COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-72/69
31 Luglio 1972

B. Bartoli, F. Felicetti and V. Silvestrini : ELECTROMAGNETIC STRUCTURE OF THE HADRONS. -

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

LNF-72/69
31 Luglio 1972

B. Bartoli, F. Felicetti and V. Silvestrini: ELECTROMAGNETIC STRUCTURE OF THE HADRONS. -

INTRODUCTION. -

Nature has provided us with good probes of the electromagnetic structure (E. M. S.) of stable particles: as we know, these probes are the charged leptons (μ, e).

By means of a high energy lepton $\underline{1}$ the structure of a target particle T, considered as a whole, can be studied by performing elastic scattering experiments of 1 on T. This line of investigation has been pursued during last 15 years, and has provided a large quantity of information about the E. M. S. of the proton, the neutron and the pion.

Complementary information can be obtained through the production of a $T\bar{T}$ pair by lepton-antilepton annihilation. Experiments in this category have become possible only in the last few years through the operation of e^+e^- storage rings at Orsay, Novosibirsk and Frascati.

As a consequence of some rather general hypotheses, the amplitudes for both the elastic scattering and the pair production processes can be written in terms of a small number of form factors, which summarize the information on the E. M. S. of the target hadron T.

The form factors are analytic functions of one single variable q^2, the square of the four-momentum transferred by the projectile lepton to the target hadron. The form factors are explored for essentially negative values of their argument in the scattering reactions, and for positive values in the pair production experiments(x).

If T were really elementary, the above categories of experiments would provide all the possible information on the E. M. S. of T. This is however not true, and we know that the impact can excite T, or produce particles on it, or breack it into its constituents. Additional information on the E. M. S. of T can be obtained by studying these processes. - the inelastic scattering processes. While in the elastic scattering the kinematics is completely known once the momentum of the final lepton (or of the recoil target particle) is measured, in inelastic processes the complete knowledge of the final state configuration cannot be achieved by detecting a single particle. The study of inelastic processes based on a detailed study of the final state becomes therefore very difficult both experimentally and theoretically.

However, some relevant insight into the E. M. S. of T can be achieved also through the partial knowledge of the final state which is obtained by measuring only the momentum of the final state lepton. These experiments - the so called inclusive experiments - have been an important field of investigation during last few years. The corresponding of inelastic processes in the time-like region are of course the production of many-hadron systems from e^+e^- interactions.

(x) - Throughout this paper, the following convention is used:

$$q^2 = q_\mu q^\mu = E^2 - |p|^2$$

so that positive q^2 are time-like, and negative q^2 are space-like.

The following four categories of experiments are the object of this purely experimental review article:

a) elastic scattering experiments of leptons on nucleons and on virtual pions;
b) inelastic scattering experiments of leptons on nuclear targets (inclusive);
c) hadron-antihadron pair production from e^+e^- interactions;
d) production of multihadron systems in e^+e^- collisions.

In addition, we will review the experimental evidence for the validity of the hypotheses which allow us to express the results of the experiments in terms of structure functions of the hadrons involved in the reactions.

PART I. - SPACE-LIKE REGION. -

I. 1. - VALIDITY OF THE UNDERLYING HYPOTHESES IN THE SPACE-LIKE REGION. -

The hypotheses which allow one to express the elastic and inelastic scattering amplitudes in terms of structure functions of the target hadrons are essentially the following[1 ÷ 3]:

a) The scattering amplitude is well described by the first order, one photon exchange (1 - PE) Feynman diagrams.
b) Charged leptons behave as pointlike Dirac particles.
c) The photon propagator is given by $(1/q^2)$, the inverse of its four-momentum squared.

The validity of more general hypotheses - Lorentz invariance, parity and time reversal symmetry, invariance under rotations in the isospin space, and gauge invariance - will be assumed to hold in Electromagnetic Interaction without discussing their experimental foundation.

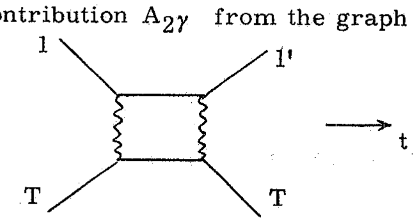
Let us instead review the experimental support to the above hypotheses a), b) and c) in the space-like region.

I. 1. 1. - One-photon exchange hypothesis. -

Due to the smallness of the electromagnetic coupling constant $\alpha = (e^2/\hbar c) \simeq (1/137)$ the first order one-photon exchange contribution $A_{1\gamma}$ (proportional to α) to the scattering amplitude in lepton scattering is expected to dominate over the second order (proportional to α^2) and higher order contributions.

Despite the technical difficulties[4], the contribution of higher order diagrams can be calculated in detail in the case of lepton-lepton scattering described by pure QED. This contribution, which depends in part on the features of the experimental apparatus, does not exceed in general a few per cent; it can therefore be treated as a correction (radiative corrections), and the 1-PE contribution to the scattering amplitude can be accurately extracted from the experimental data. In the case of lepton-hadron scattering, the problem is treated in a similar way: at each order, the knowledge of the hadron structure which is needed for the calculation is derived from the results of the lower order approximation.

Some doubts, however, have been raised about the correctness of this procedure. In particular, it has been pointed out that the interference term $2A_{1\gamma}$ Re $A_{2\gamma}$ (x) between the 1-PE term and the contribution $A_{2\gamma}$ from the graph



(x) - Due to gauge invariance and current hermiticity $A_{1\gamma}$ is real.

when evaluated with the above procedure could result in large uncertainty, since possible resonances could enhance $A_{2\gamma}$ [5, 6]. Since the magnitude of $A_{2\gamma}$ has been determined by several experimental investigations, we can consider these as tests of the correct evaluation of $A_{2\gamma}$ with the standard radiative correction techniques. These experiments are based on cross-section measurements, on the comparison of 1^--p with 1^+-p elastic scattering cross sections, and on polarization measurements.

a. - Cross-section measurements. Rosenbluth plots. -

Lorentz invariance, gauge invariance and the 1-PE hypothesis lead to the form of the electron nucleon elastic scattering cross-section known as the Rosenbluth formula[3]:

$$\frac{d\sigma}{d\Omega} = A\left[\frac{G_E^2 + \tau G_M^2}{1+\tau}\,\mathrm{cotg}^2\,\theta/2 + 2\tau G_M^2\right] \tag{1}$$

where

$$A = \left(\frac{e^2}{2E_o}\right)^2 \frac{1}{\sin^2\theta/_2\,\left(1+\frac{2E_o}{M}\sin^2\theta/_2\right)} = \left(\frac{d\sigma}{d\Omega}\right)_{NS}\,\mathrm{tg}^2\,\theta/_2 \tag{2}$$

$$\tau = \frac{-q^2}{4M^2} \tag{3}$$

θ is the electron scattering angle, E_o the energy of the incident electron and M the nucleon mass. The form factors G_E and G_M are real functions of q^2 only, related to the Dirac and Pauli form factors F_1 and F_2 (see Section I. 2.).

Plots of $R = ((1/A)(d\sigma/d\Omega))$ versus $\mathrm{cotg}^2\,\theta/_2$ at fixed q^2 are known as Rosenbluth plots and are straight lines of slope $(G_E^2 + \tau G_M^2)/(1+\tau)$ and intercept $2\tau\,G_M^2$. Deviations of the experimental points from the Rosenbluth plot would imply the breakdown of one of the hypotheses, the weakest of which is the 1-PE approximation.

Experimental tests of the Rosenbluth formula have been extensively made[7]. We report an example in Fig. 1.

The Rosenbluth behaviour has also been tested in μ-p scattering[8], as shown in Fig. 2.

Agreement has always been found within the errors. It is worth recalling, however, that only the real part of a possible 2-PE amplitude $A_{2\gamma}$ gives an α^3 contribution to the cross-section, and in addition even a non vanishing $\mathrm{Re}A_{2\gamma}$ does not necessarily destroy the Rosenbluth behaviour[6].

b. - Comparison of 1^+-p and 1^--p cross-sections. -

While the modulus squared of both the $A_{1\gamma}$ and the $A_{2\gamma}$ amplitudes are obviously even in the charge of the incident lepton 1, the interference term is odd:

$$\frac{d\sigma^{\pm}}{d\Omega} = A_{1\gamma}^2 + \left|A_{2\gamma}\right|^2 \pm 2A_{1\gamma}\,\mathrm{Re}\,A_{2\gamma} \tag{4}$$

σ^+ and σ^- are the cross sections for $e^+(\mu^+)$ and $e^-(\mu^-)$ scattering in the same kinematical situation.

The comparison of the $e^+(\mu^+)$ and $e^-(\mu^-)$ scattering cross-sections has been performed by many authors[9 ÷ 15].

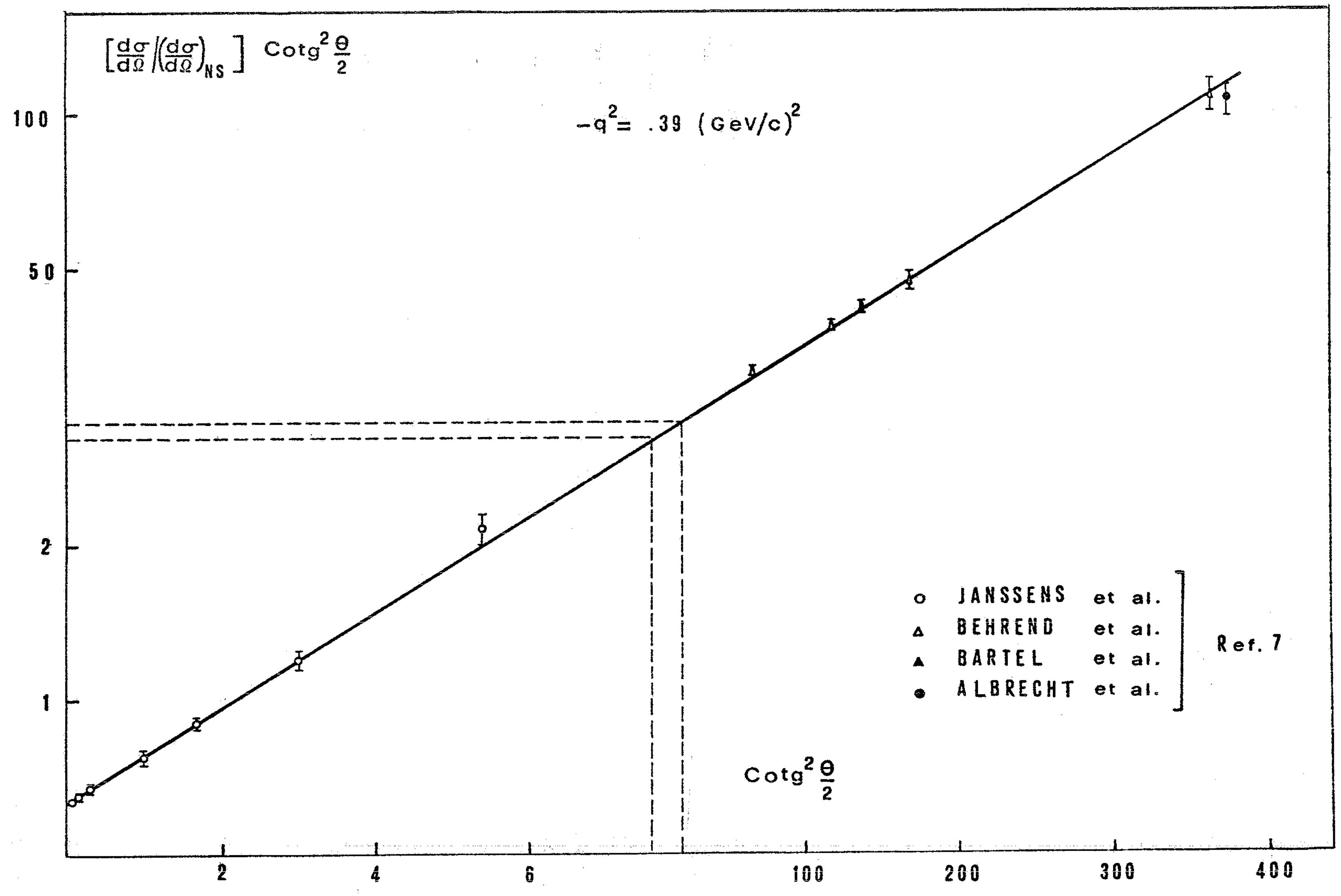


FIG. 1 - Typical Rosenbluth straight-line plot for electron-proton elastic scattering at $-q^2 = 0.39\ (GeV/c)^2$. The quantity $R = \left[(d\sigma/d\Omega)/(d\sigma/d\Omega)_{NS}\right]$ $cotg^2\ \theta/2$ is plotted against $cotg^2\ \theta/2$.

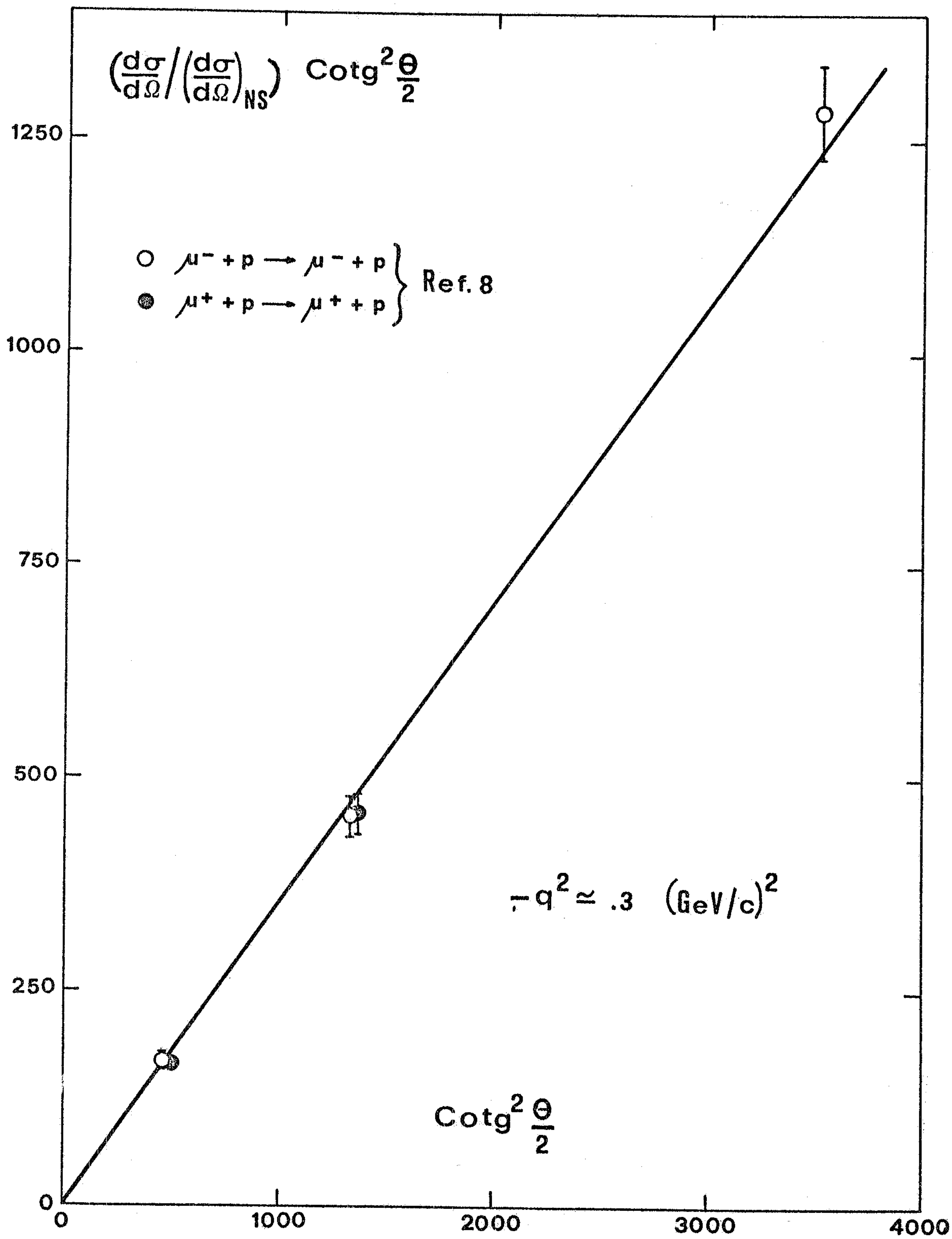


FIG. 2 - Typical Rosenbluth straight-line plot for muon-proton elastic scattering at $-q^2 \simeq 0.39$ (GeV/c)^2. The quantity R = $\left[(d\sigma/d\Omega)/(d\sigma/d\Omega)_{NS}\right]$ cotg^2 $\theta/2$ is plotted against cotg^2 $\theta/2$.

The summary of all the experimental results available is given in Fig. 3 and 4, in which $R = (\mathrm{Re}\,A_{2\gamma} / A_{1\gamma}) = ((1/2)\,(\sigma^{+} - \sigma^{-})/(\sigma^{+} + \sigma^{-}))$ is plotted versus $-q^2$. For electrons, R appears to be smaller than $\sim 1\%$ for $0.01 < -q^2 < 1$ $(\mathrm{GeV}/c)^2$, and smaller than $\sim 2\%$ up to $-q^2 \simeq 5 (\mathrm{GeV}/c)^2$. In the case of muons(8), only $-q^2 \lesssim 1 (\mathrm{GeV}/c)^2$ have been explored with a lower accuracy.

c. - Polarization measurements. -

In the 1-PE approximation, gauge invariance and the hermiticity of the E. M. current restrict the elastic l-p scattering cross-section to have no spin dependence. In other words, both the polarization P of the recoil proton and the asymmetry A in the scattering of leptons from a polarized target must vanish.

Due to the presence of an imaginary part of a two photon amplitude, however, A and P could be different from zero. If T-invariance holds, then A = P are linear homogenous functions of the imaginary part of the two-photon exchange spin amplitudes(16).

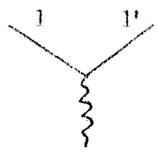
Both A and P have been experimentally investigated in e-p scattering(17 ÷ 21). The results are summarized in Fig. 5.

No evidence for A or P different from zero appears in the explored q^2 range $0.2 < -q^2 < 1$ $(\mathrm{GeV}/c)^2$. The experimental accuracy is typically $1 \div 3\%$.

I. 1. 2. - Tests of the photon propagator and of the point-like leptons. -

We now consider the experimental tests of the two hypotheses b) of the point-like leptons and c) of the $1/q^2$ photon propagator. These experiments go usually under the name of tests of the validity of QED (quantum electrodynamics).

Here, we review only those which are relevant in the spirit of testing the leptons as good probes of the E. M. structure of hadrons; i. e., we are interested in the experimental proof of the hypothesis that the vertex function and photon propagator in the graph

(5) 

(1 and 1', the initial and final lepton, being both on their mass-shell) can be written according to the rules of pure QED, i. e. γ_μ and $1/q^2$(22) respectively.

The most general modifications of QED allowed in this case by gauge and Lorentz invariance are

(6) $$\gamma_\mu \longrightarrow \gamma_\mu F_1(q^2) + k\sigma_{\mu\nu}\, q^\nu F_2(q^2)$$

(7) $$\frac{1}{q^2} \longrightarrow \frac{M(q^2)}{q^2}$$

The restrictions on F_1, F_2 and M required by general principles(23) are not relevant in this context.

The g-2 experiments(24) allow us to conclude that $kF_2(q^2) = 0$(x) with, from our point of view, absolute precision.

(x) - The measurement of a static quantity like g-2 gives information also on the values of the form factors at $q^2 \neq 0$. For details see, for instance, K. Okamoto, Nuclear Phys. B4, 226 (1967).

