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 e^+e^- ANNIHILATION AND ELECTROPRODUCTION

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CONSERVATION CONSTRAINTS FOR e^+e^- ANNIHILATION AND ELECTROPRODUCTION

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It is the purpose of this work to investigate some consequences of energy-momentum conservation [1] for hadron production in e^+e^- annihilation and electroproduction. It is shown that under certain assumptions the distribution function for e^+e^- annihilation,

$$f_{\gamma}^h = \frac{1}{\sigma_{e^+e^-}^{\text{tot}}(s)} \frac{d\sigma}{d^3p/E}$$

scales as $s \rightarrow \infty F^h(x)/s$, where $x = 2p q/s$, and that the n -particle invariant distribution function defined analogously (when averaged over all angles) scales as $s^{-n} F(x_1 \dots x_n)$ where $x_i = 2E_i/\sqrt{s}$. These results agree exactly with naive dimensional counting reflecting the absence of any intrinsic dimensional parameters. When applied to the total cross section, this argument implies that $\sigma_{e^+e^-}^{\text{tot}}(s) \rightarrow \text{constant}/s$. The statement that the

structure functions for annihilation $\bar{F}_{1,2}(x)$ scale (in the Bjorken limit) is contained in the above results [2].

The variable x when crossed over into the "scattering" region, $\gamma(q) + p \rightarrow \text{anything}$, becomes the Bjorken variable x_B . Therefore, if as is generally believed, $F_{1,2}(x)$ ($x \geq 1$) are indeed the analytic continuations of $\bar{F}_{1,2}(x)$ ($x \leq 1$), then Bjorken scaling follows [2]. Notice that for annihilation it is ($s f_{\gamma}^h$) which scales unlike the case of particle production in scattering where f_{ab}^h scales. The difference arises because in the latter case there exists a dimensional parameter — the well-

know transverse momentum cutoff.

We also find that the average energy grows as \sqrt{s} (for large s) and the multiplicities become independent of energy (although unlikely, a logarithmic growth cannot be excluded on general grounds). Similar considerations for electroproduction imply that not only does $f_{\gamma p}^h(s, x_B, x_B, p_{\perp})$ scale (i.e. become independent of s) but may also become independent of x_B .

We first consider single hadron production in e^+e^- annihilation

$$\gamma(q) \rightarrow \text{hadron}(p) + x.$$

Energy conservation reads

$$\sqrt{s} = \sum_h \int p E dE d\Omega f_{\gamma}^h. \quad (1)$$

The angular integration is trivial if one considers the spin averaged cross-section (call this function $\bar{f}_{\gamma}^h(s, E)$). In terms of the scaling variable $x = 2E/\sqrt{s}$, eq. (1) becomes

$$1 = \frac{\pi}{2} s \cdot \sum_h \int_{2m_h/\sqrt{s}}^1 \sqrt{x^2 - \frac{4m_h^2}{s}} x dx \bar{f}_{\gamma}^h(s, x). \quad (2)$$

The sum extends over *stable* hadrons only (hence is finite). If we assume that

$$\bar{f}_{\gamma}^h(s, x) \xrightarrow{s \text{ large}} s^{-\alpha} (\ln s)^{\beta} F^h(x), \quad (3)$$

uniformly in x^{\dagger} , then equation (2) gives

$$\alpha = 1, \beta = 0 \quad \text{and} \quad 1 \approx \frac{\pi}{2} \sum_h \int_0^1 x^2 dx F^h(x) \quad (4)$$

\dagger Footnote see next page.

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The average multiplicity is given by

$$\langle n(s) \rangle \approx \pi \int_0^1 \frac{dx}{2m/\sqrt{s}} x F(x). \quad (5)$$

Since $F(x)$ is proportional to a measurable cross-section, we expect that this integral exists, in which case $\langle n(s) \rangle$ tends to a *constant* for large s . However, if one is willing to have $F(x)$ diverge as x^{-2} as $x \rightarrow 0$, then $\langle n(s) \rangle$ can grow logarithmically. On physical grounds we find the latter behaviour unattractive (firstly because it will give unrealistically large cross sections for $x \rightarrow 0$) and secondly because in the case of scattering (with transverse momentum cutoff) one gets the $\ln s$ growth of multiplicities *not* by having the scaled function diverge but by requiring that it has a *finite* limit in the pionization region ($x \rightarrow 0$).

For the average energy/particle, we have

$$\langle E(s) \rangle \approx \frac{\pi\sqrt{s}}{2\langle n(s) \rangle} \int_0^1 dx x^2 F(x),$$

and is thus seen to grow like \sqrt{s} (up to possible $\ln s$ terms). This behaviour is similar to that found in strong interactions, while Bjorken and Brodsky [3] find $\langle E(s) \rangle \rightarrow \text{constant}$ and $\langle n(s) \rangle \sim \sqrt{s}$.

