_0^\tau \omega_r d\tau = \omega_r \frac{\tau}{T}, \quad (13)$$

where T is the revolution period of the particle and τ denotes the transit time through the perturbed region ($\tau \ll T$). Neglecting periodic terms of ω_r (which cannot be responsible for the resonant spin flip) eq. (12) expressed in terms of the three orthogonal components of the polarization vector in the reference frame Σ' becomes

$$\begin{aligned} \dot{S}_u &= \Delta\omega S_v - \langle \omega_r \rangle S_z, \\ \dot{S}_v &= -\Delta\omega S_u, \\ \dot{S}_z &= \langle \omega_r \rangle S_u, \end{aligned} \quad (14)$$

where S_u and S_v are the horizontal components of \mathbf{S} . The system of eqs. (14) must be solved with the initial conditions $S_u(0)=0$; $S_v(0)=0$; $S_z(0)=1$. One is primarily interested in the variation in time of the vertical component S_z of the polarization vector.

It is known that both the quantum-mechanical⁶⁾ and the classical²⁾ formulation of the problem contained in system (14) lead to the same expression for S_z : the expectation value of the spin in the z -direction is expressed by

$$S_z = 1 - 2|g|^2, \quad (15)$$

where g is the solution of the second order differential equation

$$\ddot{g} - i\Delta\omega \dot{g} + \frac{1}{4} \langle \omega_r \rangle^2 g = 0, \quad (16)$$

with the initial conditions $g(0)=0$; $|\dot{g}(0)| = \frac{1}{2} \langle \omega_r \rangle$.

Assuming $\langle \omega_r \rangle = \text{constant}$, one obtains for S_z

$$S_z = 1 - \frac{\langle \omega_r \rangle^2}{(\Delta\omega)^2 + \langle \omega_r \rangle^2} \{1 - \cos[(\Delta\omega)^2 + \langle \omega_r \rangle^2]^{\frac{1}{2}} t\}, \quad (17)$$

so that

$$\frac{(\Delta\omega)^2 - \langle\omega_r\rangle^2}{(\Delta\omega)^2 + \langle\omega_r\rangle^2} \cong S_z \cong 1. \quad (18)$$

From eq. (18) one can conclude that the vertical polarization remains unaffected when

$$\langle\omega_r\rangle^2 \ll (\Delta\omega)^2. \quad (19)$$

We will now investigate the polarization vector behaviour when the radiation loss causes an electron which has been lost from the phase stable position, to spiral into the above-mentioned perturbed magnetic field. Let us assume that at every passage of the particle through the perturbing field region the coordinate axes of Σ and Σ' coincide. Let us imagine, for concreteness, that at every passage the u -axis is directed along the direction of motion (and the v -axis is directed along the direction of the perturbing field). From eq. (12) it follows that in the part of the orbit that lies within the unperturbed azimuthal region (where $\omega_r=0$) the polarization vector motion is described by the system

$$\dot{S}_u = \Delta\omega S_v; \quad \dot{S}_v = -\Delta\omega S_u; \quad \dot{S}_z = 0. \quad (20)$$

Likewise, in the part of the orbit that lies within the perturbed azimuthal region (where $\omega_r \neq 0$) the polarization vector evolution is governed by the system

$$\dot{S}_u = -\omega_r S_z; \quad \dot{S}_v = 0; \quad \dot{S}_z = \omega_r S_u. \quad (21)$$

The solutions of systems (20) and (21) are the following: If S_{in} denotes the value of S at the entrance position of the $\omega_r=0$ region, one sees that at the end of such a region

$$\begin{aligned} S_{u_{out}} &= S_{u_{in}} \cos(\Delta\omega \cdot T) + S_{v_{in}} \sin(\Delta\omega \cdot T), \\ S_{v_{out}} &= -S_{u_{in}} \sin(\Delta\omega \cdot T) + S_{v_{in}} \cos(\Delta\omega \cdot T), \\ S_{z_{out}} &= S_{z_{in}}. \end{aligned} \quad (22)$$