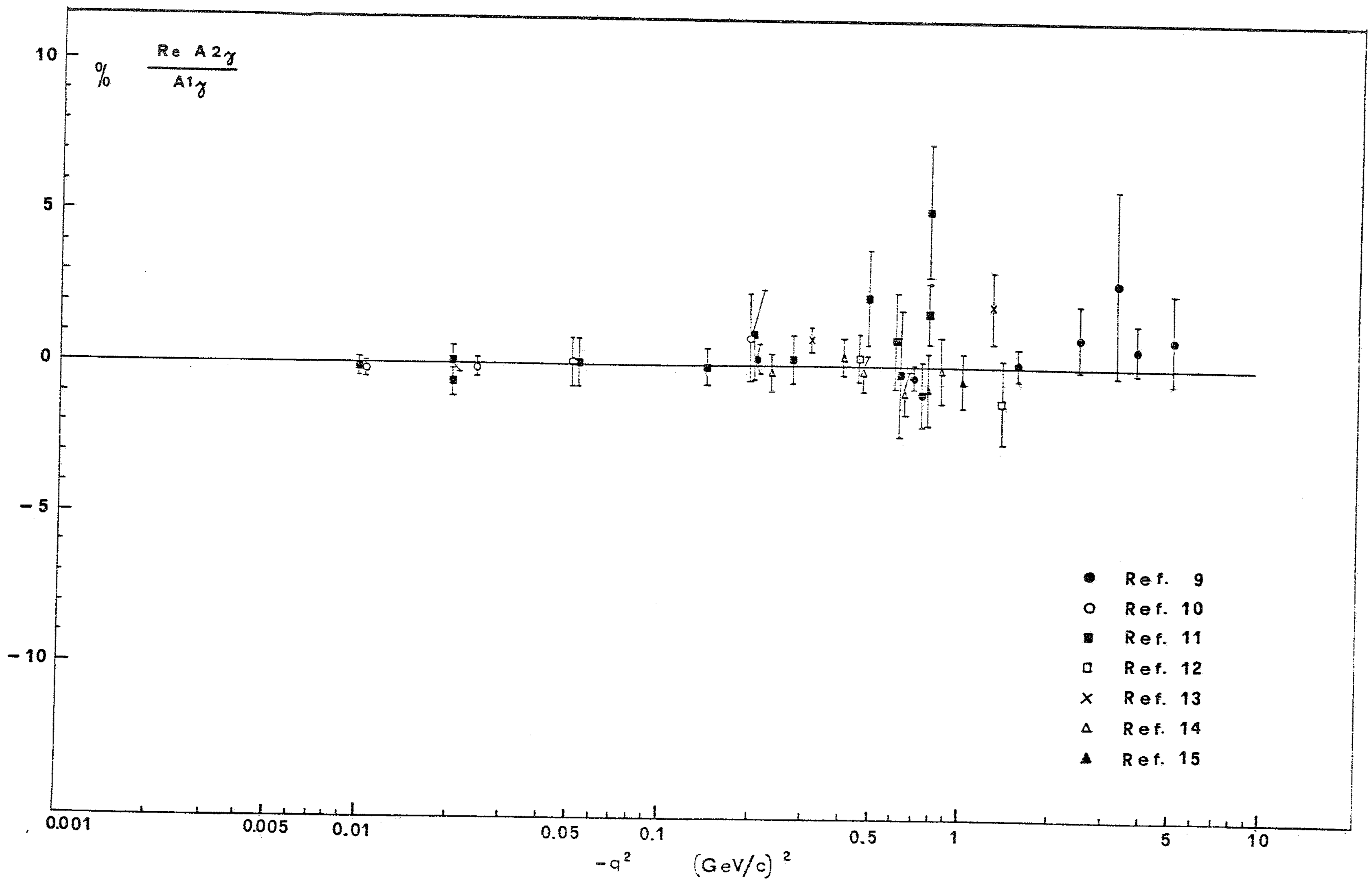


FIG. 3 - Comparison of the e^--proton and e^+-proton cross-sections. The ratio of the real part of the 2-photon exchange amplitude to the 1-photon exchange amplitude $(\mathrm{Re}\, A_{2\gamma}/A_{1\gamma}) = (1/2)(\sigma^+ - \sigma^-)/(\sigma^+ + \sigma^-)$ is plotted versus $-q^2$

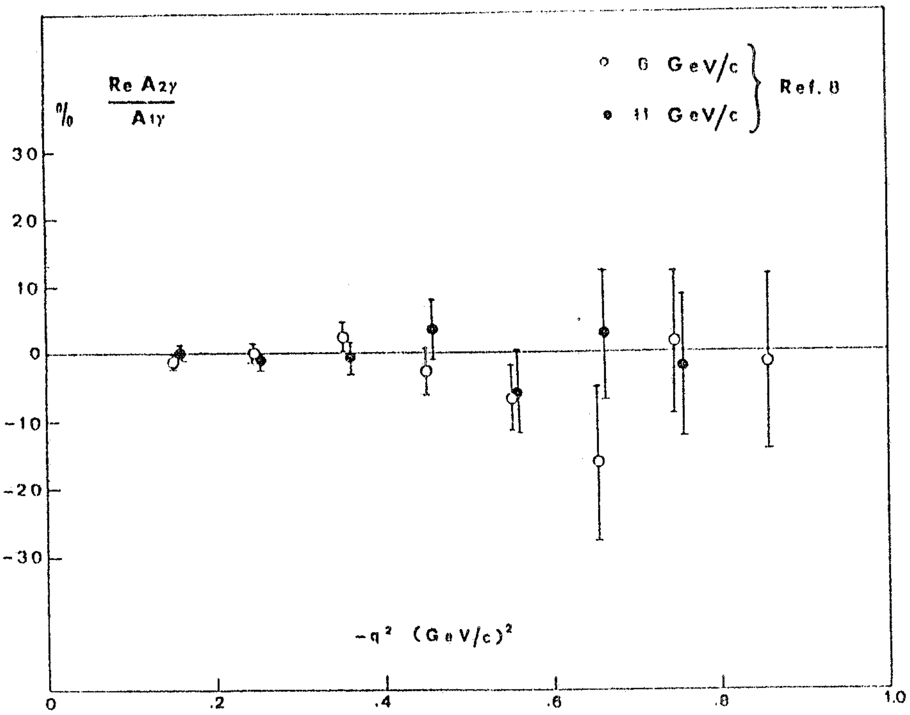


FIG. 4 - Comparison of the μ^--proton and μ^+-proton cross-sections. The ratio of the real part of the 2-photon exchange amplitude to the 1 - photon exchange amplitude (Re $A_{2\gamma}$ /$A_{1\gamma}$) = $(1/2)(\sigma^+ - \sigma^-)/(\sigma^+ + \sigma^-)$ is plotted versus $-q^2$. Data at two different primary muon energies are shown: open circles 6 GeV, black circles 11 GeV.

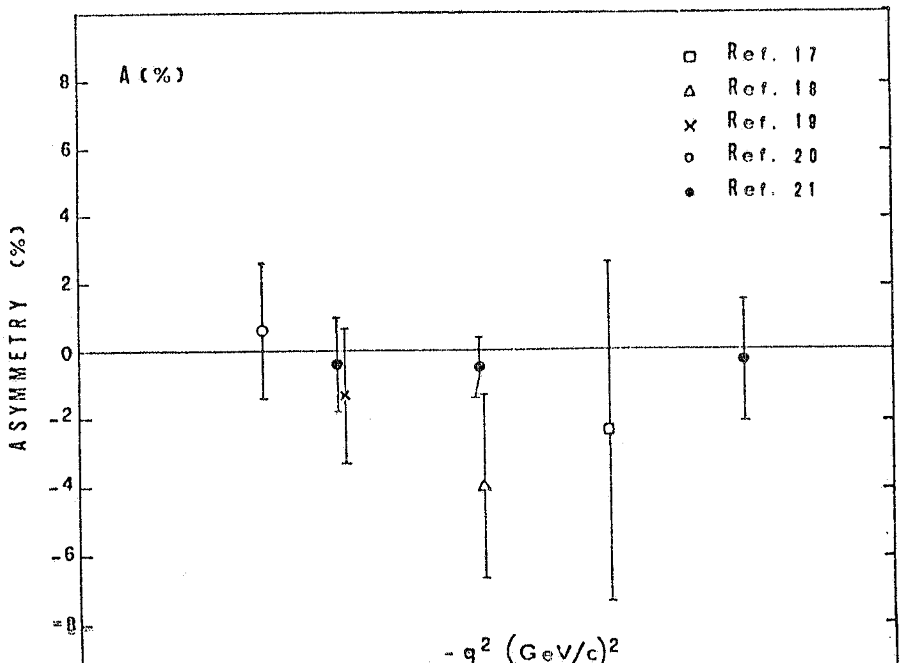
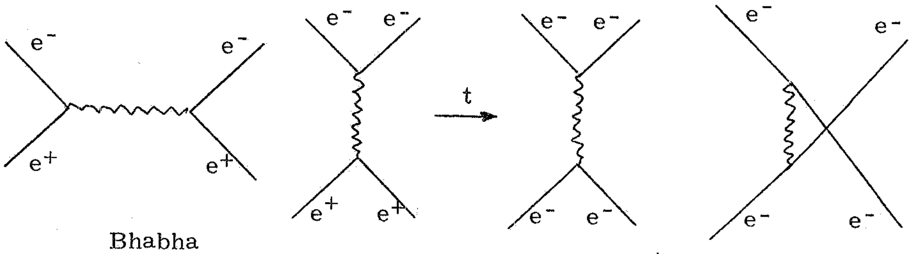


FIG. 5 - Values of the recoil proton polarization or the asymmetry in the scattering from polarized protons for elastic electron-proton scattering.

Thus, the most general form of (6) becomes

(8) $$\gamma_\mu \longrightarrow F_1(q^2)\,\gamma_\mu$$

For space-like values of q^2, $F_1(q^2)$ and $M(q^2)$ have been measured, in the case of electrons, by means of the e^--e^- (Møller)[25] and e^+e^- (Bhabha)[26] elastic scattering. The Feynman graphs which describe these processes to first order are



Bhabha

Møller

The amplitude corresponding to each graph is obviously proportional to $M(q^2)F_1^2(q^2)$. Since q^2 is not the same for the two graphs contributing to each process, the ratio $R = (\sigma_{exp})/(\sigma_{th})$, in terms of which the experimental data are conveniently expressed, is not proportional to $M^2(q^2)F_1^4(q^2)$. However, in the kinematical regions explored by the experiments, one graph (the one corresponding to the lower value of the momentum transfer squared, q_e^2, which is space-like also in the Bhabha scattering), usually dominates the other so that R is approximately proportional to $M^2(q_e^2)F_1^4(q_e^2)$. In Fig. 6, R (for both Møller and Bhabha scattering) is shown as a function of $-q_e^2$. In the Stanford I[27] and II[28] experiments the absolute value of the cross-section is not measured: therefore the average value of R has been normalized to 1. In the Frascati data[29 ÷ 31] and in the Orsay point[32], in addition to the statistical errors (bars), the systematic uncertainty is also displayed (boxes), including the overall normalization uncertainty in each experiment.

The above data form e^--e^- and e^+-e^- storage ring experiments allow us to conclude that the possible form factor of the electron is tested to be equal to 1 to within $\sim 1 \div 2\%$ (F_1^4 within $4 \div 8\%$) up to $-q^2 \leq 1.5\ (\mathrm{GeV}/c)^2$; and within $\sim 5\%$ (F_1^4 within $\simeq \pm 20\%$) up to about $2.5\ (\mathrm{GeV}/c)^2$. $M(q^2)$, appearing in R only squared, is measured with about twice as large a relative error.

In the case of muons, no scattering measurement on lepton targets has been performed. However, the muon structure in the space-like region has been investigated by comparing the muon-proton elastic cross-section $d\sigma_\mu/d\Omega$ with the electron-proton elastic scattering $d\sigma_e/d\Omega$ in the same kinematical situation.

In the ratio $\Sigma_{\mu,e} = (d\sigma_\mu/d\sigma_e)$ the proton structure contribution cancels out (and the same is true for a possible modification of the photon propagator $M(q^2)$), so that $\Sigma_{\mu,e}$ turns out to be proportional to $F_\mu^2(q^2)/F_e^2(q^2)$. The results of this comparison[33, 34] in the case of lepton-proton elastic scattering is shown in Fig. 7. The ratio $F_{\mu,e} = F_\mu(q^2)/F_e(q^2)$ is displayed as a function of $-q^2$ ($0.1 \lesssim -q^2 \lesssim 1\ (\mathrm{GeV}/c)^2$). $F_{\mu,e}$ does not appear to have any appreciable q^2 dependence within the errors; its absolute value, however, appears to be $\simeq 0.96$. This 4% discrepancy, six standard deviations outside the statistical errors, corresponds to $\sim 8\%$ difference in the cross-sections. The authors[33, 34] do not exclude a possible systematic normalization error of this size in either the muon or the electron experiments.

Similar tests have also been performed by comparing μ-p with e-p inelastic scattering experiments. These inelastic processes allow us to compare the muon with the electron structure even better than elastic scattering experiments. In fact a larger category of kinematical situations can be explored in this case. In particular the large values of the inelastic cross-sections allow us to have a statistically significant measurement of $F_{\mu,e}$ up to $-q^2 \simeq 3(\mathrm{GeV}/c)^2$. The result is[35, 36] that $F_{\mu,e}$ can be fitted with a straight line $N(1-q^2 \cdot K^2)$, with $N = 0.946 \pm 0.042$ and $K^2 = (0.021 \pm 0.021)\ (\mathrm{GeV}/c)^{-2}$.

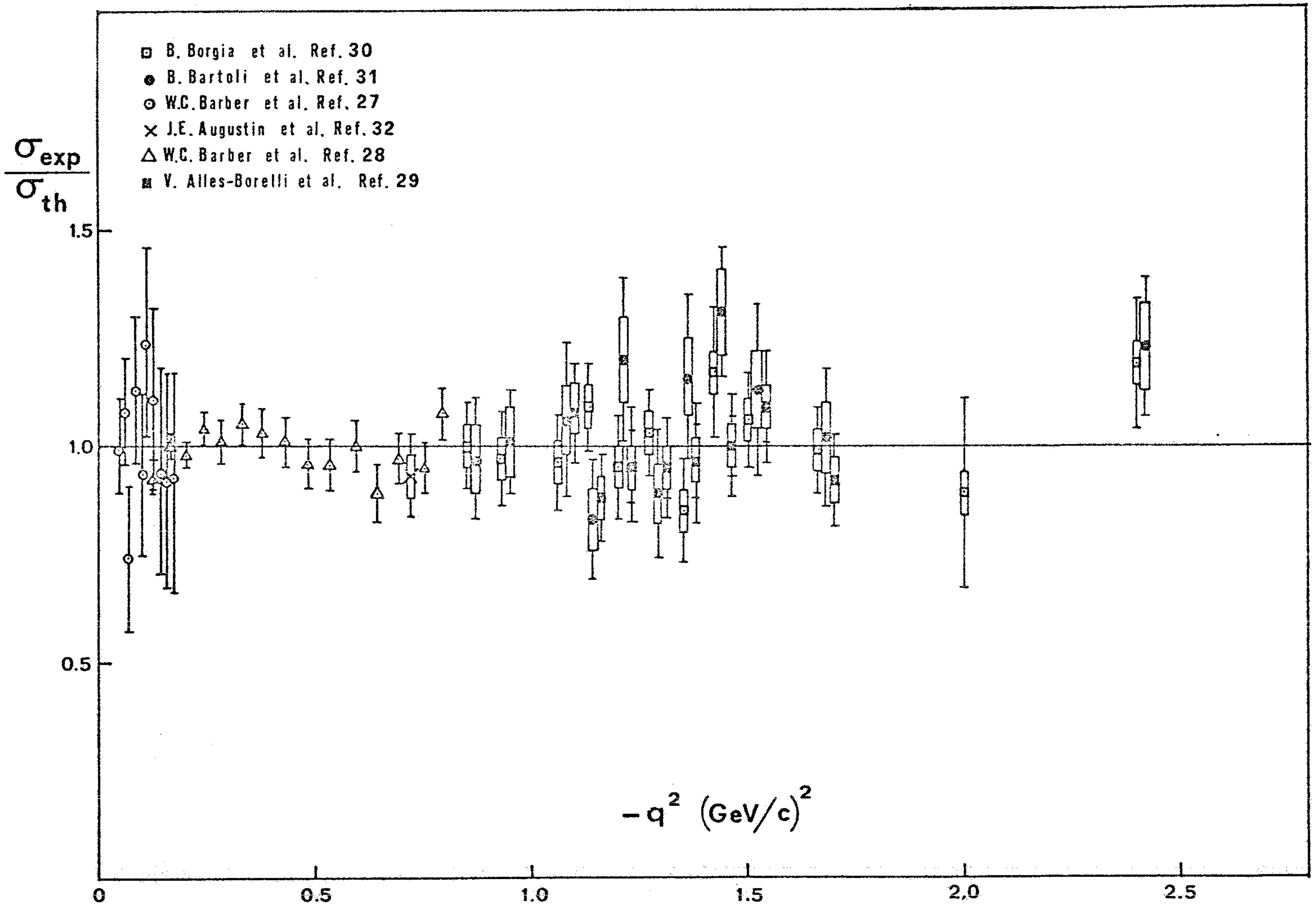


FIG. 6 - Comparison of experimental data on e^-e^- and e^+e^- elastic scattering with QED predictions as a function of the space-like momentum transfer $-q^2$. The results are expressed in terms of the ratio σ_{exp}/σ_{th}. Statistical errors are shown as bars, while boxes represent the systematic uncertainty.

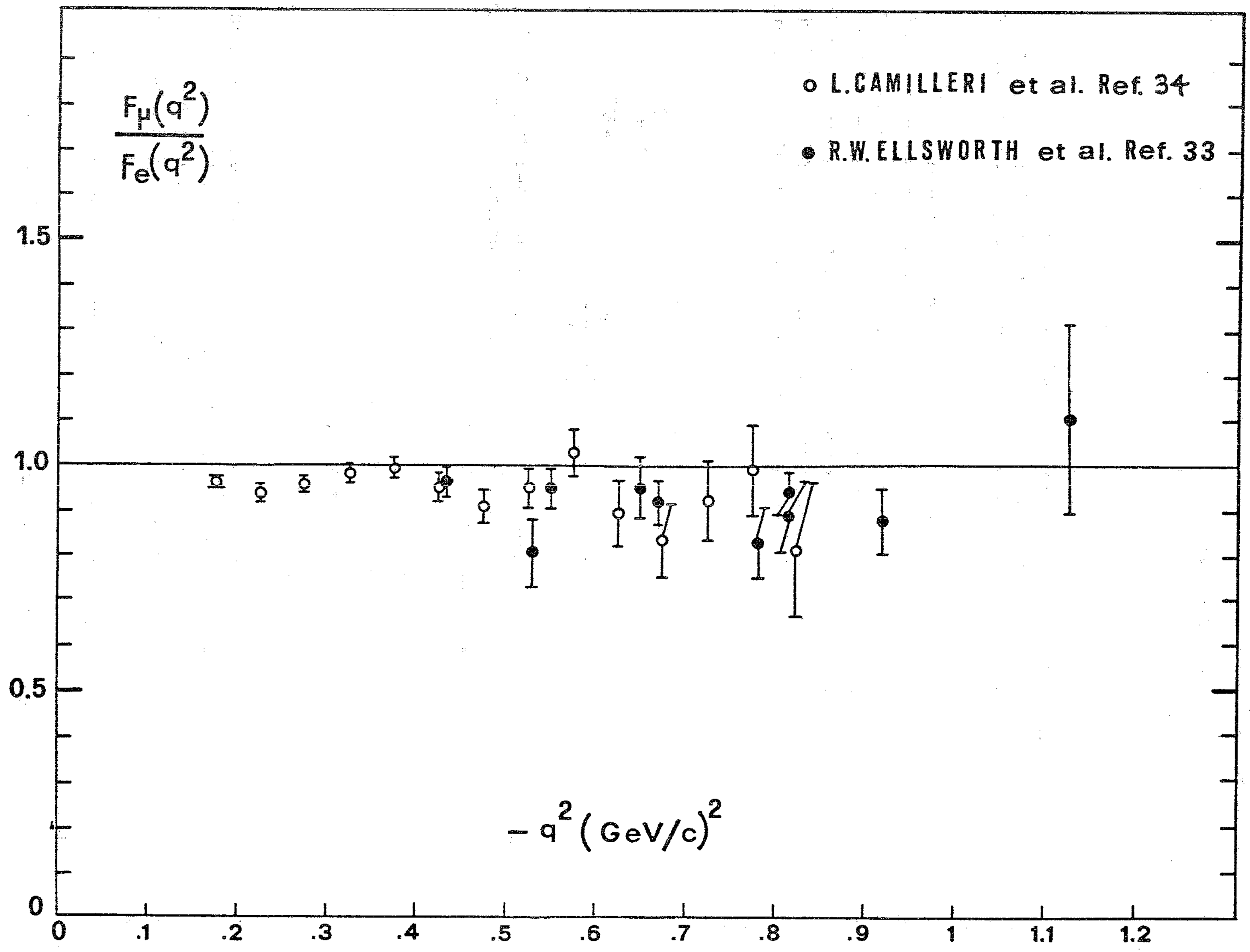


FIG. 7 - Comparison of muon-proton and electron-proton elastic scattering. The ratio $F_{\mu,e} = F_\mu(q^2)/F_e(q^2)$ is displayed as a function of $-q^2$. $F_\mu(q^2)$ and $F_e(q^2)$ are the vertex modification functions.

Again the slope of $F_{\mu,e}$ is compatible with zero, the absolute value (although smaller than one) being consistent with 1 within the errors.

I. 2. - NUCLEON FORM FACTORS. -

The most general form of the matrix element of the E. M. current of a nucleon between two states of four-momentum p and p' ($p^2 = p'^2 = M^2$) is[1,2]

$$\langle p' | J_\mu | p \rangle = \langle p' | F_1(q^2)\gamma_\mu + k\sigma_{\mu\nu} q^\nu F_2(q^2) | p \rangle \tag{9}$$

where $q_\mu = p'_\mu - p_\mu$ is the difference between the final and initial nucleon four-momenta; k is the anomalous magnetic moment μ-1. F_1 and F_2 (the so-called Dirac and Pauli form factors) are restricted by the requirement of the hermiticity of the E. M. current to be real for $q^2 < 0$ (space-like region).

As a consequence of the hypotheses discussed in the previous section, the cross-section for lepton-nucleon scattering turns out to be a rather simple expression of E (energy of the incident lepton), θ (scattering angle), F_1^2 and F_2^2. By performing at least two cross-section measurements at the same value of q^2, but at different angles and energies, F_1^2 and F_2^2 can in principle be evaluated, although it is quite a complicated numerical procedure[1]. This procedure becomes quite simple if we solve in terms of two other functions, G_E^2 and G_M^2. G_E and G_M, the so called "Electric" and "Magnetic" form factors, are related to F_1 and F_2 as follows:[37]

$$G_E = F_1 - \tau k F_2 \qquad F_1 = \frac{\tau G_M + G_E}{1+\tau}$$
$$G_M = F_1 + k F_2 \qquad F_2 = \frac{G_M - G_E}{(1+\tau)k} \qquad \left(\tau = -\frac{q^2}{4M^2}\right) \tag{10}$$

As usual, from here on we will attach when needed a second index P or N to the form factors according to whether they refer to the proton or to the neutron. The isovector and isoscalar form factors are also conveniently introduced

$$\begin{aligned} G_{MV} &= G_{MP} - G_{MN} \\ G_{MS} &= G_{MP} + G_{MN} \\ G_{EV} &= G_{EP} - G_{EN} \\ G_{ES} &= G_{EP} + G_{EN} \end{aligned} \tag{11}$$

In terms of G_E and G_M the elastic scattering cross-section assumes the simple form (1), and G_E^2 and G_M^2 are simply related to the slope and intercept of the Rosenbluth plots. It is worth noticing, however, that as $\tau = (-q^2/4M^2)$ grows larger than 1, the cross-section becomes less and less dependent on G_E^2. As a consequence, at large values of $-q^2$ G_E^2 is difficult to measure.