The n -particle distribution functions defined as

$$f_{\gamma}^{(h_1 \dots h_n)} = \frac{1}{\sigma_{e^+e^-}^{\text{tot}}(s)} \frac{d\sigma^{(h_1 \dots h_n)}}{\prod_{i=1}^n [d^3p_i/E_i]}, \quad (7)$$

when averaged over all angles, satisfy the energy momentum constraint, if they scale as

$$\bar{f}^{(h_1 \dots h_n)}(s, x_1 \dots x_n) \sim \frac{1}{s^n} F^{(h_1 \dots h_n)}(x_1 \dots x_n), \quad (8)$$

where the F 's satisfy the recursion relation

† Such an asymptotic expansion can always be made; however, that the dominant term is uniformly the same is an extra assumption. That this technicality is indeed crucial can be seen in a model by Bjorken and Brodsky [3], where results different from ours are obtained only by violating it. In this model the "wee" region ($x \lesssim 2E_0/\sqrt{s}$) is the important one and the function f behaves differently in the wee and the non-wee regions. This has been accomplished by an extra intrinsic dimensional parameter E_0 .

$$\begin{aligned} [1 - \frac{1}{2}(x_1 + x_2 + \dots + x_n)] F^{(h_1 \dots h_n)}(x_1 \dots x_n) = \\ = \frac{\pi}{2} \sum_{h'} \int_0^1 dx' x'^2 F^{(h_1 \dots h_n, h')}(x_1 \dots x_n, x'). \end{aligned} \quad (9)$$

This equation is obviously consistent with factorization if the x_i 's are small.

Energy-momentum conservation for electroproduction also yields rather interesting results. The invariant distribution function for $\gamma(q) + \text{proton}(p) \rightarrow \text{hadron}(p') + \text{anything}$ is again defined as

$$f_{\gamma p}^h = \frac{1}{\sigma_{\gamma p}^{\text{tot}}(Q^2, x_B)} \frac{d\sigma_{\gamma p}^h}{(d^3p'/E')},$$

and satisfies

$$(q+p)^\mu = \sum_h \int \frac{d^3p'}{E} p'^\mu f_{\gamma p}^h. \quad (10)$$

The $\mu = 0$ component gives

$$1 = \frac{\pi}{2} \sum_h \int_{-1}^{+1} dx_F \bar{f}_{\gamma p}^h(s, x_B, x_F, \langle p_\perp \rangle), \quad (11)$$

where $x_B = 2pq/Q^2$, $x_F = 2p'_\parallel/\sqrt{s}$ and $\bar{f}(\)$ is obtained by integrating f over p_\perp (under the standard assumption of a transverse momentum cutoff). One obtains another rule by multiplying eq. (10) by q^μ :

$$\begin{aligned} (x_B - 2) = \frac{\pi}{2} \sum_h \int_{-1}^{+1} dx_F \left\{ (x_B - 2) - \right. \\ \left. - \frac{x_F x_B}{x_F} \bar{f}_{\gamma p}^h(s, x_B, x_F, \langle p_\perp \rangle) \right\} \end{aligned} \quad (12)$$

Repeating arguments similar to those discussed above, eq. (11) shows that \bar{f} scales (i.e., is independent of s) [4]. While this equation only says that \bar{f}^h , when integrated over x_F and summed over h , is independent of x_B , we conjecture that in fact the *individual* \bar{f}^h are independent of x_B . From a comparison of eqs. (11) and (12) we further suggest that the way the latter equation is satisfied is that $[\sum_h \bar{f}^h]$ is symmetric under $x_F \leftrightarrow -x_F$ ††. Moreover, if we assume that $[\sum_h \bar{f}^h]$ has a structure in x_F similar to that found

†† Footnote see next page.

in strong interaction (since they satisfy identical sum rules this is not unreasonable), then there is some evidence for the symmetry in x_F in $\pi^- p$ reaction [6].

The other conservation constraints (due to charge, baryon number and strangeness) can be used in conjunction to derive other useful sum rules to which we hope to return elsewhere.

In conclusion, we have obtained some interesting consequences from momentum conservation constraints for e^+e^- annihilation as well as for electroproduction, which show different scaling properties. Under certain assumptions, we were able to show that for annihilation naive dimensional counting is indeed correct, whereas for electroproduction a different function scales due to the presence of a dimensional

parameter in the form of p_{\perp} cutoff. In addition, we have given support for, through not a proof of, factorization and symmetry properties of the electroproduction distribution functions.

Some of the results can be derived in specific models (e.g. partons, light cone, multiperipheral models, etc.). However their generality can be appreciated from our analysis.

References

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†† No data are available to test the independence of \bar{f} on x_B . There is some evidence for symmetry in x_F for the proton spectra from Desy [5] which is at a rather low value of s ($\simeq 7 \text{ GeV}^2$) and $Q^2 \approx 1.15 \text{ GeV}^2$.