In the same manner, at the exit position of the $\omega_r \neq 0$ region S_{out} is given in terms of S_{in} by the transformation

$$\begin{aligned} S_{u_{out}} &= S_{u_{in}} \cos(\omega_r \tau) - S_{z_{in}} \sin(\omega_r \tau), \\ S_{v_{out}} &= S_{v_{in}}, \\ S_{z_{out}} &= S_{u_{in}} \sin(\omega_r \tau) + S_{z_{in}} \cos(\omega_r \tau). \end{aligned} \quad (23)$$

After passage through the two successive field regions, i.e. after one complete revolution of the particle, S_{out} is connected with S_{in} by the transformation

$$\begin{pmatrix} S_{u_{out}} \\ S_{v_{out}} \\ S_{z_{out}} \end{pmatrix} = \begin{pmatrix} \cos(\Delta\theta) & 0 & -\sin(\Delta\theta) \\ 0 & 1 & 0 \\ \sin(\Delta\theta) & 0 & \cos(\Delta\theta) \end{pmatrix} \begin{pmatrix} S_{u_{in}} \\ S_{v_{in}} \\ S_{z_{in}} \end{pmatrix},$$

$$\times \begin{pmatrix} \cos(\Delta\varphi) & \sin(\Delta\varphi) & 0 \\ -\sin(\Delta\varphi) & \cos(\Delta\varphi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} S_{u_{in}} \\ S_{v_{in}} \\ S_{z_{in}} \end{pmatrix}, \quad (24)$$

where (see fig. 2)

$$\Delta\varphi = \Delta\omega \cdot T; \quad \Delta\theta = \omega_r \tau. \quad (25)$$

Starting with proper initial conditions, the motion of the spin vector for successive revolutions may be followed by applying at each revolution the transformation (24). At each step of the calculation, the $\Delta\varphi$ and $\Delta\theta$ values must be changed because:

- for the $\Delta\varphi$ value: the energy value of the particle decreases by a small amount corresponding to the radiation loss per revolution and, consequently,
- for the $\Delta\theta$ value: the equilibrium orbit moves a small distance inward, corresponding to the orbit contraction per revolution.

The dependence of $\Delta\varphi$ on the number N of revolution after spiraling is started is

$$\Delta\varphi = 2\pi n \left[\frac{L}{E_{res}} \left(\frac{a}{\sigma} - N \right) + \frac{\Delta E}{E} \right], \quad (26)$$

where

L is the radiation loss per revolution at $E \approx E_{res}$;
 σ is the corresponding closed orbit contraction per revolution (the center of the resonance is assumed to lie at $\Delta r = -a = -N_{max}\sigma$);

$\Delta E/E$ is the fractional displacement in energy of an off-momentum particle.

$\Delta\theta$ is expressed by

$$\Delta\theta \approx (1 + \gamma_{res} G) \frac{B_r(\Delta r) \cdot l}{B_0 R}, \quad (27)$$

where

$$\Delta r = -N\sigma \quad \text{in the absence of radial betatron oscillations;} \quad (28)$$

$B_r(\Delta r)$ is given by eq. (8) and (8');

$B_0 R$ is the magnetic rigidity;

l is the length of each current strip.

5. Numerical results for the Frascati electron synchrotron

The analysis outlined above has been applied to the Frascati electron synchrotron. It is felt that the promising numerical results obtained here could also be achieved for other electron accelerators, and, in particular, also for alternating gradient synchrotrons.