Elastic scattering experiments of electrons on nucleons have been performed for more than 15 years.

A summary of the proton results is presented in Figs. 8, 9, 10, 11, 12 and 13[52,53].

In Fig. 8 $G_{EP}(q^2)$ is shown. In Fig. 9, $G_{MP}(q^2)$ for $-q^2 \lesssim 150\ f^{-2}$ is presented. When $-q^2$ is larger than $\sim 50\ f^{-2}$, the errors on G_{EP} become very large, and no measurement is available for $-q^2 \gtrsim 100\ f^{-2}$. At higher values of $-q^2$, the G_{EP} contribution to the cross-section becomes in fact so small that only G_{MP} can be determined. This is usually done by assuming in this high $-q^2$ region the same relation between G_{MP} and G_{EP} which experimentally approximately holds at lower momentum transfers, namely $G_{EP} \simeq (G_{MP}/\mu_P)$

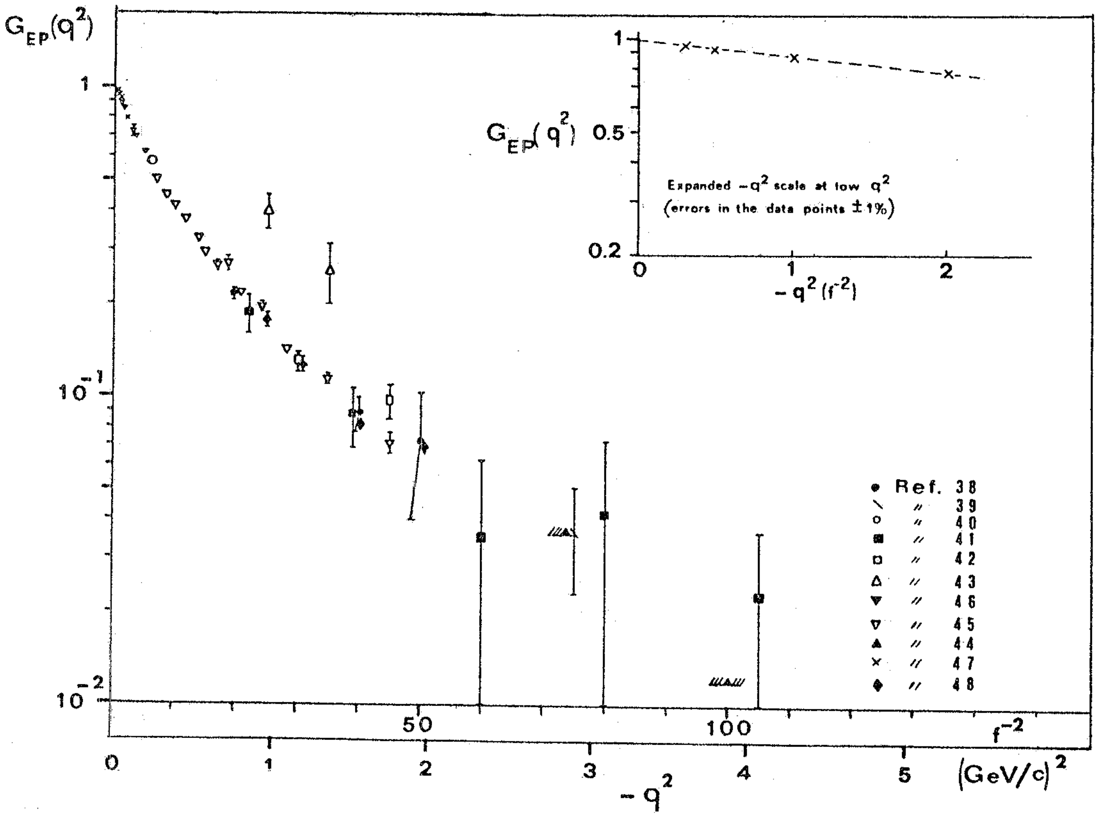


FIG. 8 - Electric form factor of the proton $G_{EP}(q^2)$ plotted agaist $-q^2$.

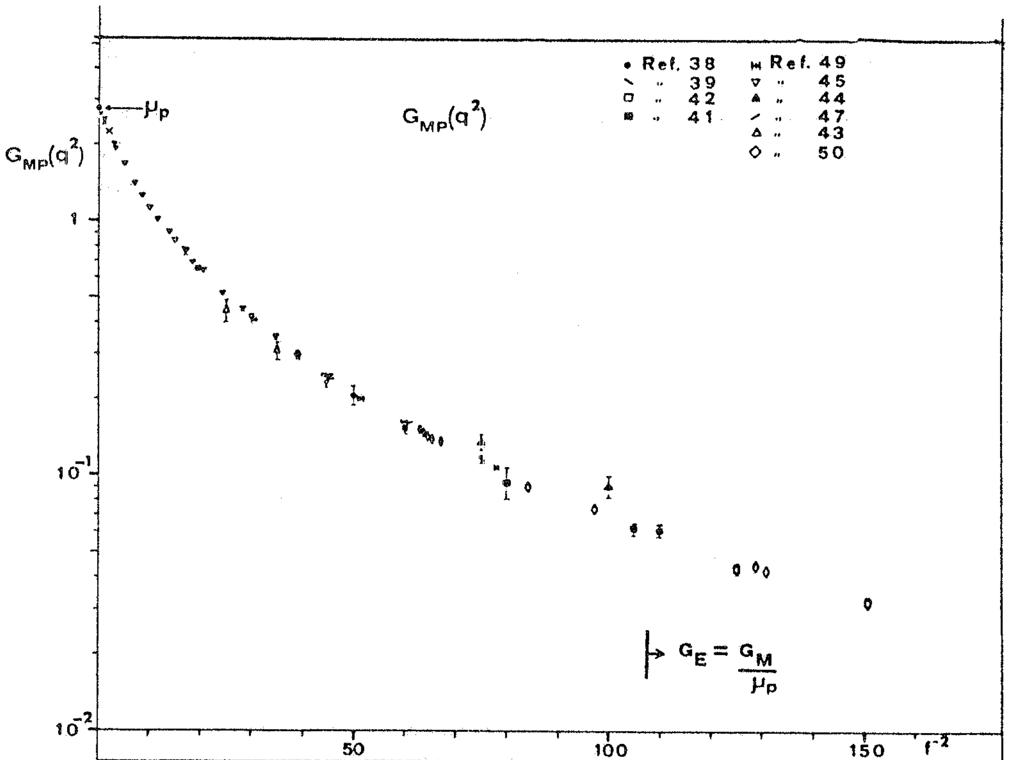


FIG. 9 - Magnetic form factor of the proton $G_{MP}(q^2)$ plotted agaist $-q^2$ up to $-q^2 \leqslant 150\ f^{-2}$. Values of $G_{MP}(q^2)$ for $-q^2 \gtrsim 100\ f^{-2}$ are obtained with the usual assumption $\mu_p G_{EP}(q^2) = G_{MP}(q^2)$.

(see the following). Actually, the cross-sections in the $-q^2$ region above $\sim 150\ f^{-2}$ depend so little on G_{EP}, that the data points on G_{MP} presented in Fig. 10 would remain the same within the errors unless G_{EP} exceeds G_{MP} by more than one order of magnitude. The fact that $G_{EP}(q^2) = G_{MP}(q^2)/\mu_P = 1$ for $-q^2 \Longrightarrow 0$ is required by the interpretation of the form factors as Fourier transforms of the charge and magnetic moment distributions of the proton(6): this picture is rigorous in the static limit $-q^2 \Longrightarrow 0$. On the contrary, at large momentum transfers the Fourier transform interpretation is far from being rigorous(37), and an attempt to explain the rapid fall-off of the form factors for increasing $-q^2$ with a hard--core of the nucleon is therefore arbitrary.

It is easily seen from Figs. 8, 9 that $G_{EP} \simeq (G_{MP}/\mu_P)$. To what extent this famous relation ("scaling law") really holds is better seen in Fig. 11, where the quantity $(\mu_P G_{EP}/G_{MP})^2$ is shown. We see that for $-q^2 \gtrsim 30\ f^{-2}$ deviations $\gtrsim 20 \div 30\%$ are experimentally found.

The scaling law for the proton form factors is to be considered an approximate mnemonic rule. Its theoretical foundation (quark model, SU_6(54)) is rather weak. In addition, in the time-like region, its validity near threshold would cause serious problems (see section II. 2.).

Another useful mnemonic rule (with no theoretical foundation) is the so-called dipole fit, i. e. $G_{MP}/\mu_P \simeq G_D \equiv (1 - (q^2/0.71\ (\mathrm{GeV}/c)^2))^{-2}$. The comparison between G_{MP}/μ_P and G_D is shown in Fig. 12, 13 where the quantity $G_{MP}/\mu_P G_D$ is presented. The dipole fit appears to hold within $\simeq 30\%$.

The situation on the neutron form factors is much less clean. This is due to the fact that free neutron targets are not available, so that the information on e-n scattering is extracted using a quite complicated and model dependent procedure from e-deuteron, (e-d), elastic and inelastic scattering. Some information at very low momentum transfers is also obtained by scattering low energy (thermal) neutrons on high-Z atoms. An excellent review of the methods used to extract G_{EN} and G_{MN} from the experimental data can be found in ref. (6).

The difficulty in the experiments of scattering of neutrons on atoms (first performed by Fermi and Marshall(55)) is to separate the coherent scattering amplitude on the nucleus from the scattering amplitude of the neutrons on the atomic electrons.

Different atomic targets and analysis methods have been used in different experiments. The results expressed in terms of $(dG_{EN}/dq^2)\,|_{q^2=0}$ are in fair agreement:

TABLE I

Author	N. Ref.	$dG_{EN}/dq^2\ (\mathrm{GeV}/c)^{-2}$
Krohn I	56	0.459 ± 0.02
Krohn II	56	0.50 ± 0.01
Melkonian	57	0.575 ± 0.019
Hughes	58	0.512 ± 0.019

dG_{EN}/dq^2 is related to the root-mean-square charge radius of the neutron(6)

$$\langle r^2_{ch} \rangle = \frac{1}{6} \left| \frac{dG_{EN}}{dq^2} \right|_{q^2=0} \tag{12}$$

which can thus be considered known - including uncertainties from the models - to within $\sim 5 \div 10\%$.

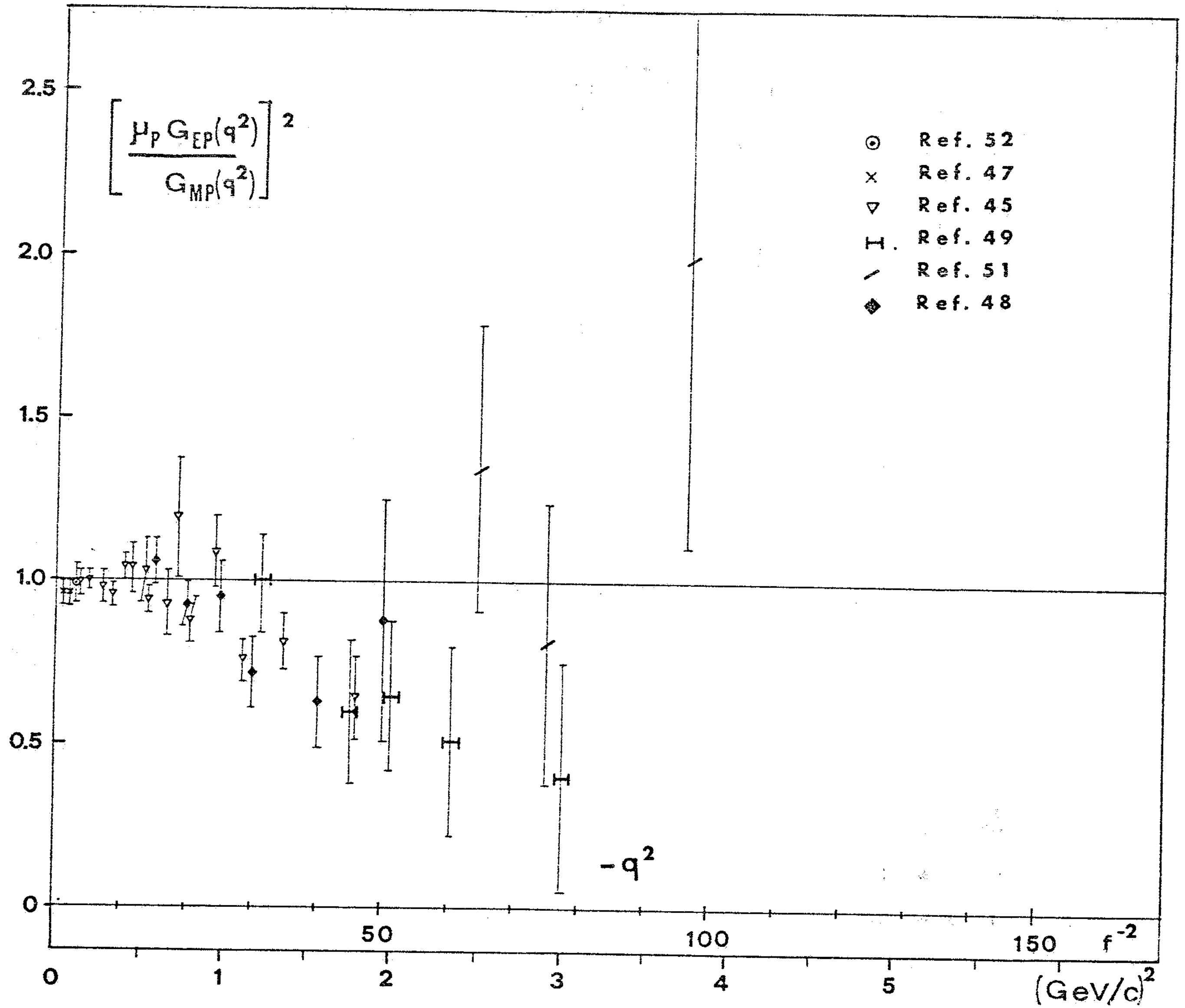


FIG. 11 - Comparison between the electric and magnetic proton form factors. The ratio $\left[\mu_p G_{EP}(q^2)/G_{MP}(q^2)\right]^2$ is shown against the momentum transfer $-q^2$.

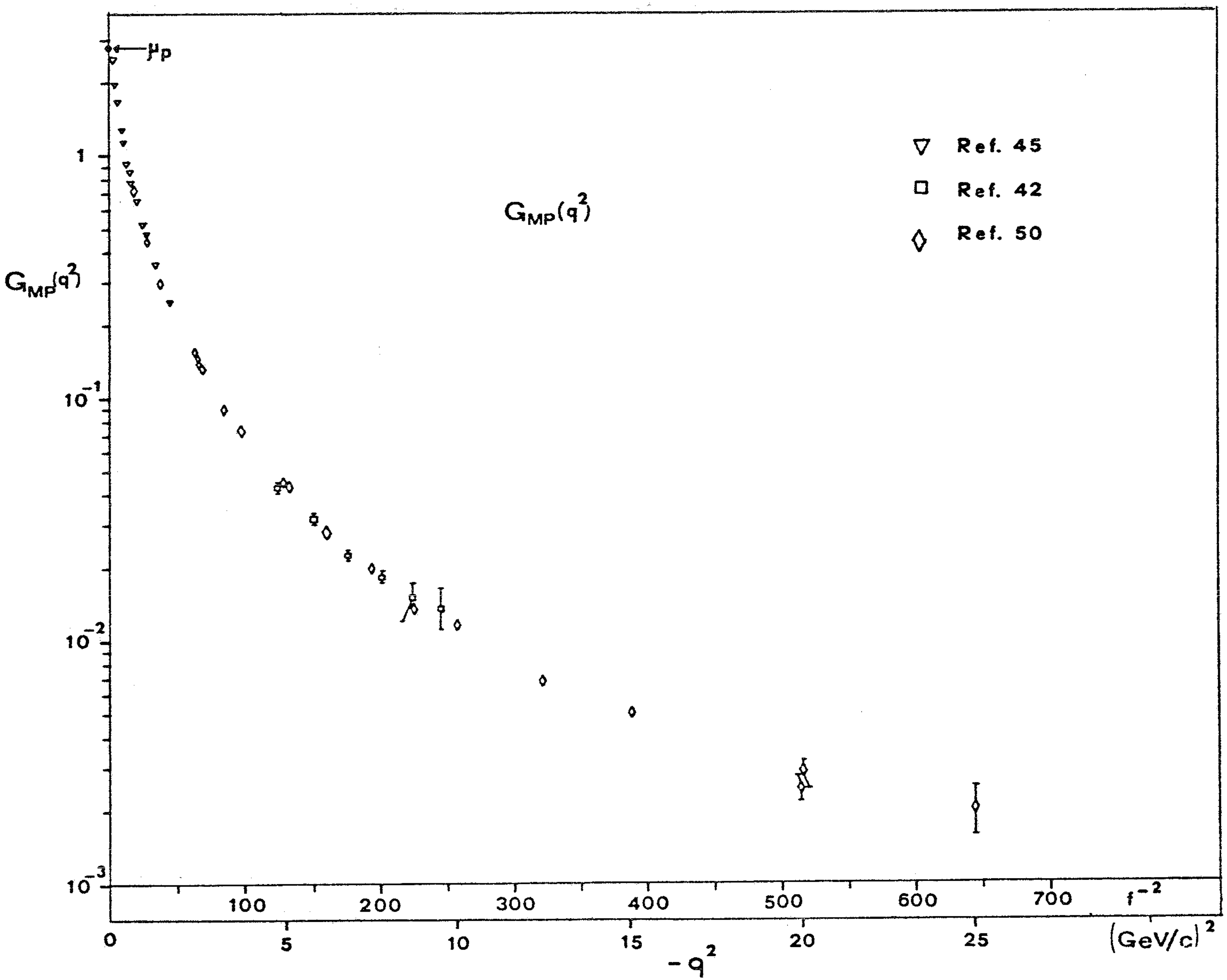


FIG. 10 - Magnetic form factor of the proton $G_{MP}(q^2)$ plotted against $-q^2$ for all the measured $-q^2$ range. Values of $G_{MP}(q^2)$ for $-q^2 \gtrsim 100\ f^{-2}$ are obtained with the usual assumption $\mu_p G_{EP}(q^2) = G_{MP}(q^2)$.

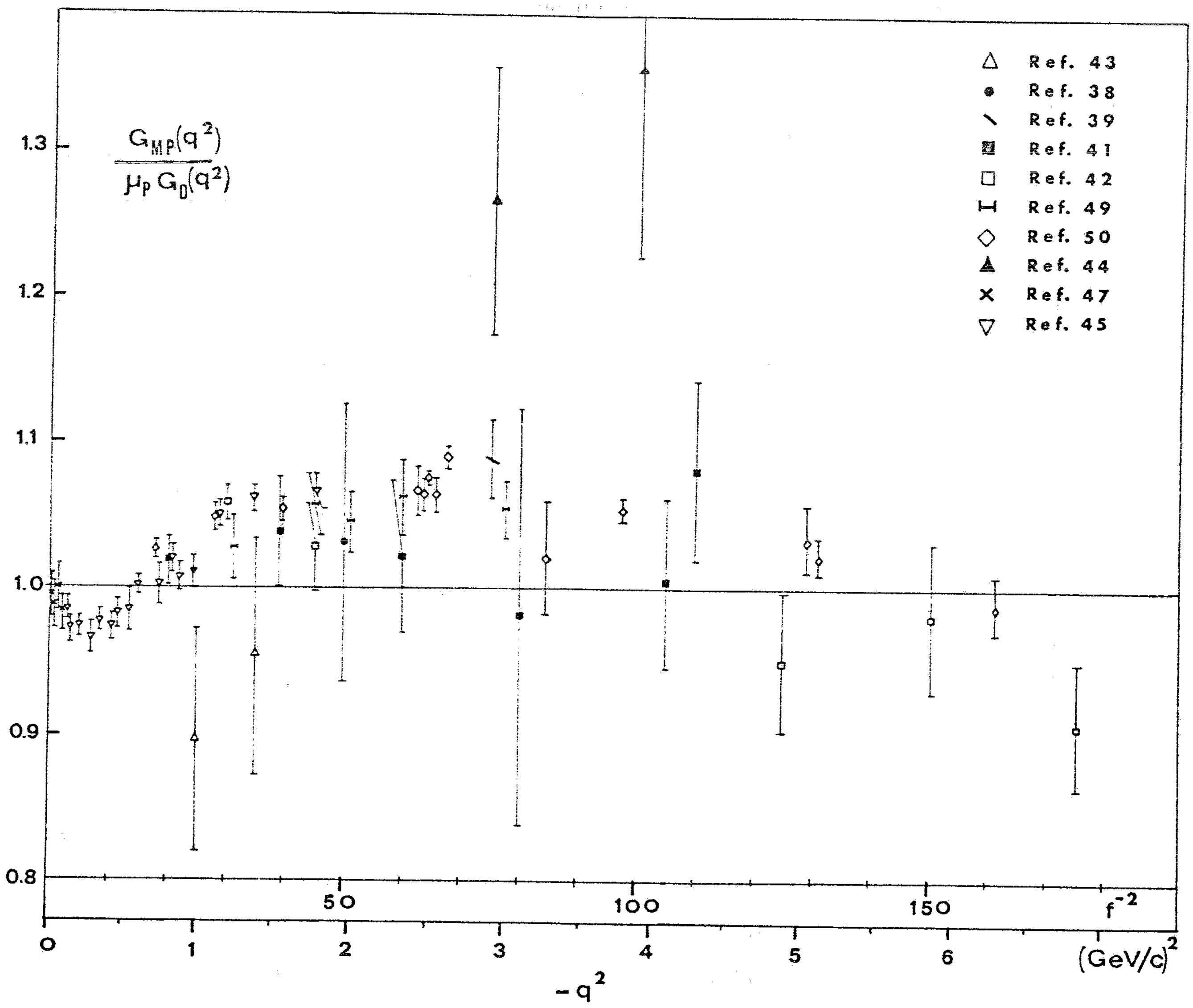


FIG. 12 - "Dipole fit" for the magnetic proton form factor. The ratio $G_{MP}(q^2)/\mu_p G_D(q^2)$ is plotted agais $-q^2$ for values of $-q^2 \lesssim 180\ f^{-2}$.

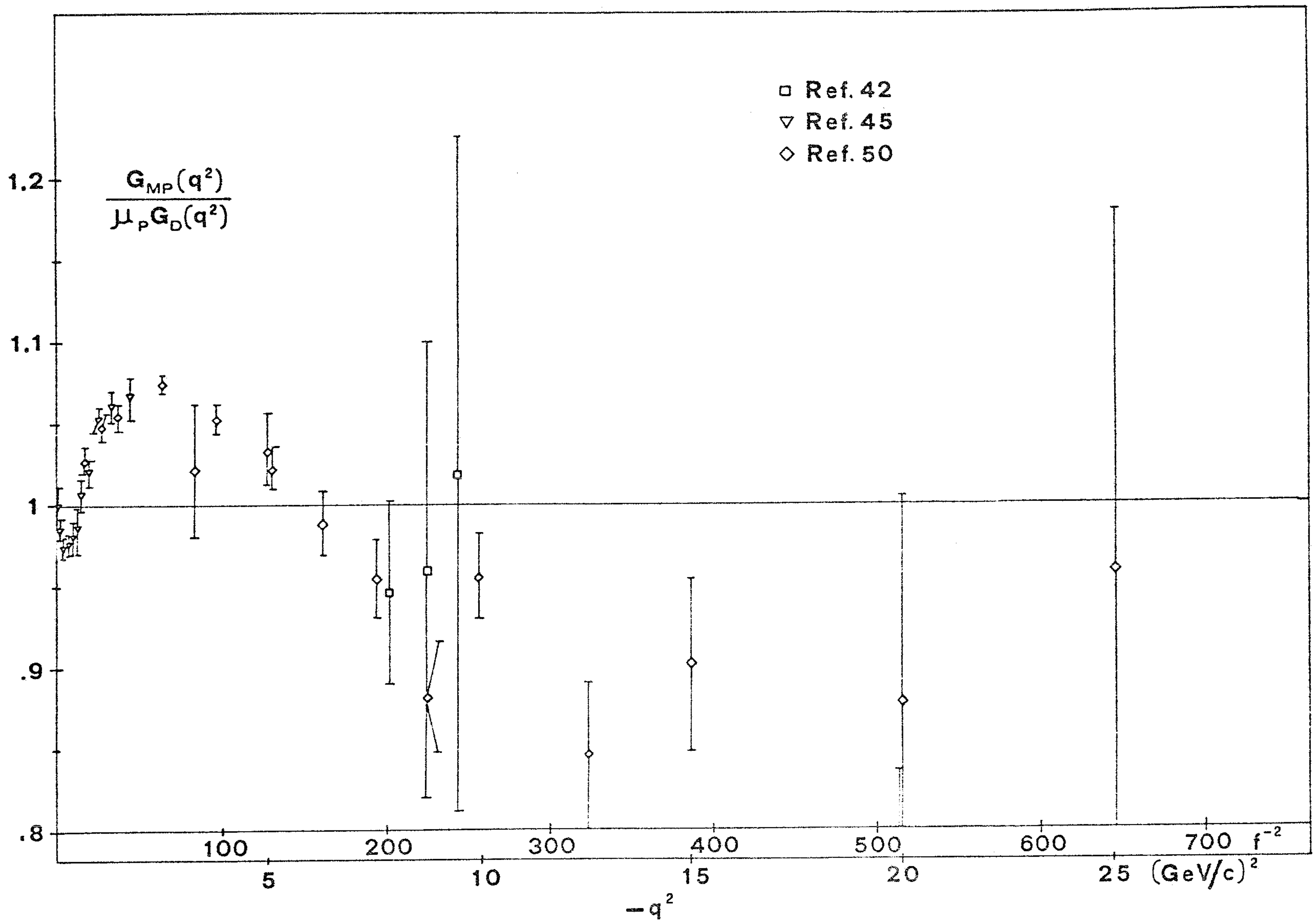


FIG. 13 - "Dipole fit" for the magnetic proton form factor. The ratio $G_{MP}(q^2)/\mu_p G_D(q^2)$ is plotted against $-q^2$ for all the measured $-q^2$ range.

At higher values of $-q^2$ ($0.3 < -q^2 \leqslant 20\ f^{-2}$) G_{EN} can be extracted from elastic e-d scattering measurements, once the electric proton form factor G_{EP} and the deuteron structure are known. In fact the e-d elastic scattering cross-section can be written as[6]:

$$(13) \qquad \left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[A(q^2) + B(q^2)\ tg^2 \frac{\theta}{2} \right]$$

$A(q^2)$ and $B(q^2)$ contain the charge, magnetic and quadrupole form factors of the deuteron, in addition to the isoscalar (the deuteron has $T = 0$) nucleon form factors. For electron scattering angles θ smaller than 15^o the term $B(q^2) tg^2\, \theta/2$ contributes less than 0.1% to the cross section and can be neglected. Equation (13) takes then the simple form.

$$(14) \qquad \frac{d\sigma}{d\Omega} \simeq \left(\frac{d\sigma}{d\Omega}\right)_{Mott} A(q^2)$$

so that cross section measurements allow one to determine $A(q^2)$. Notice however that for $-q^2 \simeq 15\ f^{-2}$, $A(q^2)$ is as small as $\sim 10^{-4}$, so that experiments become very difficult.

In turn

$$(15) \qquad A(q^2) = \frac{G^2_{Ed}(q^2)}{1+\tau} + \frac{8}{9}\ \tau^2\ \frac{G^2_{Qd}(q^2)}{1+\tau} + \frac{2}{3}\ \tau^2\ \frac{G^2_{Md}}{1+\tau}$$

where

$$(16) \qquad \left[\begin{array}{l} G_{Ed}(q^2) = 2\ G_{ES}(q^2)\ F_{Ed}(q^2) \\ G_{Qd}(q^2) = 2\ G_{ES}(q^2)\ F_{Qd}(q^2) \\ G_{Md}(q^2) = 2\ (Md/M)\ G_{MS}(q^2)\ F_{Md}(q^2) \end{array} \right.$$

F_{Ed}, F_{Qd} and F_{Md}, which describe the deuteron structure, can be calculated from the non-relativistic deuteron wave functions for the S and D states. Several models are available for this purpose (Hamada-Johnston[59], Mc Gee[60], Feshbach-Lomon[61], etc.). Relativistic corrections have been evaluated by F. Gross[62].

The experimental values of $A(q^2)$ are compared with the different models. It turns out that no model fits the data if G_{EN} is put equal to zero. This leads to $G_{EN} \neq 0$, the actual value of G_{EN} depending however on the wave function of the deuteron used in the analysis.

A review of the results for G_{EN} obtained with this method can be found in ref. (63). In Fig. 14 the results obtained with the Feshbach-Lomon model are shown[63]. Relativistic corrections are included[62]; the four-pole fit[64] for the electric proton form factor is used (to extract G_{EN} from G_{ES}, G_{EP} must be known, see (11)). The data points are from ref. 63, 65, 66, 67, 68, 69. The dashed curve is $G_{EN} = \mu_N\ \tau\, G_{EP}$[70], which corresponds to the assumption $F_{1N} = 0$; the dash-dot curve is $G_{EN} = (\mu_N \tau / 1+4\tau)\ G_{EP}$[71]; while the solid curve is the best fit to the data points with a curve $G_{EN} = (\mu_N \tau / 1+b\tau)\ G_{EP}$, where the parameter b turnsout to be $b = 5.6$.

At higher values of q^2 ($-q^2 \gtrsim 15\ f^{-2}$) the neutron form factors can be determined by means of inelastic electron-deuteron scattering experiments.

At sufficiently large momentum transfers the nucleons can be treated as approximately free (impulse approximation), but with a momentum distribution given by the deuteron ground state wave functions (Again the knowledge of the deuteron wave function is needed)[6].

Inelastic e-d scattering experiments have been performed, detecting either only the scattered electron (non-coincidence experiments), or the scattered electron in coincidence with the proton or the neutron (coincidence experiments). Small corrections are needed in this case due to the tail of the deuteron wave function and the final state interaction.

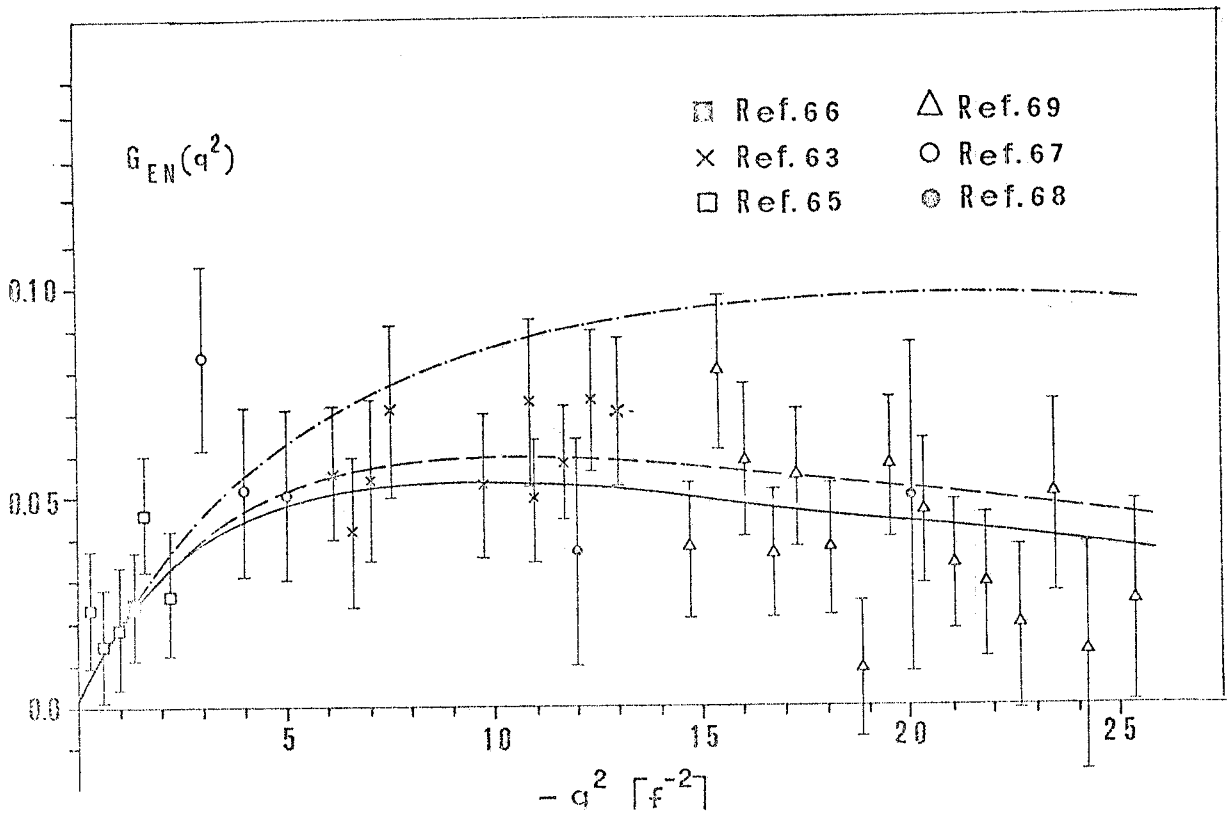


FIG. 14 - The neutron electric form factor $G_{EN}(q^2)$ derived from elastic electron-deuteron scattering measurements. The Feshbach-Lomon deuteron wave function was used. The dashed line is $G_{EN}=\mu_n\tau G_{EP}$, which corresponds to the assumption $F_{1n}=0$; the dashdot curve is $G_{EN}=(\mu_n\tau/1+4\tau)G_{EP}$; the solid curve is the best fit to the data points with a curve $G_{EN}=(\mu_n\tau/1+b\tau)G_{EP}$ (b, free parameter, turns out to be b = 5.6).

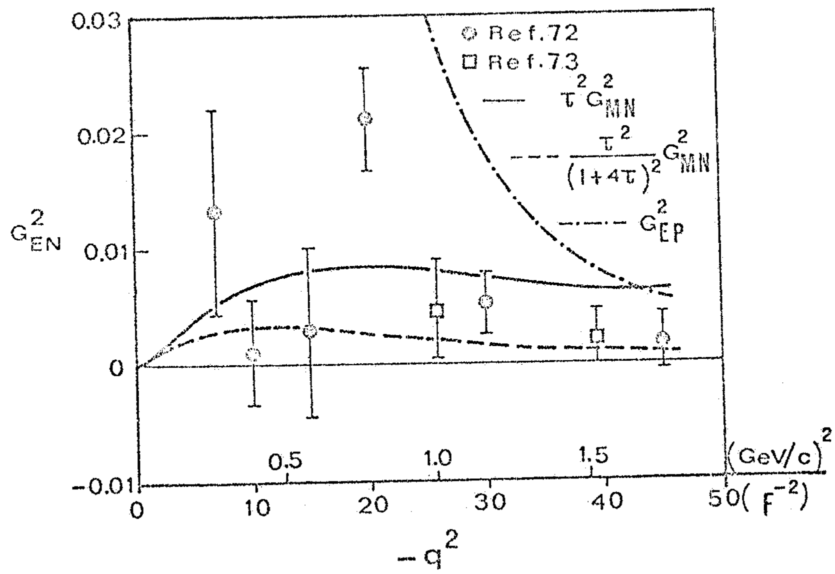


FIG. 15 - G_{EN}^2 as determined from inelastic electron-deuteron scattering is plotted against $-q^2$.

In Fig. 15, 16 G_{EN} and G_{MN} as determined in inelastic e-d scattering measurements are shown. In Fig. 15 G_{EN} is shown, and compared with the same models as in Fig. 14. In Fig. 16 G_{MN} is shown. The data of some old experiments, not shown directly in the figure, have been used in more recent papers to improve the quality of their data, and are therefore quoted under the reference of the most recent analysis. The points at $-q^2 > 50\ f^{-2}$ are upper limits.

In Fig. 17 the ratio $\mu_P G_{MN}/\mu_N G_{MP}$ is shown. We see that also the "scaling law"(54), $G_{MN}/\mu_N = G_{MP}/\mu_P$ approximately holds.

I. 3. - PION FORM FACTOR. -

The most general form of the matrix element of the E. M. current of a spin-0 particle B is

(17) $$\langle p' \left| J_\mu^B \right| p \rangle = F\,(q^2)(p'_\mu + p_\mu)$$

As a consequence of (17) and of the usual hypotheses, the cross-section for elastic scattering of electrons on B can be written as(6)

(18) $$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot F^2(q^2)$$

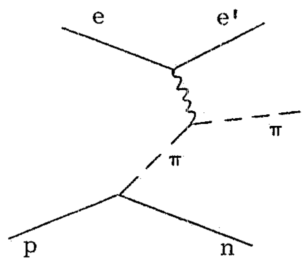
where $F(q^2)$, the form factor of the target particle, is a function of q^2 only and is real in the space-like region.

Elastic scattering experiments of electrons on free pions are however not possible, and elastic scattering experiments of pions on electrons have not yet been performed. The information available on $F_\pi(q^2)$ in the space-like region has been obtained by scattering electrons on virtual pions, namely through electroproduction experiments of pions on nucleons near threshold. In this case, however, the connection between experimental data and F_π is much more complicated than (18), and is in addition model dependent.

This is due to two facts:

a) In electroproduction processes (e. g. $e^- + p \longrightarrow e^- + \pi^+ + n$) the contribution from the graph

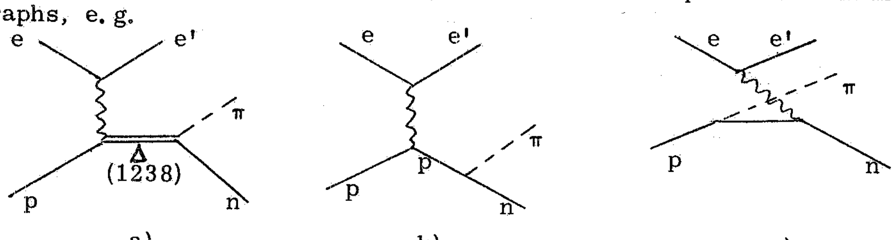
(19) "pion-pole"



(which is relevant for the determination of F_π) is due to an off-mass-shell pion interacting with the nucleon.

b) The contribution from the graph (19) must be separated from the contribution of other graphs, e. g.

(20)



a) b) c)

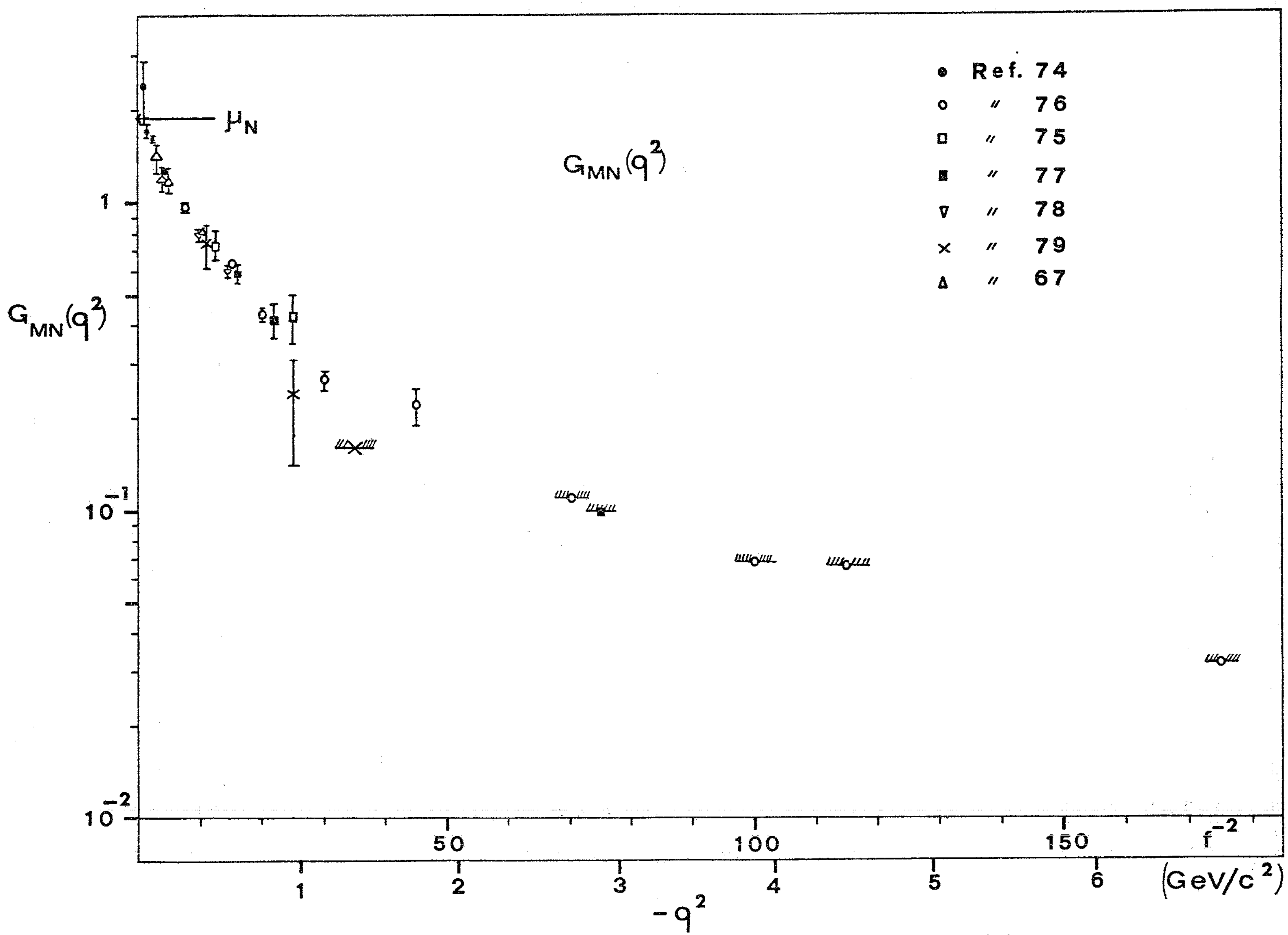


FIG. 16 - G_{MN} as determined from inelastic electron-deuteron scattering. The symbols represent upper limits,