The maximum value of γG that can be reached in the 1 GeV Frascati synchrotron is $\gamma G = 2.25$, so that the $\gamma G = n = 2$ resonance is the most advantageous im-

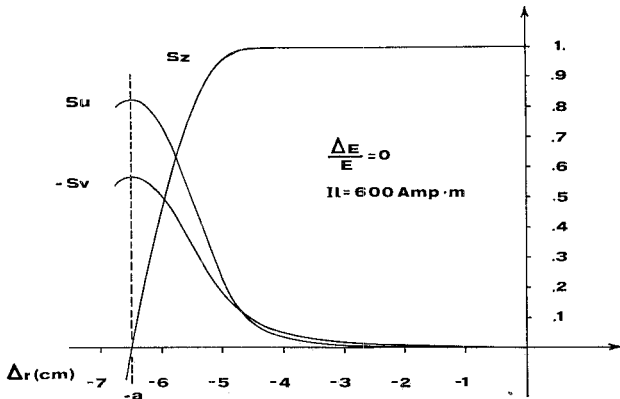


Fig. 3. Spin vector evolution of the "best" particle at the azimuthal location of the perturbing field.

perfection resonance for our purposes. The corresponding resonance energy is $E_{res} = 880$ MeV. The following parameters have been used in the calculations:

Beam parameters at $E = 880$ MeV	Strip pair parameters
$\pm(\Delta E/E)_{max} = \pm 1 \times 10^{-3}$,	$a = 6.5$ cm,
$L = 15$ keV,	$h = 1.5$ cm,
$\sigma = 0.17$ mm,	$d = 0.5$ cm.

Thus, the beam must be accelerated up to the energy $E = E_{res} + La/\sigma = 885.75$ MeV; the corresponding number of revolutions available to deliver the spin vector to the horizontal plane is $N_{max} = 383$.

The optimum $I \cdot l$ value required for a particle leaving the stable position with $\Delta E/E = 0$ ("best" particle) is

$$I \cdot l = 600 \text{ Am.}$$

In fig. 3 is shown the evolution of the spin vector of the "best" particle during spiraling. The ratio $\langle \omega_r \rangle / \Delta \omega$ has been computed (at $\Delta r = -1$ cm), and is $< 0.6 \times 10^{-2}$, so condition (19) is largely satisfied for particles that

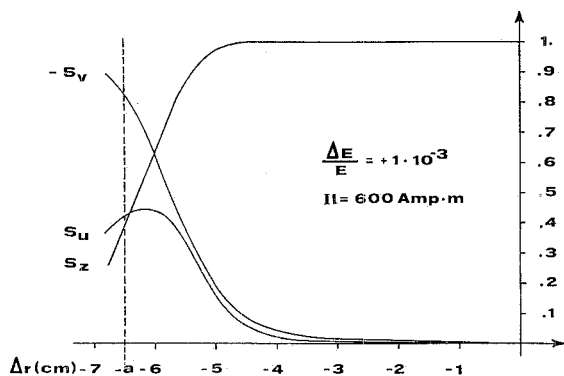


Fig. 4. Spin vector evolution of an off-momentum particle ($\Delta E/E = +1 \times 10^{-3}$).

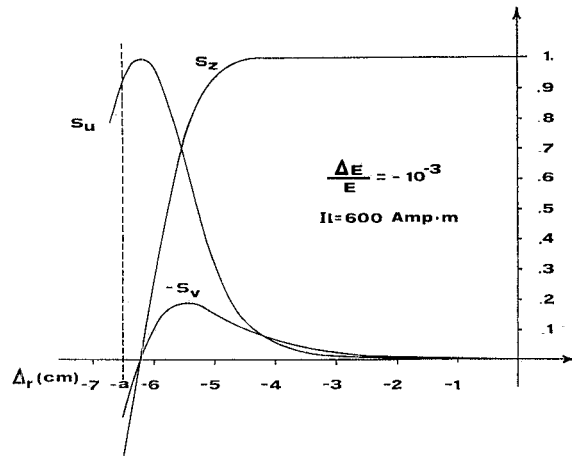


Fig. 5. Spin vector evolution for $\Delta E/E = -1 \times 10^{-3}$.

are not yet lost from the phase stable position during the spill-out time.