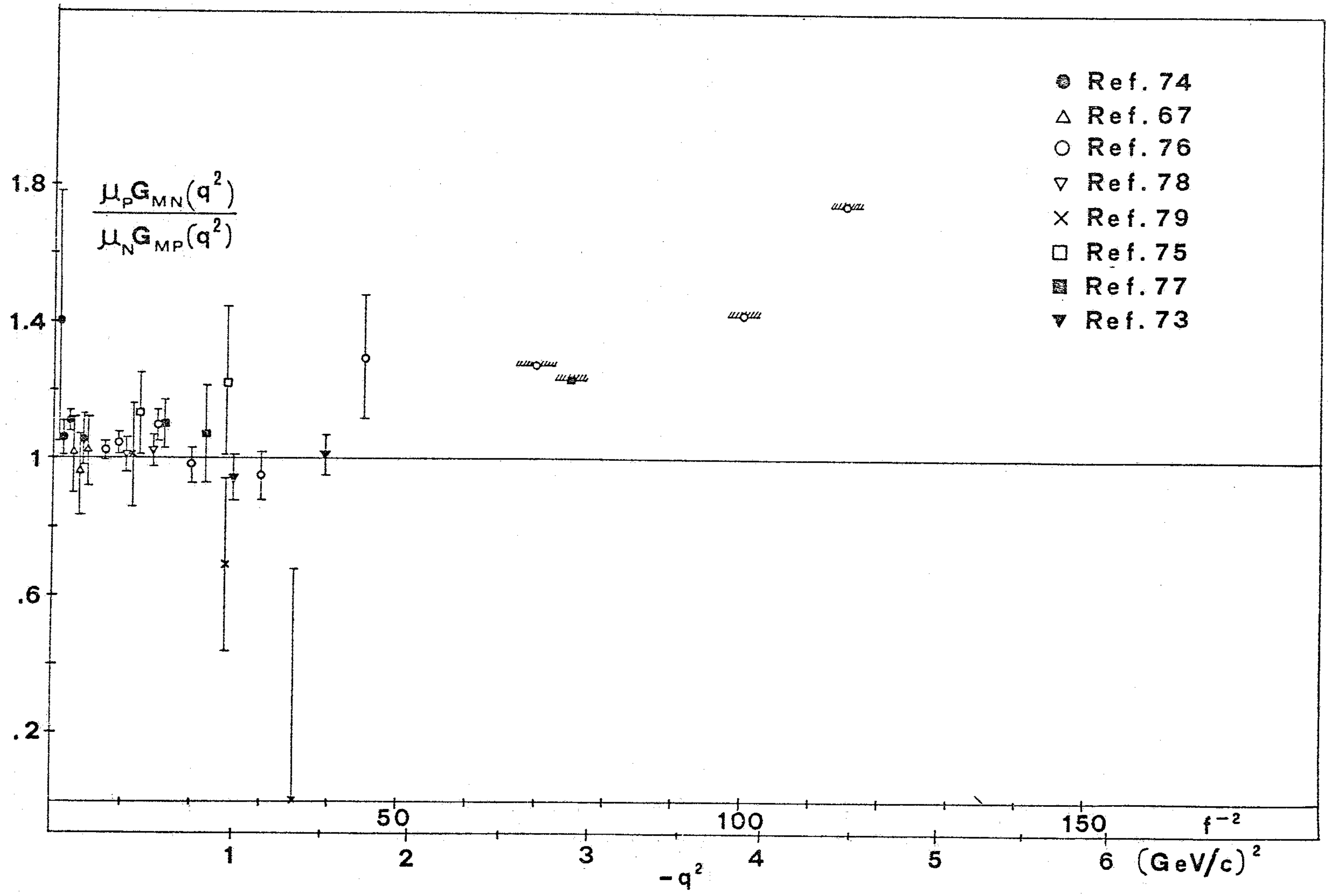


FIG. 17 - Test of the "scaling law": $\mu_p G_{MN}/\mu_N G_{MP}$ is plotted as a function of $-q^2$. The symbols represent upper limits.

whose contributions are expected to be important in the electroproduction process near threshold.

The electroproduction cross section - under the 1 – PE assumption - can be written as[80]

$$\frac{d^3\sigma}{dE'\, d\Omega_e\, d\Omega^x_\pi} = \Gamma_{ph} \frac{d}{d\Omega^x_\pi} \tag{21}$$

E' = energy of the scattered electron in the lab. system
$d\Omega_e = \sin\theta_e\, d\theta_e\, d\varphi_e$ element of solid angle of the scattered electron in the lab. system
$d\Omega^x_\pi = \sin\theta^x\, d\theta^x\, d\varphi^x_\pi$ element of solid angle of the produced pion in the c. m. system of the hadrons in the final state.

The factor Γ_{ph} contains the electrodynamics of the process (electron-photon vertex and photon propagator) and $d\sigma / d\Omega^x_\pi$ is the cross-section for pion photoproduction by virtual (polarized) photons[80]:

$$\frac{d\sigma}{d\Omega^x_\pi} = A + \varepsilon B + \varepsilon C \sin^2\theta^x_\pi \cos^2\varphi^x_\pi + \sqrt{2\varepsilon\,(1+\varepsilon\,)}\, D \sin 0^x_\pi \cos\varphi^x_\pi \tag{22}$$

where ε , the polarization of the virtual photon, is given by

$$\varepsilon = \left(1 + \frac{2\,|\vec{p}\,|^2}{|q^2|} \cdot \tan^2 \frac{\theta_e}{2}\right)^{-1} ,$$

with $\vec{p}$ three momentum of the photon in the lab. system. The functions A, B, C and D contain the information on the structure of the hadrons involved in the process, i. e. they depend on the form factors at the vertexes $[\gamma\pi\pi]$, $[\gamma NN]$, $[\gamma N\Delta]$ ($\Delta \equiv 1238 \;\; 3/2 \;\; 3/2$ resonance). The first term A is the differential cross-section for unpolarized transverse virtual photons; the second term is the contribution from virtual longitudinal photons; the third term from linearly polarized transverse photons; and the last term represents the interference between the transverse and the longitudinal amplitudes.

In order to extract from the experimental data the pion form factor $F_\pi(q^2)$, i. e. the $[\gamma\pi\pi]$ vertex function, the contribution from the 1-pion-exchange pole diagram (19) must be isolated. For this purpose a detailed phenomenological theory of electroproduction is needed. Many different approaches have been tried by several authors. Dispersion relations[81÷86], isobaric models[87], current algebra techniques and the PCAC assumption[88] are the main ingredients in the calculations. A rather complete list of references can be found in ref. (80). The experimental information comes mainly from reactions

$$e + p \longrightarrow e + n + \pi^+ \tag{23}$$

$$e + p \longrightarrow e + p + \pi^0 \tag{24}$$

in which one of the hadrons in the final state is detected in coincidence with the scattered electron. Reaction (24) is particularly relevant to a detailed phenomenological understanding of the electroproduction process since in this case the pole-diagram (19) does not contribute; in particular the so called "transition" form factor $G_{M\Delta}(q^2)$ to the first Δ(1238) resonance (namely the $[\gamma N\Delta]$ vertex function) can be evaluated. This makes easier the interpretation of the more complicated data from reaction (23) in terms of $F_\pi(q^2)$.

In Fig. 18, a summary of the experimental information available on $F_\pi(q^2)$ is shown. In several cases the experimental data have been analyzed using different theoretical models: this gives an idea of the dependence of $F_\pi(q^2)$ on the method of analysis.

An additional uncertainty connected with the models, which does not show up as a

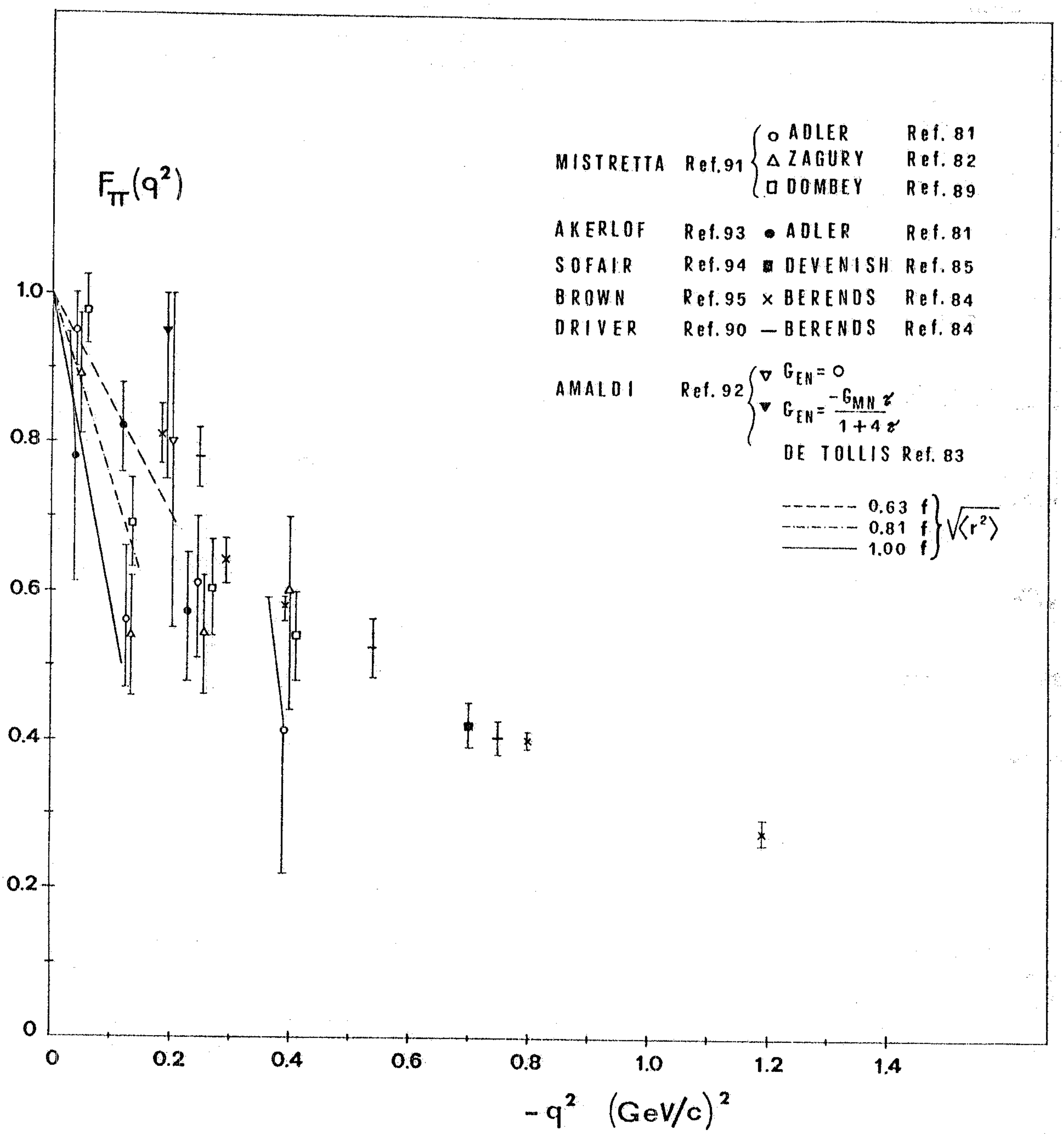


FIG. 18 - The pion form factor $F_{\pi}(q^2)$ as determined from electroproduction experiments. In correspondence with each point we quote both the reference of the experiment (e. g. Mistretta ref. (91)) and of the model used for the analysis (e. g. Dombey, ref. (89). The slopes expected at $q^2 = 0$ for three different values of the pion radius $\sqrt{\langle r^2 \rangle}$ are also shown.

difference in data points coming from different types of analysis, is the uncertainty in the foundation of hypotheses which are common to different models.

An important example of this kind has been pointed out by Dombey & Read[89, 96], and is related to our ignorance of the lower vertex of diagram (19). In current algebra techniques (PCAC), the pion field is related to the divergence of the axial weak current associated with the nucleon. This brings about the axial form factor of the nucleon G_A. Usually $G_A = G_{EP}$ is assumed. If one however does not make this more or less arbitrary assumption (which is actually justified by chiral symmetry), at each value of q^2 a large band of possible values of $F_\pi(q^2)$ can fit the experimental data very well by properly choosing an associated value for G_A.

In Fig. 18 three slopes expected at $q^2 = 0$ for different pion radii $\sqrt{\langle r^2 \rangle}$ are also shown. It appears that at this stage $\sqrt{\langle r^2 \rangle}$ is still poorly determined by electroproduction measurements.

I. 4. - THE Δ(1238) TRANSITION FORM FACTOR $G_{M\Delta}$. -

When the invariant mass of the π-nucleon system in the pion electroproduction process is near the Δ (1238) mass, the reaction is dominated by the production of the Δ resonance.

This is expecially true for reaction (24) to which diagram (19) does not contribute.

If we treat the Δ-resonance as a particle, then the cross section can be written as[97]

$$\frac{d\sigma}{d\Omega_e} = \sigma_{NS} \frac{|q^2|}{4M^2} \frac{1}{\varepsilon} \cdot \left[|G_{M\Delta}|^2 + |G_{E\Delta}|^2 + \frac{2|q^2|}{|\vec{P}|^2} \varepsilon |G_{C\Delta}|^2 \right] \tag{25}$$

where

$$\sigma_{NS} = \frac{\alpha \cos^2 \frac{\theta_e}{2}}{4E^2 \sin^4 \frac{\theta_e}{2} \left[1 + 2(E/M) \sin^2 \frac{\theta_e}{2} \right]}$$

E energy of the incident electron; $\vec{P}$ three-momentum of the virtual photon in the Δ rest frame. $G_{M\Delta}(q^2)$, $G_{E\Delta}(q^2)$ and $G_{C\Delta}(q^2)$ correspond to the M_1, E_2 and C_2 (Coulomb octupole) which can excite the $J = 1/2^+$ to $J = 3/2^+$ transition. Both on theoretical[98] and experimental grounds[99] $G_{E\Delta}$ and $G_{C\Delta}$ are expected to be much smaller than $G_{M\Delta}$, and are usually neglected in the analysis of experimental data. In this case equation (25) assumes a very simple form allowing one in principle to measure quite easily $|G_{M\Delta}|^2$. Notice in particular that according to (25), $|G_{M\Delta}|^2$ can be measured in experiments in which only the scattered electron is detected.

In practice, however, there are problems connected with the contribution of non-resonant background and with the large width of the Δ-resonance. A detailed phenomenological theory of electroproduction is again needed, although the situation is much simpler than for the determination of the pion form factor F_π.

The information from coincidence experiments is in practice needed for the multipole analysis of the electroproduction data, in which case $G_{M\Delta}$ is simply connected to the q^2 behaviour of the M_1 multipole contribution[97].

The experimental situation for $G_{M\Delta}$ is summarized in Fig. 19. The value at $q^2 = 0$ determined from photoproduction data is 3.00 ± 0.01[97]. With increasing $-q^2$ $G_{M\Delta}$ drops more rapidly than the proton form factors, as shown in Fig. 20 where the ratio of $G_{M\Delta}$ to three times the dipole fit is presented.

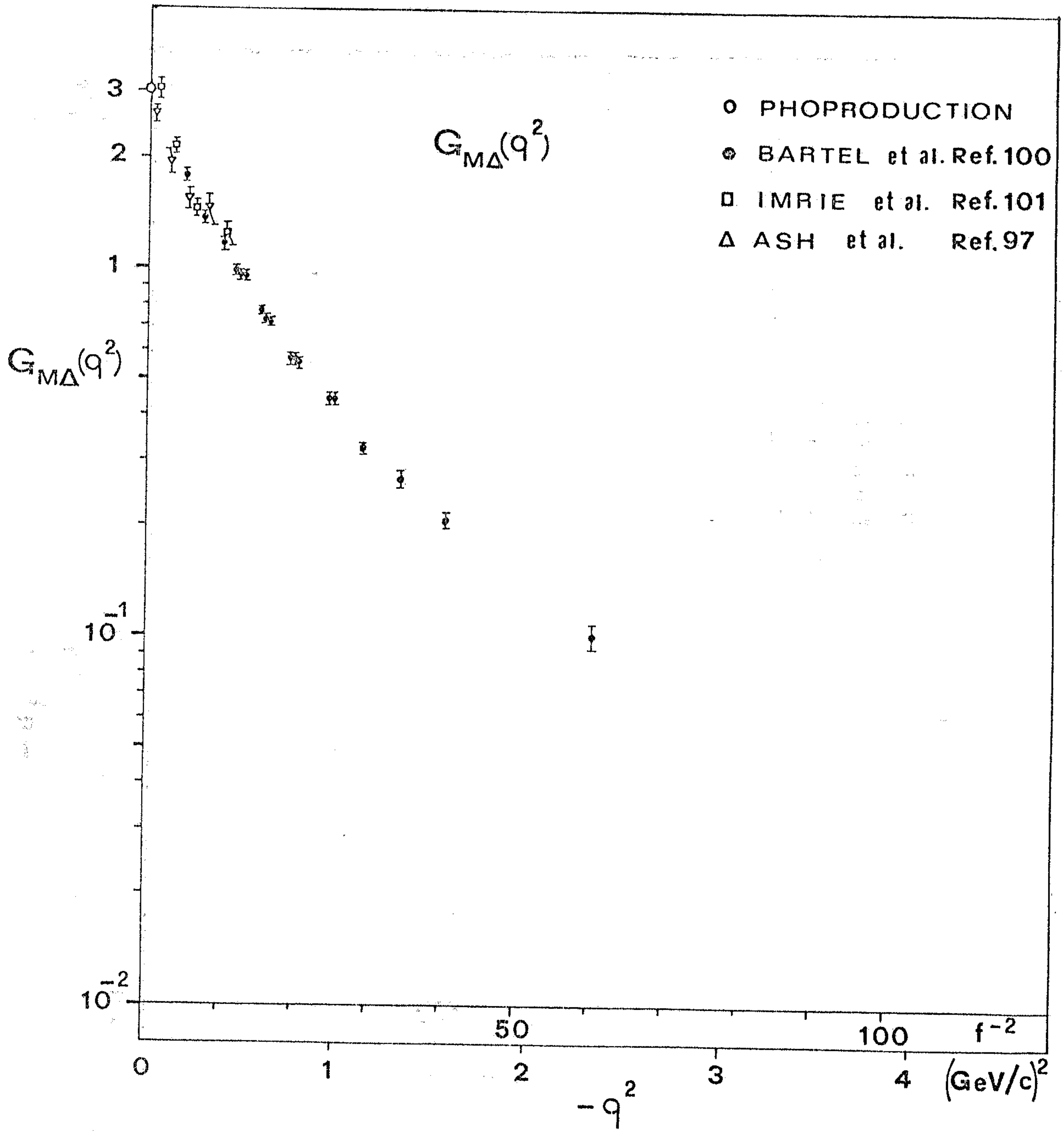


FIG. 19 - The transition form factor $G_{M\Delta}$ to the first nucleon isobar $\Delta(1238)$ as determined from electroproduction experiments. The point at $q^2 = 0$ comes from photoproduction experiments; see ref. (97).

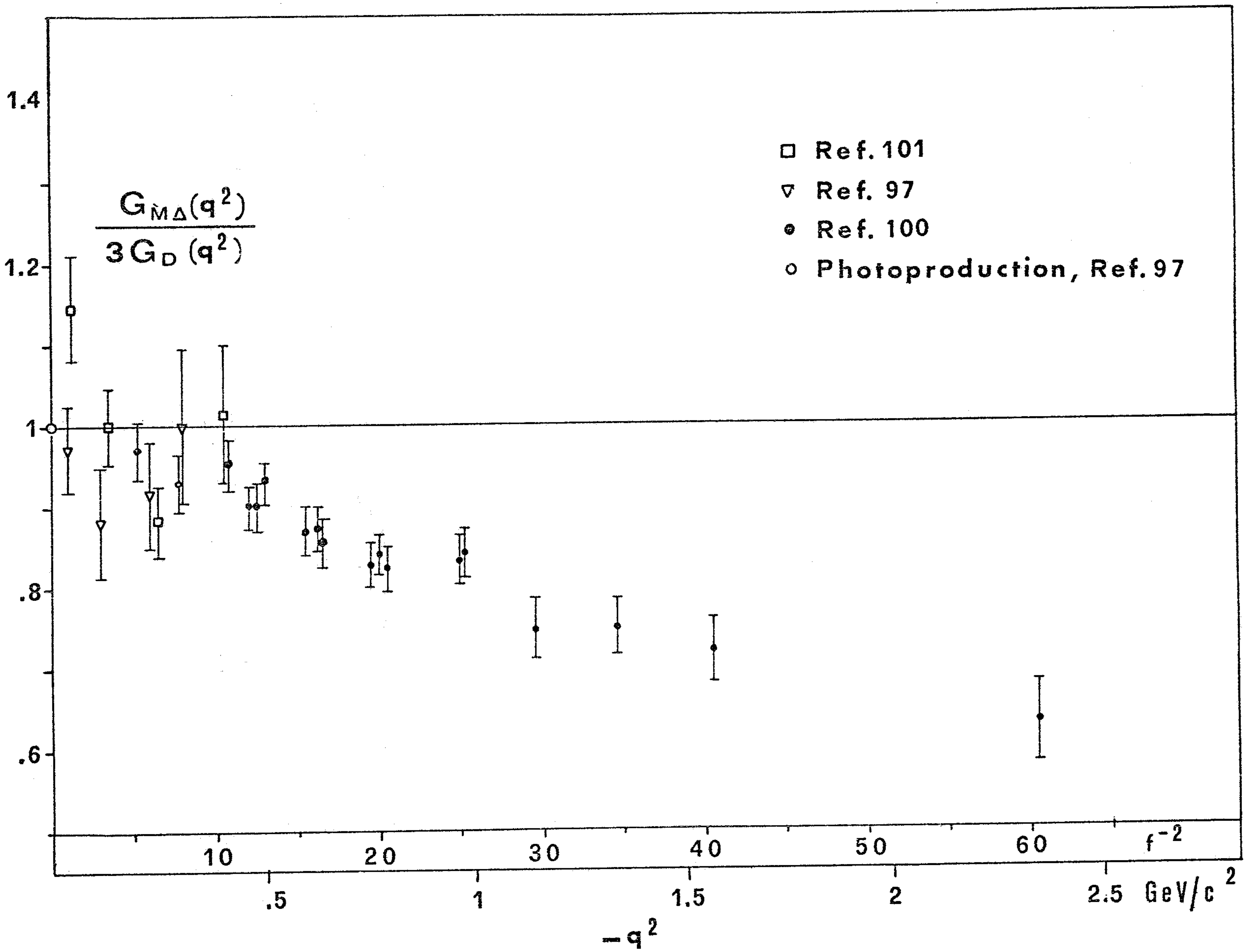


FIG. 20 - The ratio $G_{M\Delta}/3G_D$ of the transition form factor $G_{M\Delta}$ for the first nucleon isobar to three-times the dipole fit is plotted as a function of $-q^2$.

I. 5. - DEEP INELASTIC ELECTRON NUCLEON SCATTERING. -

An extensive systematic investigation of electron nucleon inelastic scattering has been carried out during last few years at SLAC by a SLAC-MIT collaboration(102). These data include measurements at large values of $|q^2|$ and correspondingly large energy transfers to the nucleon (deep inelastic region) and are of the inclusive type, i. e. only the scattered electron was detected. Some coincidence measurements - with the detection of part of final state hadronic products - have also been performed(103): however, these investigations are only now beginning and the results cannot yet properly be included in a review paper.

Also, since the inclusive - type data come essentially from one single experiment(104), we report here only a short summary of the results.

The cross-section for inclusive electron-nucleon inelastic scattering can be written as

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha}{4\pi^2}\,\frac{W^2 - M^2}{2M\,|q^2|}\,\frac{E'}{E}\,\frac{2}{1-\varepsilon}\left[\sigma_t + \varepsilon\sigma_s\right] \tag{26}$$

The symbols have the same meaning as in (21), (22); W, the missing mass of the unobserved hadronic final state, is given by $W^2 = 2M(E-E') + M^2 - q^2$. The cross-sections for transverse and longitudinal polarized virtual photons, σ_t and σ_s, are functions of the invariants q^2 and W (or of q^2 and $\nu = E-E'$).

As $q^2 \longrightarrow 0$, $\sigma_s \longrightarrow 0$ and $\sigma_t(q^2, \nu) \longrightarrow \sigma_\gamma(\nu)$, where $\sigma_\gamma(\nu)$ is the photo-absorbtion cross section for real photons of energy ν .

Alternatively, $d^2\sigma/d\Omega dE'$ can be written in terms of the "structure functions" W_1 and W_2

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{e^4}{4E^2}\,\frac{\cos^2\theta/2}{\sin^4\theta/2}\left[W_2 + 2W_1\tan^2\theta/2\right] \tag{27}$$

W_1 and W_2 are related to σ_t and σ_s by

$$\begin{cases} W_1 = K\sigma_t \\ W_2 = K\dfrac{|q^2|}{|q^2| + \nu^2}(\sigma_t + \sigma_s) \\ K = \dfrac{W^2 - M^2}{8\pi^2 M\alpha} \end{cases} \tag{28}$$

and to the experimental cross section by

$$\begin{aligned} W_1 &= \left[\frac{d^2\sigma}{d\Omega\, dE'}\right]_{exp}\left(\frac{d\sigma}{d\Omega}\right)^{-1}_{Mott}\left[(1+R)\frac{|q^2|}{|q^2| + \nu^2} + 2\tan^2\theta/2\right]^{-1} \\ W_2 &= \left[\frac{d^2\sigma}{d\Omega\, dE'}\right]_{exp}\left(\frac{d\sigma}{d\Omega}\right)^{-1}_{Mott}\left[1 + 2\left(\frac{1}{1+R}\right)\frac{|q^2| + \nu^2}{q^2}\tan^2\theta/2\right]^{-1} \end{aligned} \qquad R = \frac{\sigma_s}{\sigma_t} \tag{29}$$

For small values of $\tan^2\theta/2$, W_2 depends much less critically than W_1 on R.

The experimental findings, which are of extreme interest and have stimulated a quantity of theoretical speculation, can be summarized in the following points:

a) R, although rather poorly measured (see Fig. 21), appears to be quite small and is compatible with being constant over the full q^2 and ν range explored. The average value is $R = 0.18 \pm 0.10$. This fact is often interpreted as an indication that the contribution to the total cross section from the scattering on virtual scalar (or pseudoscalar) mesons is small. Actually in the infinite momentum limit when all the particles involved in the reaction travel along the beam direction (a situation which is not necessarily satisfied in the experiment) helicity cannot be conserved if a longitudinal photon is absorbed by a boson.

b) When W is well above the resonance region, $d^2\sigma/d\Omega dE'$ has, at fixed W, a rather weak dpendence on q^2. This is shown in Fig. 22, where $\Gamma = \left[(d^2\sigma/d\Omega dE')/(d\sigma/d\Omega)_{Mott}\right]$ is plotted as a function of $-q^2$ for different values of W. With increasing W, Γ becomes flatter. This is one of the facts which has suggested the idea of the proton built up of point-like constituents[105] (partons) which, due to the above observation a), should be mostly fermions.

c) νW_2, determined in the hypothesis that R is constant and equal to 0.18 ± 0.10 (W_2 however depends rather weakly on R, so that this hypothesis is not very drastic), appears to depend on the ratio $\omega = 2M\nu/q^2$ rather than on q^2 and ν separately. The experimental situation is presented in Fig. 23, where νW_2 is shown as a function of $-q^2$ for various values of ω. At $\omega = 4$, νW_2 is clearly independent of q^2 within the errors (see also Fig. 24). At different values of ω, the data suggest that a plateau is being reached at higher values of $|q^2|$. This behaviour was first suggested by Bjorken[106] ("scaling law") to occur in the so-called Bjorken limit $|q^2| \longrightarrow \infty$, $\nu \longrightarrow \infty$ (ω constant) and the fact that this limit appears to be reached at relatively small values of $|q^2|$ and ν was considered surprising. Other variables (e.g. $\omega' = \omega + (M^2/q^2) = 1 + (W^2/q^2)$) in terms of which the scaling behaviour is reached even earlier have also been proposed[107].

From an intuitive point of view, the scaling law of the structure functions can be understood as follows. Consider a function of the type (one dimensional light cone singularity).

$$f(x)\, \delta\, (x - t) \tag{30}$$

where x is the direction of the incoming virtual photon of momentum $q_\mu \simeq (\nu + (M/\omega), 0, 0, \nu)$. The Fourier transform of (30) is

$$\int f(x)\, \delta\, (x - t)\, e^{i\nu t - i(\nu + \frac{M}{\omega})x}\, dxdt = \int f(x)\, e^{-i\frac{M}{\omega}x}\, dx = F(\omega)$$

Therefore a function of the type (30) has a scaling Fourier transform. Bjorken first pointed out that the e. m. current commutators, whose Fourier transforms are closely connected with the structure functions, must be dominated, in the infinite momentum (Bjorken) limit by singularities on the light cone ($x_\mu x^\mu = 0$). The experimental result on the scaling law was therefore surprising mainly because scaling is reached at relatively low energy.

The main trend of the present theoretical work on scaling[108] is essentially along two lines: a) techniques to evaluate the light-cone singularities of the e. m. current commutators and b) parton models, in which the proton structure assumes quite naturally the form of objects moving in the infinite momentum limit with the velocity of light (the simplest form of the charge distribution of the proton being $\Sigma Q_i\, \delta\, (x_i - t)$, with Q_i the parton charges).

Both these approaches have a non negligible predictive power: in the first case the transformation properties of the currents under different groups (Poincaré group, SU_2, SU_3, etc.) can be used to relate cross-sections and to predict sum rules; in the second case many predictions can be made once the partons are identified, e. g., with the quarks.

Experimentally, some data on the comparison[102] of deep inelastic scattering on protons and neutrons are also available.

Finally, the results of ref. (36) (see section I. 1.) allow one to conclude that deep inelastic scattering data of muons on proton are consistent with the electron data.

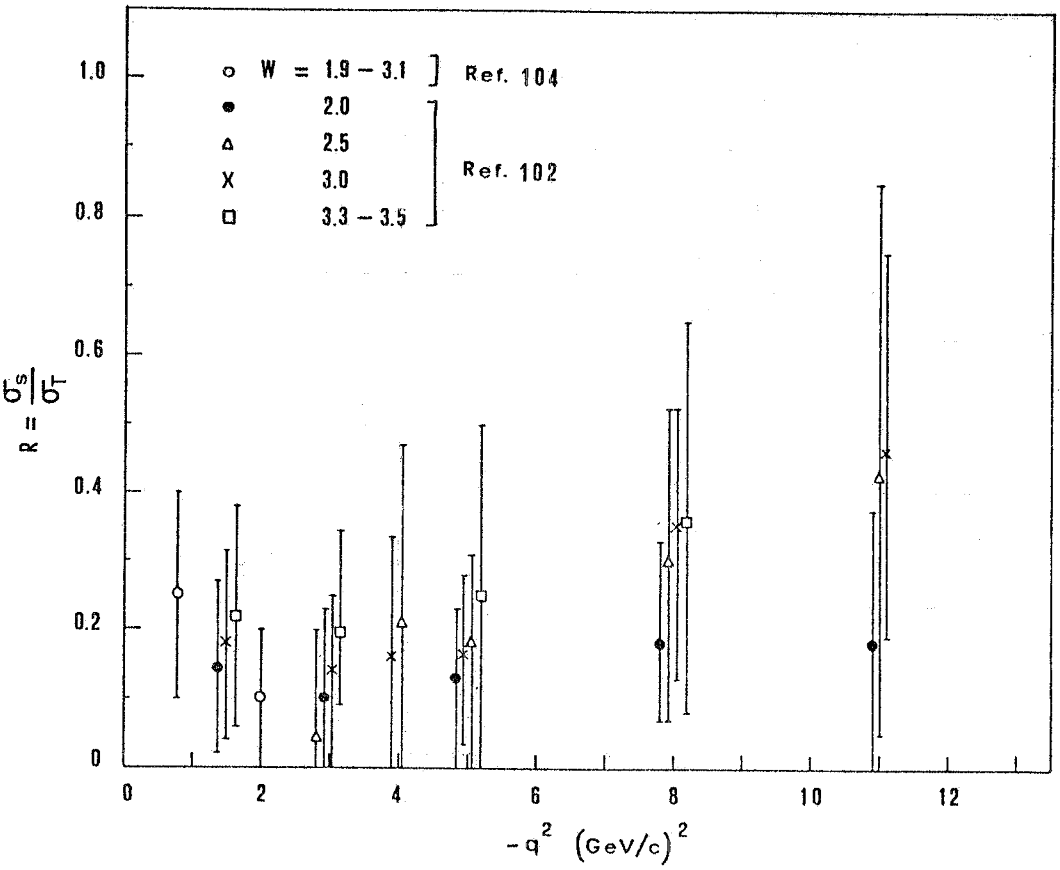


FIG. 21 - The ratio σ_S/σ_T (σ_S and σ_T defined as in (26) plotted as a function of $-q^2$, for different values of W. W is the invariant mass of the unobserved hadronic final state.

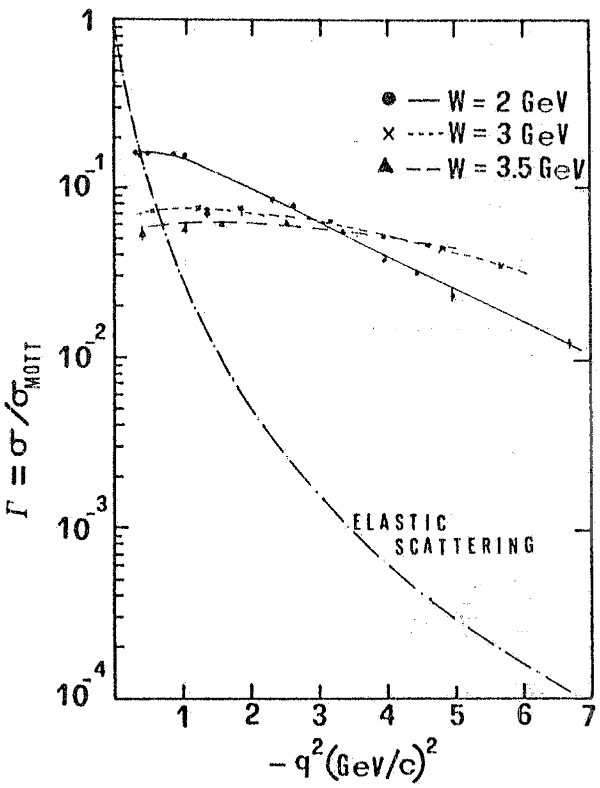


FIG. 22 - The ratio of the experimental cross-section to σ_{MOTT} in deep inelastic electron-proton scattering is plotted as a function of $-q^2$ for different values of W. For comparison, the same ratio is also shown for elastic electron-proton scattering.

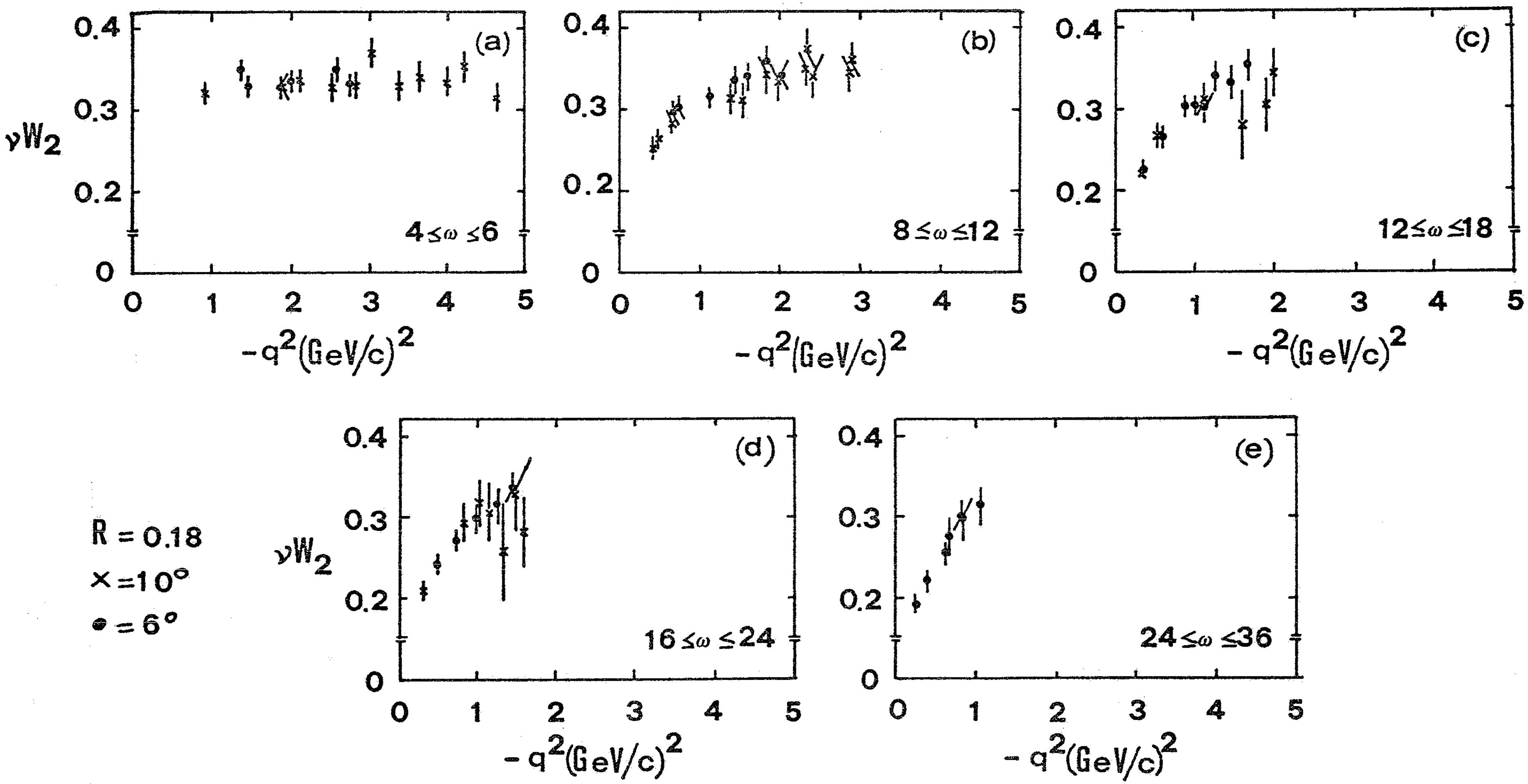


FIG. 23 - The structure function νW_2 is plotted as a function of $-q^2$ for different values of $\omega = (-q^2/2M\nu)$. R=0.18 was assumed, see ref. (102).

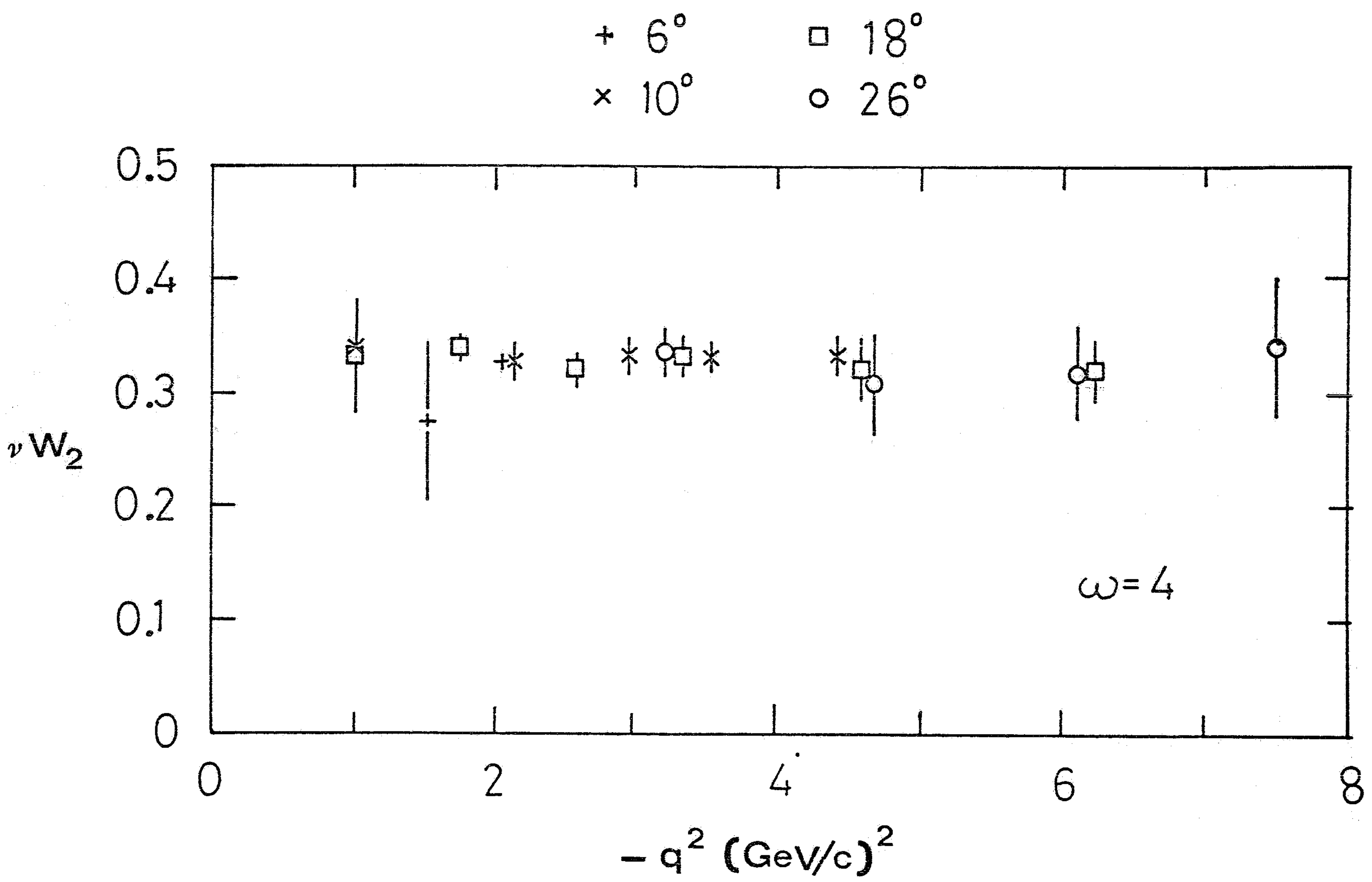


FIG. 24 - The structure function νW_2 is plotted as a function of $-q^2$ for $\omega = 4$, see ref. (102). $R = (\sigma_S/\sigma_T) = 0.18$ was assumed.

PART II. - TIME-LIKE REGION. -

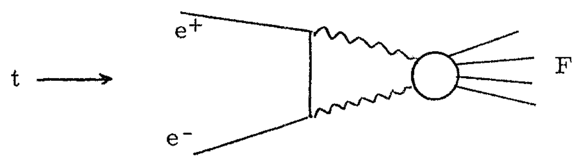
The investigation of e. m. structure of hadrons in the time-like region has only recently begun with the operation of e^+e^- storage rings and has been pursued up to now only in few and rather small laboratories, essentially at Orsay, Novosibirsk and Frascati.

The experimental information is very important to an understanding of the physics of elementary particles, but is still quite meagre. This is not only due to the fact that this kind of physics has a short history and a rather limited geography, but also to the fact that hadrons are produced in e^+e^- interactions with very small cross-sections (in the range of nanobarns - 10^{-33} cm^2 - when the total energy is above 1 GeV). In addition, in storage rings, beams of $\sim 10^{11}$ particles are sent one against another, to be compared with the use of beams against targets in the conventional machines. The fact that the beam-beam impact occurs $\sim 10^7$ times per second does not compensate the difference in intensity. For this reason, very small beam dimensions are generally used in storage rings to obtain a high target density, which, while making the machine operation quite delicate and difficult, brings the counting rate just to the limit of experimental feasibility. Counting rates of a few events per day, or even per week, are not unusual in typical experiments.