Particles having different momenta have different spin vector evolutions, as shown in figs. 4 and 5, that refer to the two limiting cases $\pm(\Delta E/E)_{max}$. At $\Delta r = -a$, the polarization vector of the "best" particle will have a known direction at each azimuthal position (the direction at the location of the perturbing magnet can be easily obtained on inspection of fig. 3). At $\Delta r = -a$, spin vectors of off-momentum particles having different $\Delta E/E$ values fan out into the small aperture of a cone having its axis coincident with the spin direction of the "best" particle. The polarization of these particles, referred to this direction, has been computed and the results are shown in fig. 6.

A single perturbing magnet having the calculated strength value $(I \cdot l) = 600$ Am gives a strong distortion of the closed orbit for the vertical betatron motion in the Frascati Synchrotron. This distortion can be greatly reduced by using - instead of a single perturbing magnet - two perturbing magnets, each having

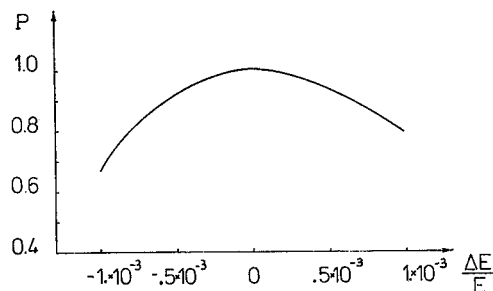


Fig. 6. Off-momentum particles polarization (at $\Delta r = -a$) computed with respect to the polarization direction of the "best" particle.

strength $\frac{1}{2}(I \cdot l) = 180$ Am, and placed in two opposite straight sections of the synchrotron (see appendix for the derivation of this strength value).

The corresponding maxima of the distorted closed orbit occur at the azimuthal positions midway between the perturbing magnets locations and are given by

$$z_{\max} \approx \frac{AR}{Q_z} \frac{\frac{1}{2}(B_r l)}{B_0 R} \frac{1}{2 \sin(\frac{1}{2} Q_z \pi)}, \quad (29)$$

where $R = 360$ cm is the equilibrium orbit radius, and $Q_z \approx 0.9$ is the number of vertical betatron oscillations per revolution. At $\Delta r = -a$, $z_{\max} \approx +5$ mm.

The minima of the distorted closed orbit occur at the longitudinal mid-points of the perturbing lenses and are given by

$$z_{\min} = z_{\max} \cos(\frac{1}{2} Q_z \pi). \quad (30)$$

At $\Delta r = -a$, $z_{\min} \approx +0.7$ mm.

Finally, it must be pointed out that the problem of the polarization vector motion has been approached by considering the action of the perturbing magnetic field(s) $B_r(\Delta r)$ during spiraling in the absence of radial betatron oscillations. In actuality, the radial position Δr of a particle at the location of either magnetic perturbation is given by

$$\Delta r \approx -\frac{1}{2} M \sigma \pm x_0 \cos(Q_x M \pi + \psi), \quad (31)$$

where

M is the number of half revolutions after spiraling is started;

x_0 is the amplitude of the radial betatron oscillation;

$Q_x \approx \frac{2}{3}$ is the number of radial betatron oscillations per revolution;

ψ is the initial phase.