II. 1. - VALIDITY OF THE ONE-PHOTON EXCHANGE, POINT-LIKE LEPTONS AN $1/q^2$ PHOTON PROPAGATOR HYPOTHESES IN THE TIME-LIKE REGION. -

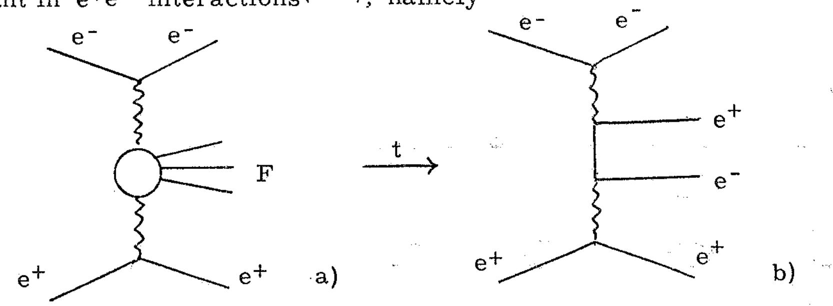
II. 1. 1. - 1-Photon exchange. -

In the time like region the smallness of the usual two-photon exchange contribution from the graph

(31) 

has not been experimentally tested, and we have therefore to trust the calculations based on the absence of strong enhancement mechanisms. It is worth noticing that in the time-like region the number of exchanged virtual photons is directly related to the charge conjugation eigenvalue C of the final state. If the charge of the produced particles is not recognized (as it was the case for all the experiments performed up to now with e^+e^- storage rings) then the interference between even and odd states of C cancels, so that the two photon exchange process can contribute only to an α^4 order.

There is however an additional two photon contribution, which is expected to be quite important in e^+e^- interactions(109), namely

(32) 

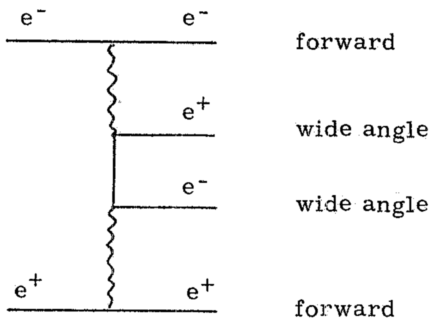
The additional α^2 factor appearing, e. g. , in diagrams (32b) is in fact expected to be at least partially compensated by the fact that this process can involve much lower momentum transfers than the usual one-photon annihilation graph.

The contribution from the above diagrams might give rise to an important background in the e^+e^- experiments. However, they deserve also some interest by themselves since they can provide useful information on the coupling of hadrons with a two photon system when F of diagram (32a) is of hadronic nature (F could be, for instance, an η or an η' particle). This will be particularly true with higher energy storage rings, since the cross-section is expected to logarithmically increase with increasing energy, while the usual annihilation cross-sections are expected to decrease with increasing energy E (as $1/E^2$ for production of point-like particle pairs, see Fig. 25).

Fortunately, processes (32a) can be separated since in the final state the incident electron and positron survive. This separation is often possible also without detecting the scattered electrons, since the angular and energy distribution for these two photon reactions is quite peculiar. For instance in reaction $e^+e^- \longrightarrow e^+e^-e^+e^-$ two leptons are expected to be emitted in a narrow cone along the beam directions, while the other two, when emitted at large angle, should have a $\Delta\phi$ distribution strongly peaked around $\Delta\phi = 0$[109]. ($\Delta\phi$ is the angle between the planes defined by the beam axis and the two emitted particles).

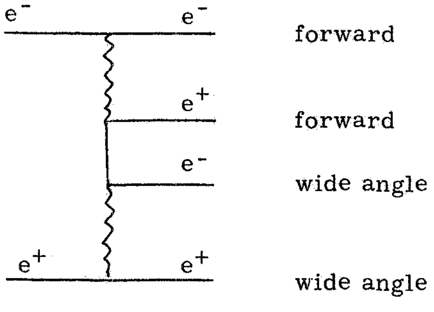
Experimentally, the $\Delta\phi$ distribution for reaction $e^+e^- \longrightarrow e^+e^-e^+e^-$ has been investigated at Novosibirsk[110]. The results are shown in Fig. 26, and are compared with the theoretical calculation of Baier and Fadin[109]. A measurement[111] of this reaction has also been performed in Frascati with Adone, where also a few candidates from reaction $e^+e^- \longrightarrow e^+e^-\mu^+\mu^-$ have been observed. In this experiment counters have been placed near the machine vacuum chamber in order to detect the electrons (and/or positron) emitted near the beam directions, using as spectrometers the magnets of the machine itself. Some additional information on the kinematics of the reaction can thus be obtained. In Fig. 27 the distribution of the 29 observed events as a function of β , the center of mass velocity of the leptons emitted at large angle, is shown. The sign of β is defined as negative when β has the same orientation of the single detected electron along the beam direction (no event in which both the small angle leptons were detected was observed). A comparison with theory is also given in Fig. 27. Although the yield of events with $\beta < 0$ depends critically on the lower experimental cut in the energy of the detected particles, it appears that there is a large contribution of events with a kinematical feature not foreseen by the standard theoretical calculations[109], whose approximations are based on the hypothesis of the dominance of the kinematical configuration

(33)



G. Parisi[112] has evaluated the contribution from the kinematical configuration

(34)



which appears to account for most of the observed events. Preliminary data from a second experiment[113], in which a lower energy cut is set on the observed wide angle electrons, show a contribution of events from the configuration (33) but also a non negligible contribution

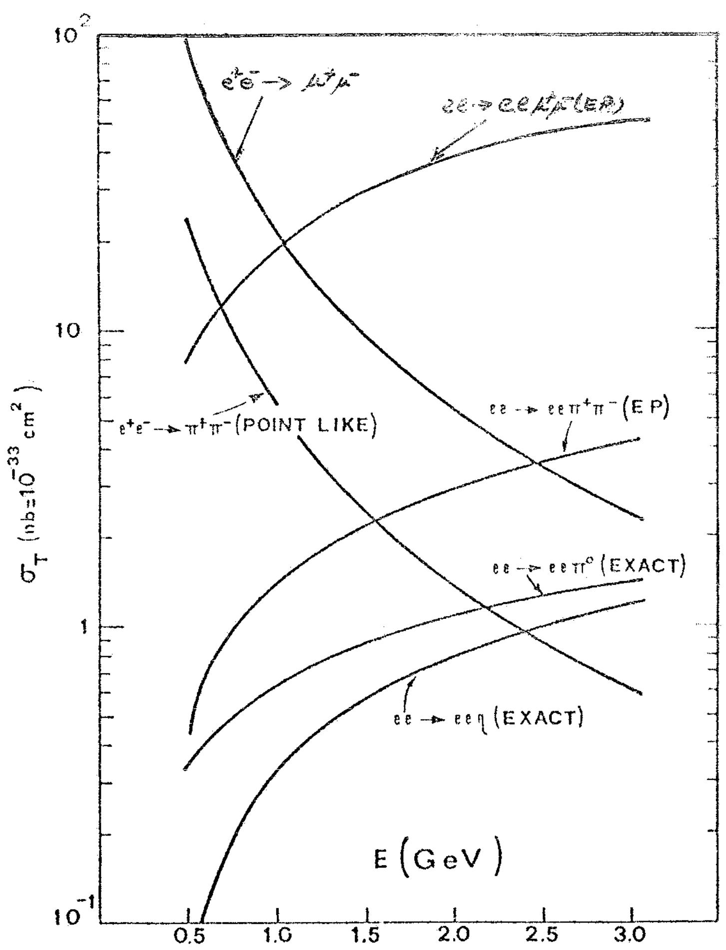


FIG. 25 - The cross-sections for different e^+e^- induced processes via two-photon interactions as calculated by S. J. Brodsky et al., ref. (109). We see that as the energy E of the electron and positron beams is above ~ 1.5 GeV, the total cross-sections for two photon-interaction processes are expected to overtake the cross-sections for the usual $e^+e^- \longrightarrow \mu^+\mu^-$ and $e^+e^- \longrightarrow \pi^+\pi^-$ annihilation process.

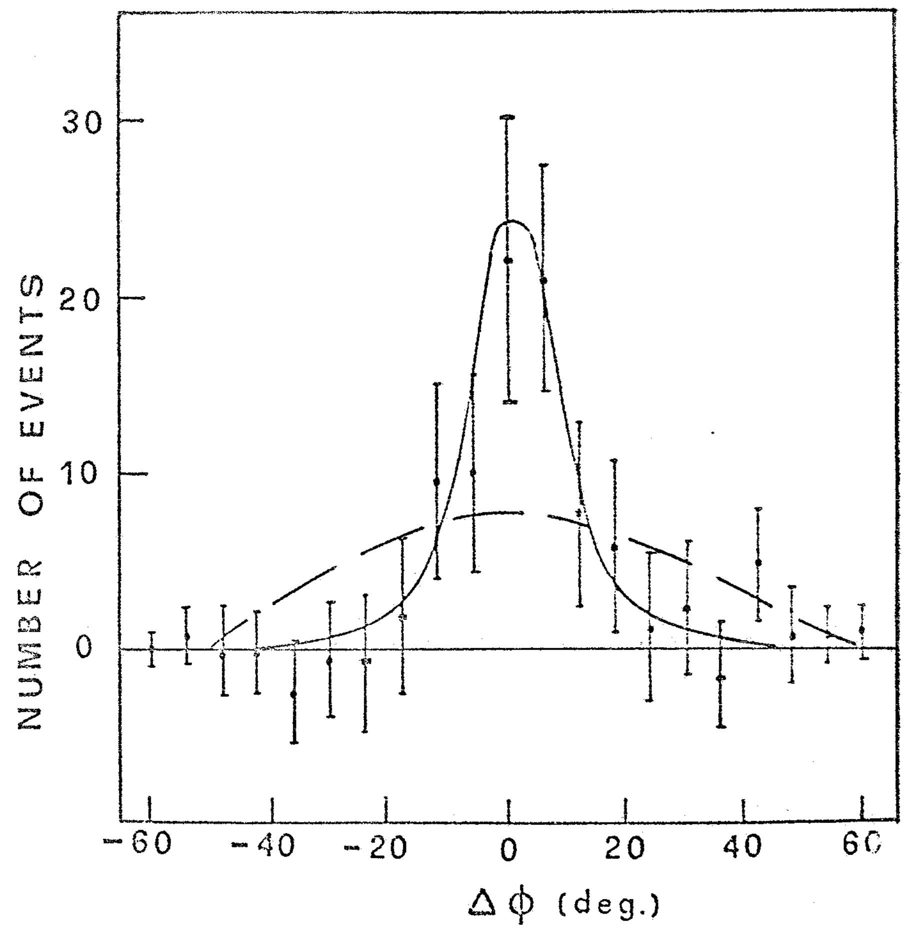


FIG. 26 - Distribution of events from reaction $e^+e^- \longrightarrow e^+e^-e^+e^-$ as a function of the angle $\Delta\phi$ defined as the angle between the two planes which contain each of the two large angle emitted electrons and the beams axis. The data were collected at three different values of the total energy (2E=1020, 1180, 1340 MeV). The full-line is the theoretical calculation of V. N. Baier and V. S. Fadin, ref. (109). The dashed line corresponds to an uniform $\Delta\phi$ distribution weighted with the detection efficiency of the apparatus.

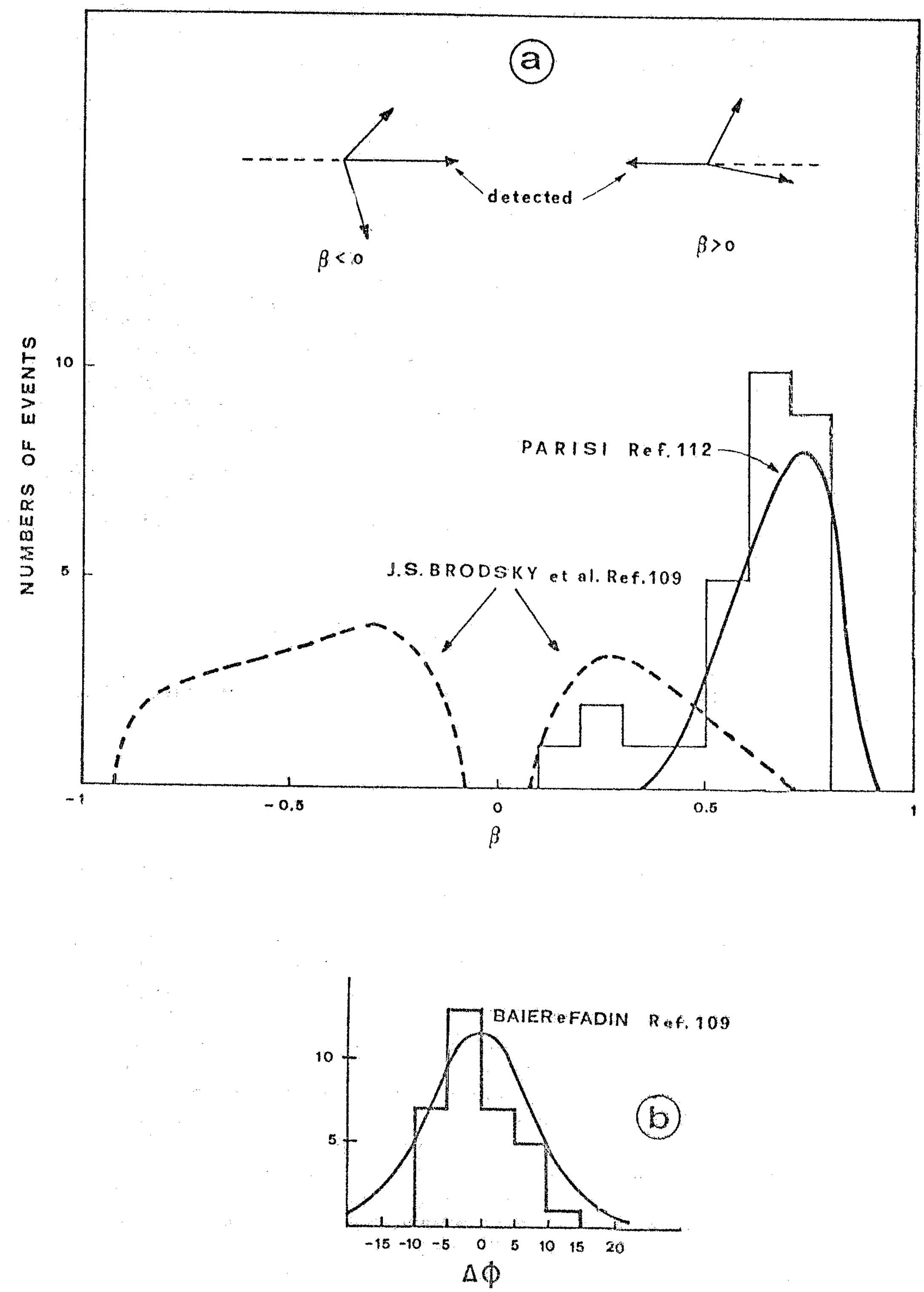


FIG. 27 - The results of the Frascati "$\gamma\gamma$-group" on reaction $e^+e^- \rightarrow e^+e^-e^+e^-$ (ref. 111). a) Distribution of the events as a function of β , the c. m. velocity of the pair of leptons emitted at large angle. The convention used for the sign of β is specified in the upper part of the drawing. The results are compared with theoretical calculations. b) Distribution of the events as a function of $\Delta\phi$, compared with the calculation of Baier and Fadin.

from configuration (34). Better theoretical calculations would therefore be welcome. Also the possibility of some higher energy storage rings to be operated both with e^+e^- and with e^-e^- appears as a convenient facility to study, in the e^-e^- mode of operation, the two-photon interactions without contamination from the annihilation channels.

In any case, there is good experimental evidence that the data reported in the next sections as hadronic events from e^+e^- interactions receive, if at all, only a small contamination from the two photon interaction graph (32a), (see section II. 5).

II. 1. 2. - Point-like leptons and $1/q^2$ photon propagator. -

Experimental tests of these hypotheses can be obtained by measuring $\mu^+\mu^-$ or e^+e^- pairs production cross-section in e^+e^- interactions. However, as we already mentioned in section I. 1. , the cross section for the reaction $e^+e^- \longrightarrow e^+e^-$ is dominated by the scattering rather than by the annihilation graph. Tests of the point-like electron in the time-like region will be possible only by measuring the cross-section for process $e^+e^- \longrightarrow e^+e^-$ with recognition of the charge of the produced electron pair, in which case the separation of the annihilation contribution will be possible. The possibility that the electron has a complex form factor F_e, even if $|F_e|^2 = 1$, can also be tested in this reaction(114).

At present, only the annihilation process $e^+e^- \longrightarrow \mu^+\mu^-$ has been experimentally investigated. A comparison of the electron and muon form factors in the time-like region is then available only at $q^2 \simeq 0.5\ (\mathrm{GeV}/c)^2$ through the measurement of the $\varrho \longrightarrow \mu^+\mu^-$ and $\varrho \longrightarrow e^+e^-$ decay rates: the result is

$$\frac{\Gamma(\varrho \longrightarrow e^+e^-)}{\Gamma(\varrho \longrightarrow \mu^+\mu^-)} = 0.97 \pm 0.17 = \frac{|F_e|^2}{|F_\mu|^2} \quad (115) .$$

The experimental situation on reaction $e^+e^- \longrightarrow \mu^+\mu^-$ is presented in Fig. 28. The results(116 ÷ 118) are expressed in terms of the ratio

$$R = \frac{\sigma_{\exp}}{\sigma_{QED}} = |F_e(s)|^2 |F_\mu(s)|^2 |M(s)|^2 .$$

The agreement is good within the large experimental errors (typically 15÷20%) up to momentum transfers as high as $s = (E_+ + E_-)^2 = q^2 = 4.4\ (\mathrm{GeV}/c)^2$.

This kind of data is often parametrized in terms of a cut-off parameter Λ^2 ($R = (1 - q^2/\Lambda^2)^2$) by assigning the form $1-(q^2/\Lambda^2)$ to either F_e or F_μ or to the photon propagator modification M. This parametrization is arbitrary, and sometimes gives rise to serious theoretical difficulties (when assigned, for instance, to M or to a lepton propagator modification(23)). However it is usually justified with the need of comparing different experiments. This attitude is misleading in our opinion, since it invites one to consider a rough experiment at high energy equivalent to a good precision low energy experiment: in fact in the cut-off philosophy deviations from QED are expected to increase with increasing energy. This might very well be wrong. Actually, a breakdown of QED <u>is</u> expected due to vacuum polarization effects originated by hadrons coupled to the virtual photon: this kind of breakdown is not expected to be more important at higher energy. This is demonstrated experimentally in Fig. 29 where we show the results of another experiment on reaction $e^+e^- \longrightarrow \mu^+\mu^-$(119) performed at Orsay at a lower energy than the experiments quoted in Fig. 28. The energy region explored is around the ϕ mass, and a vacuum polarization effect shows up, although at the limit of the experimental errors.

The relevant point is the comparison between experiment and theory; and the pertinent parameter, at whatever energy, is the precision of the experiment rather than the cut-off parameter Λ.

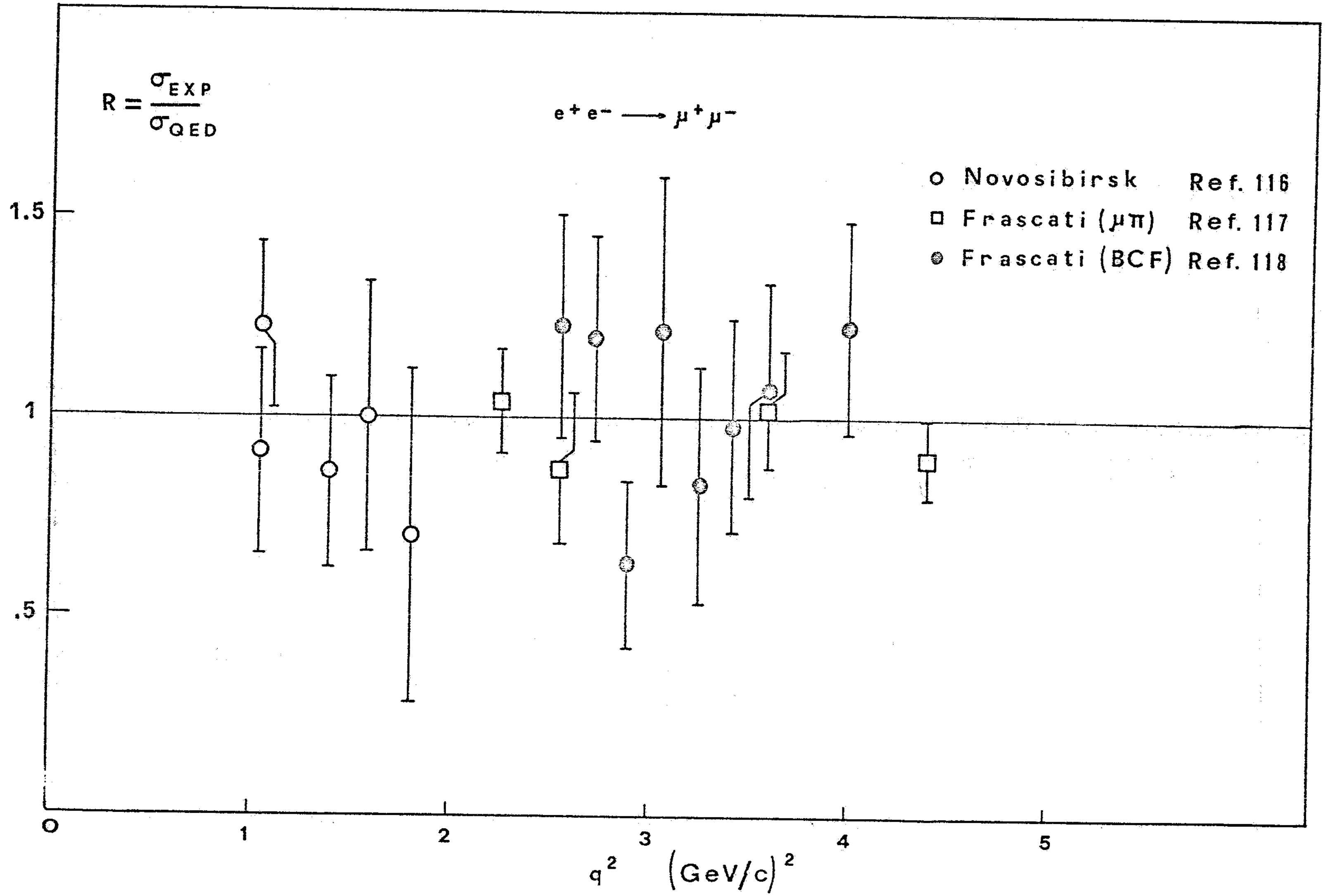


FIG. 28 - Results on the reaction $e^+e^- \longrightarrow \mu^+\mu^-$, expressed in terms of the ratio $R = (\sigma_{exp}/\sigma_{QED})$. Themachine luminosity is monitored in this case with the wide angle e^+e^- elastic scattering.