It will be necessary to examine – very briefly – the situation to see if our previous conclusions need any revision. We will do so in the resonant conditions ($\Delta \phi \rightarrow 0$), and under the assumption that $Q_x = \frac{2}{3}$. The angular kick received by the magnetic moment from the two perturbing magnets in the course of $1 + \frac{1}{2}$ revolutions is

$$\begin{aligned} \sum_{m=0}^2 \Delta \theta_m &\approx (1 + \gamma G) \frac{\frac{1}{2} l}{B_0 R} \\ &\times \sum_{m=0}^2 B_r [-\frac{1}{2} M \sigma + x_0 \cos(\frac{2}{3} m \pi + \psi)]. \end{aligned} \quad (32)$$

We expand $B_r(\Delta r)$ in the neighbourhood of $-\frac{1}{2} M \sigma$, as follows,

$$\begin{aligned} B_r [-\frac{1}{2} M \sigma + x_0 \cos(\frac{2}{3} m \pi + \psi)] &= B_r(-\frac{1}{2} M \sigma) \\ &+ \left(\frac{dB_r}{dr} \right)_{\Delta r = -\frac{1}{2} M \sigma} x_0 \cos(\frac{2}{3} m \pi + \psi) + \text{higher order terms.} \end{aligned} \quad (33)$$

Substituting eq. (33) in eq. (32) one finds

$$\begin{aligned} \sum_{m=0}^2 \Delta \theta_m &\approx (1 + \gamma G) \frac{\frac{1}{2} l}{B_0 R} \left[3 B_r(-\frac{1}{2} M \sigma) \right. \\ &\left. + \left(\frac{dB_r}{dr} \right)_{\Delta r = -\frac{1}{2} M \sigma} x_0 \sum_{m=0}^2 \cos(\frac{2}{3} m \pi + \psi) \right]. \end{aligned} \quad (34)$$

Whatever the initial phase value

$$\sum_{m=0}^2 \cos(\frac{2}{3} m \pi + \psi) \equiv 0,$$

so we may conclude that in first order approximation the spin motion is unaffected by the radial betatron motion.

6. Conclusion

It has been shown that an imperfection resonance can be driven in an electron synchrotron in such a way that the initial vertical polarization of the beam results, without significant loss of polarization in a longitudinal polarization at the resonance radius and at specific azimuthal locations.

Sincere thanks are due to Prof. G. Bologna and to Prof. A. Reale for their interest in this effort.

Appendix

Effect of two perturbing magnets placed in two opposite straight sections of the Frascati synchrotron. The periodicity of the particular configuration consisting in two perturbing magnets having the same strength $\frac{1}{2}(B_r \cdot l)$ placed in two opposite azimuthal locations of the ring contains the second harmonic.

Under these circumstances the important terms in eq. (2) will be both the first and the last perturbing term, and the resonance $\gamma G = 2$ will be driven essentially by their second harmonic components. We must, therefore, determine the new $2 \cdot \frac{1}{2}(I \cdot l)$ value necessary for optimum conditions of rotating the vertical polarization into the horizontal plane.

The shape of the closed orbit z for the vertical betatron motion due to the presence of the dipole component of the perturbing fields is represented in fig. 7.

The expression for z in the part of the orbit that lies outside the perturbed azimuthal regions is

$$z = z_{\max} \sin \left[Q_z \Theta + (1 - Q_z) \frac{\pi}{2} \right], \quad (A1)$$

where z_{\max} is given by eq. (29), and $A = 1.213$.

Θ is the azimuthal angle measured from the location of either perturbing magnet.

Remembering that outside the perturbed azimuthal

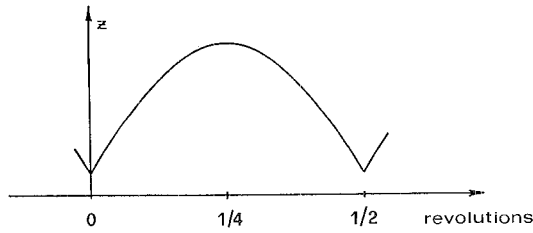


Fig. 7. Distortion of the closed orbit (for the vertical betatron motion) generated by two narrow perturbations of the radial field, spaced 180° in azimuth.

regions $B_r/B_0 = -n_0 z/R$ (n_0 is the field index), and substituting in eq. (2) the second harmonic Fourier variation of B_r/B_0 and of \dot{z} , one obtains the equation of the spin motion in the reference frame Σ

$$\frac{d\mathbf{S}}{d\Theta} = \gamma G \mathbf{S} \times \mathbf{k} + (1 + \gamma G) b_2 \cos(2\Theta) \mathbf{S} \times \mathbf{n} + \gamma G \alpha_2 \sin(2\Theta) \mathbf{S} \times \mathbf{w}, \quad (\text{A2})$$

where

$$\begin{aligned} b_2 &= \frac{1}{\pi} \int_0^{2\pi} \frac{B_r}{B_0} \cos(2\Theta) d\Theta \\ &= \frac{\frac{1}{2} B_r \cdot l}{\pi B_0 R} \left[2 + \frac{1}{2} Q_z^2 \left(\frac{1}{2 - Q_z} + \frac{1}{2 + Q_z} \right) \right], \quad (\text{A2}') \\ \alpha_2 &= \frac{1}{\pi} \int_0^{2\pi} \frac{dz/d\Theta}{R\lambda} \sin(2\Theta) d\Theta \\ &= \frac{\frac{1}{2} B_r \cdot l}{\pi B_0 R} \left(\frac{1}{2 - Q_z} + \frac{1}{2 + Q_z} \right). \end{aligned}$$

(Use of the approximate formula $Q_z^2 = n_0 \lambda$ has been done).

Under resonant conditions ($\gamma G \approx 2$) it is possible to replace each oscillating vector

$$(1 + \gamma G) b_2 \cos(2\Theta) \mathbf{S} \times \mathbf{n},$$

and

$$\gamma G \alpha_2 \sin(2\Theta) \mathbf{S} \times \mathbf{w},$$

in eq. (A2) by two vectors, constant (during one revolution) and equal in magnitude, rotating in opposite directions in the horizontal plane with angular velocity $\Omega = 2\omega_c$, in such a way that the sum of these two

rotating vectors equals the given oscillating vector. Only the component vector rotating in the same direction as the precession of the electronic spin magnetic moment is able to perturb the spin motion. It is convenient therefore to assume that only this rotating component exists [for every oscillating vector in eq. (A2)], the other component being unable to cause any resonant effect. Consequently, in the reference frame Σ' the two small horizontal perturbing terms are vectors having constant direction \mathbf{j} and magnitude

$$\frac{1}{2}(1 + \gamma G) b_2 \quad \text{and} \quad \frac{1}{2} \gamma G \alpha_2.$$

Eq. (A2), when transformed to the reference frame Σ' , becomes

$$\frac{d\mathbf{S}}{d\Theta} = (\gamma G - 2) \mathbf{S} \times \mathbf{k} + \varepsilon \mathbf{S} \times \mathbf{j}, \quad (\text{A3})$$

where

$$\varepsilon = \frac{1}{2} [(1 + \gamma G) b_2 + \gamma G \alpha_2], \quad (\text{A3}')$$

and \mathbf{j} is a unit space vector.

The sum of the angular kicks received per revolution by the magnetic moment rotating about \mathbf{j} is equal to

$$\begin{aligned} 2\pi\varepsilon &= \frac{2 \cdot \frac{1}{2} (B_r \cdot l)}{B_0 R} \left\{ (1 + \gamma G) \left[1 + \frac{1}{4} Q_z^2 \left(\frac{1}{2 - Q_z} + \frac{1}{2 + Q_z} \right) \right] \right. \\ &\quad \left. + \frac{1}{2} \gamma G \left(\frac{1}{2 - Q_z} + \frac{1}{2 + Q_z} \right) \right\}. \quad (\text{A4}) \end{aligned}$$

Numerically,

$$2\pi\varepsilon \approx 1.67 \Delta\theta,$$

[see eq. (27)], so that we can conclude that two magnets having strength $\frac{1}{2}(I \cdot l) = 180$ Am each perform the same function as a single magnet having strength $(I \cdot l) = 600$ Am.

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