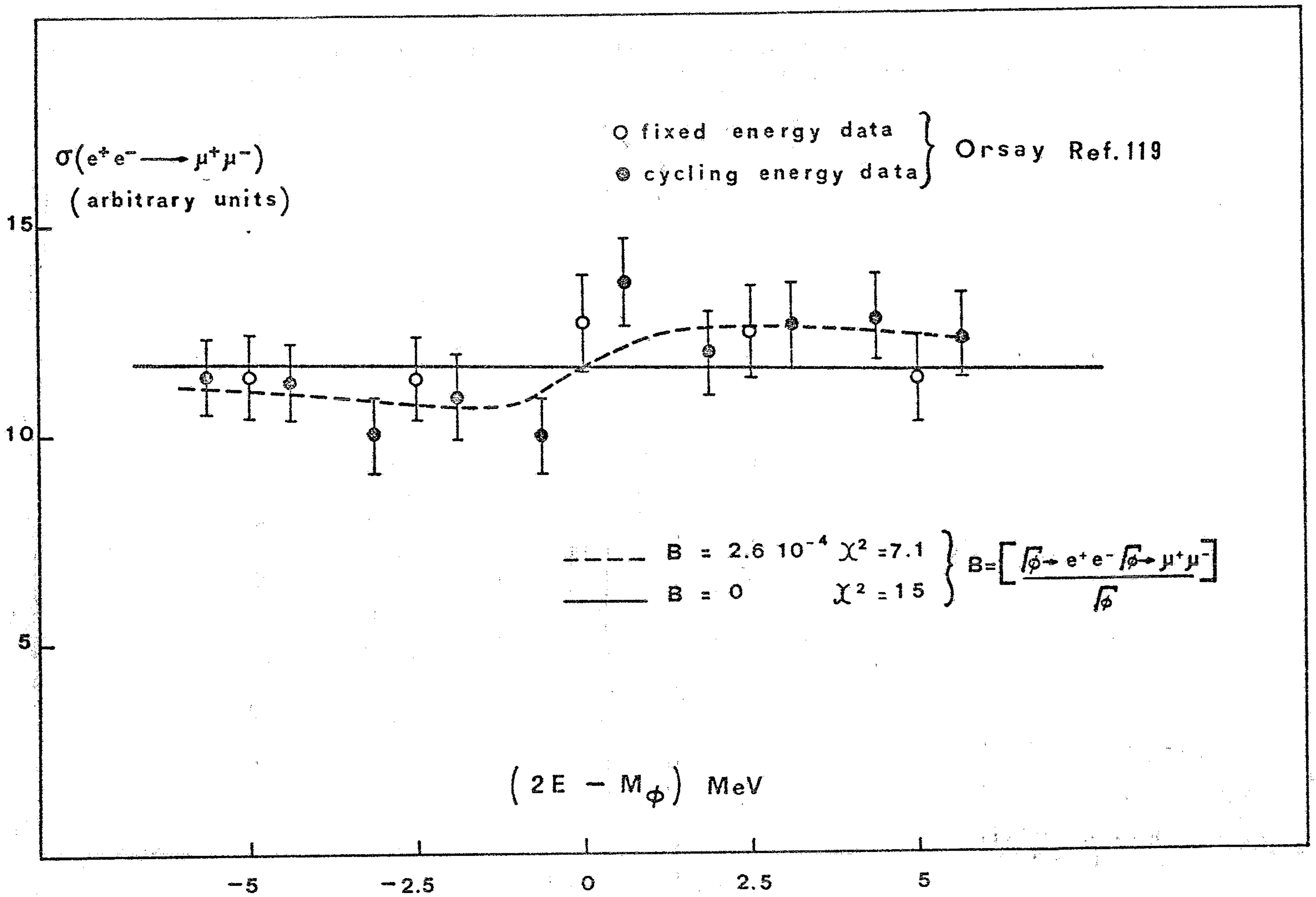


FIG. 29 - Results of the Orsay group showing the vacuum polarization effect in the reaction $e^+e^+ \longrightarrow \mu^+\mu^-$. Open circles refer to events collected at five fixed energies E. Black circles refer to events collected with a continuous cycling of the beams energy in a range of total c. m. energy 2E of $\pm$ 6 MeV around the ϕ meson mass. The full-line corresponds to no polarization effect; the dashed-line represents a best fit to the data with vacuum polarization effect included.

II. 2. - PROTON FORM FACTORS. -

A first measurement of the cross-section for reaction

(35) $$e^+e^- \longrightarrow p\bar{p}$$

has been recently performed at Frascati by the Naples group(120). 21 ± 5 events from reaction (35) have been observed at $q^2 = 4.4$ (GeV/c)^2. The separation of background from reaction (35) is achieved using energy, E, and dE/dx measurements in thick scintillation counters, time-of-flight determination, and collinearity of the observed tracks as measured in optical spark chambers. The sample of events is then quite clean in spite of the very low counting rate ($\sim$ 1 event/ (2 ÷ 4 days). 14 of the events show the antiproton annihilation star, as expected on the basis of the known detection efficiency of the apparatus.

To determine from the observed number of events the total cross-section, extrapolation over the full solid angle of the counting rate is needed (the experimental apparatus covers $\sim$0.6 of the total 4π solid angle, although the detection efficiency is equal to 1 only around 90°). For this purpose, the angular distribution of the events must be known.

The equivalent of the Rosenbluth formula in the time-like region is(121)

(36) $$\frac{d\sigma}{d\Omega} = \frac{\pi}{8}\,\alpha^2\,\bar{\lambda}^2\,\beta\left[\left|G_M\right|^2(1+\cos^2\theta) - \frac{1}{\tau}\left|G_E\right|^2\sin^2\theta\right]$$

$\sigma \simeq 10^{-32}\left(\left|G_M\right|^2 - \frac{1}{2\tau}\left|G_E\right|^2\right)\,\mathrm{cm}^2$

$\tau = -q^2/4M^2$

β, θ: velocity and angle of emission of the proton.

The angular distribution depends on $|G_M|^2/|G_E|^2$ and some hypothesis is needed, unless the experiment covers a wide enough θ region so that $|G_E|^2$ and $|G_M|^2$ can be separately determined, which is not the case for the Naples experiment.

However this experiment has been performed near threshold where one would expect the angular distribution to be isotropic, i.e. $|G_E|^2 = |G_M|^2$.

In fact, unless $G_E = G_M$ at threshold ($\tau = -q^2/4M^2 = -1$), F_{1p} and F_{2p} (see section I. 2.) become infinite producing an unwanted divergence in the e.m. current of the proton. With the hypothesis of an isotropic production distribution the authors obtain a value of $\sigma_{e^+e^-\to p\bar{p}} = (4.6 \pm 1)10^{-34}$ cm^2 ($\sigma_{e^+e^-\to p\bar{p}} = (6.2 \pm 1.8)10^{-34}$cm^2 using only the events with the detected annihilation star). In Fig. 30 these values are compared with previously available upper limits from reaction $\bar{p}+p \to e^+e^-$(122, 123) and with calculations for a few particular choices for G_E and G_M.

In the above hypothesis $|G_E| = |G_M|$, the measured value of the cross-section corresponds to $|G_E| = |G_M| = 0.19 \pm 0.03$.

This experiment is obviously only a first approach to a new field of investigation. New storage rings experiments permitting the determination of $|G_E|$ and $|G_M|$ separately (and also the form factors of unstable barions) are planned both in Frascati and Stanford.

II. 3. - THE PION FORM FACTOR. -

The pion form factor is simply related in the time-like region to the cross section for reaction

(37) $$e^+e^- \longrightarrow \pi^+\pi^-\ ,$$

by the relations(121)

(38) $$\frac{d\sigma_{\pi^+\pi^-}}{d\cos\theta} = \frac{\pi\alpha^2}{4}\,\frac{\beta_\pi^3}{q^2}\left|F_\pi(q^2)\right|^2\sin^2\theta \qquad ; \qquad \sigma_{\pi^+\pi^-} = \left(\frac{\pi\alpha^2}{3}\right)\frac{\beta_\pi^3}{q^2}\left|F_\pi(q^2)\right|^2$$

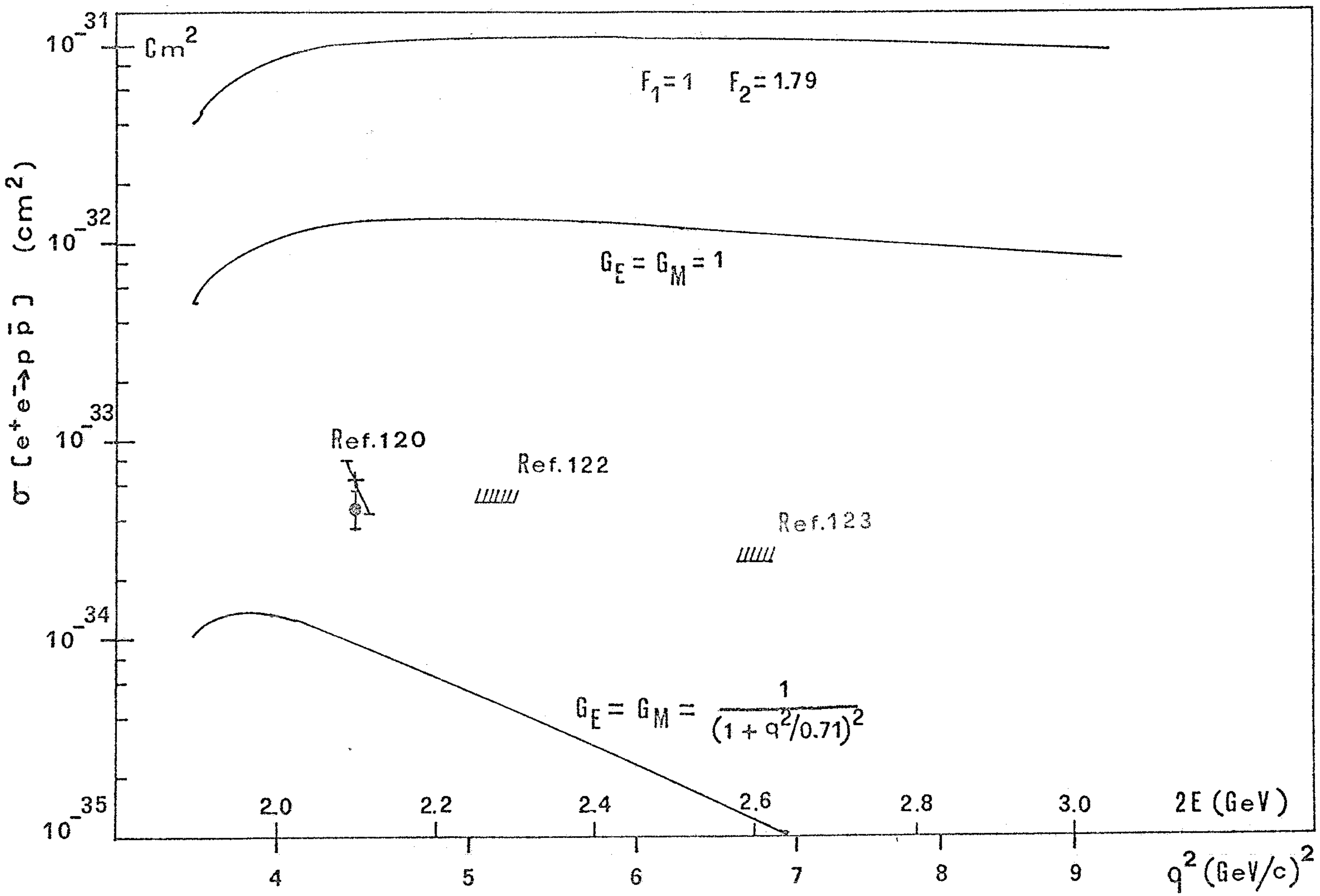


FIG. 30 - The result of the Naples group (ref. 120) on the reaction $e^+e^- \longrightarrow p\bar{p}$. The black circle ∅ represent the cross-section as determined using all the detected events; the cross ⁎ represent the cross-section as determined using only the events in which the antiproton annihilation star is detected. Previously available upper limits ////// from reaction $p\bar{p} \longrightarrow e^+e^-$ are also shown (ref. 122 and 123). The full-lines represent theoretical predictions for some particular values of the form factors.

A measurement of $\sigma(e^+e^- \longrightarrow \pi^+\pi^-)$ therefore permits one to measure $|F_\pi(q^2)|^2$. Notice that a $\pi^+\pi^-$ system with $l = 1$ (as it must be in the one-photon-exchange hypothesis) must have $T = 1$ so that $F_\pi(q^2)$ is related to the coupling of isovector photons with hadrons. Actually, F_π coincides exactly with F_V below threshold for 4π production and in practice below $q^2 = 1(\mathrm{GeV}/c)^2$. Phase-space factors are expected to strongly depress the $e^+e^- \longrightarrow 4\pi$ channels below $q^2 = 1(\mathrm{GeV}/c)^2$, as confirmed by the first experimental measurements: see section II. 5.

Measurements of $\sigma(e^+e^- \longrightarrow \pi^+\pi^-)$ at $q^2 \lesssim 1(\mathrm{GeV}/c)^2$ have been quite extensively performed during last $\sim$5 years at Novosibirsk(124) and Orsay(125). The phenomenology is dominated by the production of the vector meson ϱ, the foreseen interference term with the ω contribution (via the electromagnetic decay mode $\omega \longrightarrow \pi^+\pi^-$) having also been observed. The results are summarized in Fig. 31, showing $|F_\pi|^2$ as a function of q^2.

The full line is the Breit-Wigner best fit to the Novosibirsk points; the dashed line is the best fit to the Orsay points including the $\omega \longrightarrow \pi^+\pi^-$ contribution. On the top of the ϱ-peak the cross-section is $\sim$1.5 μbarn. In terms of ϱ parameters, the results can be summarized as follows:

TABLE II

	Orsay	Novosibirsk
m_ϱ (MeV)	775.4 ± 7.3	754 ± 9
Γ_ϱ (MeV)	149 ± 23	105 ± 20
$\dfrac{\Gamma_{\varrho \to e^+e^-}}{\Gamma_{\varrho\,\mathrm{total}}}$	$(4.0 \pm 0.5)10^{-5}$	$(5 \pm 1)10^{-5}$
$\Gamma_{\varrho \to e^+e^-}$ (KeV)	6.1 ± 0.7	5.2 ± 0.5

The difference in the parameters obtained at Novosibirsk appears to be essentially due to the fact that the ω contribution is not taken into account. Actually, the best fit to the Orsay points (which provides the additional information $(\Gamma_{\omega \to 2\pi}/\Gamma_{\omega \to \mathrm{total}})^{1/2} = (0.2 \pm 0.05)$, the phase angle between the ω and ϱ amplitudes being $\phi_{\omega\varrho} = 87°.5 \pm 15°.4$) appears to also fit perfectly the Novosibirsk points.

The fact that the cross-section for $\pi^+\pi^-$ production can be accounted for by the ϱ and ω contributions alone (actually the ϱ, ω and ϕ account for all the hadronic production below 1 GeV) was a strong support in favour of the "vector dominance" hypothesis(126). The data above 1 GeV, however, give evidence in favour of a non negligible contribution from other mechanisms(x).

Above 1 GeV, the results are presented in Fig. 32. We see that for $1 < q^2 < 4$ $(\mathrm{GeV}/c)^2$ $|F_\pi|^2$ is larger than the expected contribution of the ϱ-tail. In this energy region, the corresponding cross-sections are of a few nanobarn ($|F_\pi|^2 = 1$ would correspond to a cross-section $\sigma(e^+e^- \longrightarrow \pi^+\pi^-) = (20 \cdot 10^{-33}\ \mathrm{cm}^2/q^2\ (\mathrm{GeV}/c)^2)$.

A separation of π's from k's has not been achieved in the Frascati points, so that the interpretation of the data of Fig. 32 as $|F_\pi|^2$ requires the hypothesis that the contribution to the counting rate from the channel $e^+e^- \longrightarrow k^+k^-$ is negligible.

(x) - (See next page).

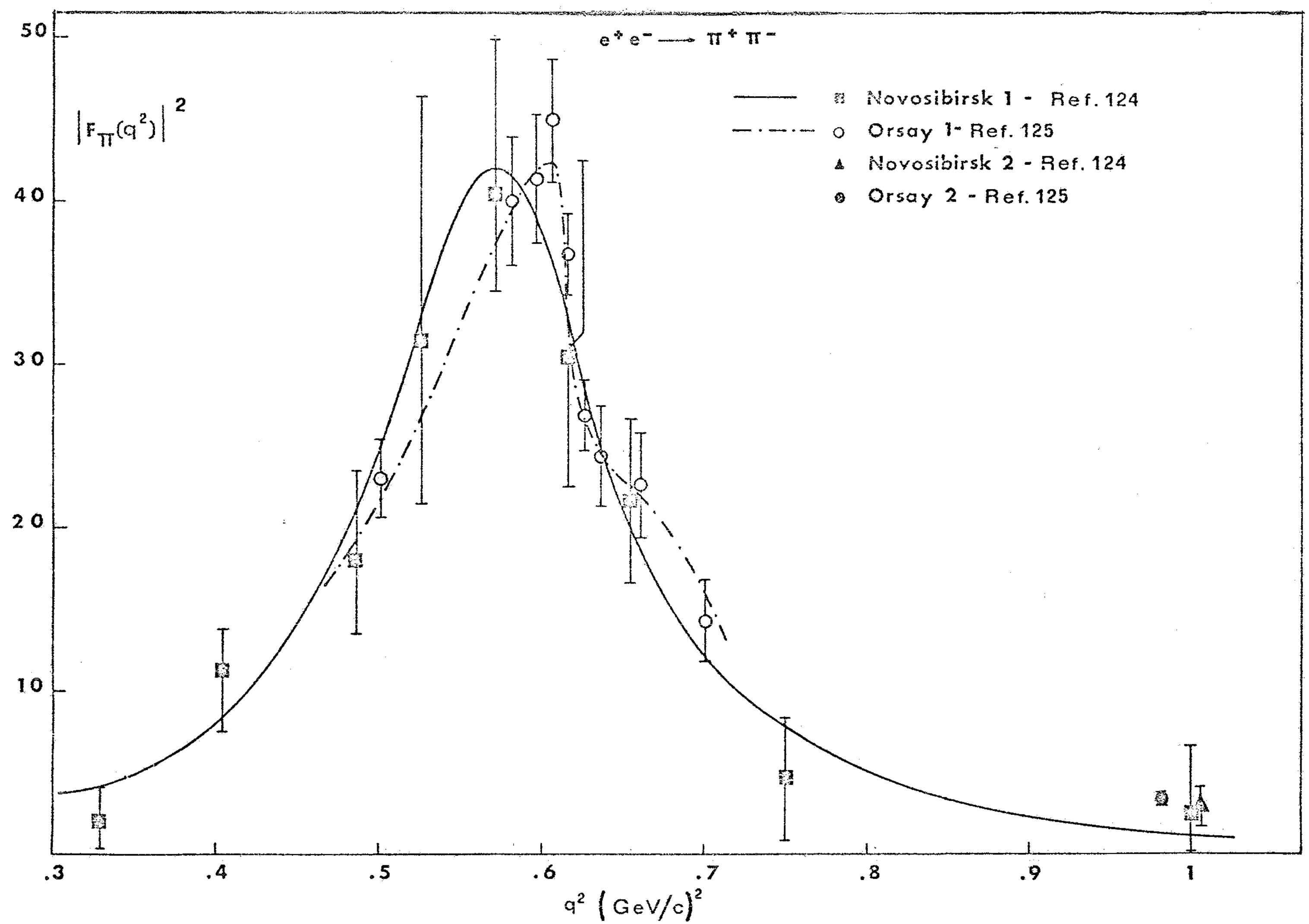


FIG. 31 - $|F_{\pi}(q^2)|^2$ as determined by the Novosibirsk and Orsay groups through measurements of reaction $e^+e^- \rightarrow \pi^+\pi^-$ at $q^2 \lesssim 1$ $(GeV/c)^2$. The full line is the Breit-Wigner best fit to the Novosibirsk points. The dashdotted line is the best fit to the Orsay points using a Gounaris-Sakurai formula and taking into account the ω contribution via the decay channel $\omega \rightarrow \pi^+\pi^-$.

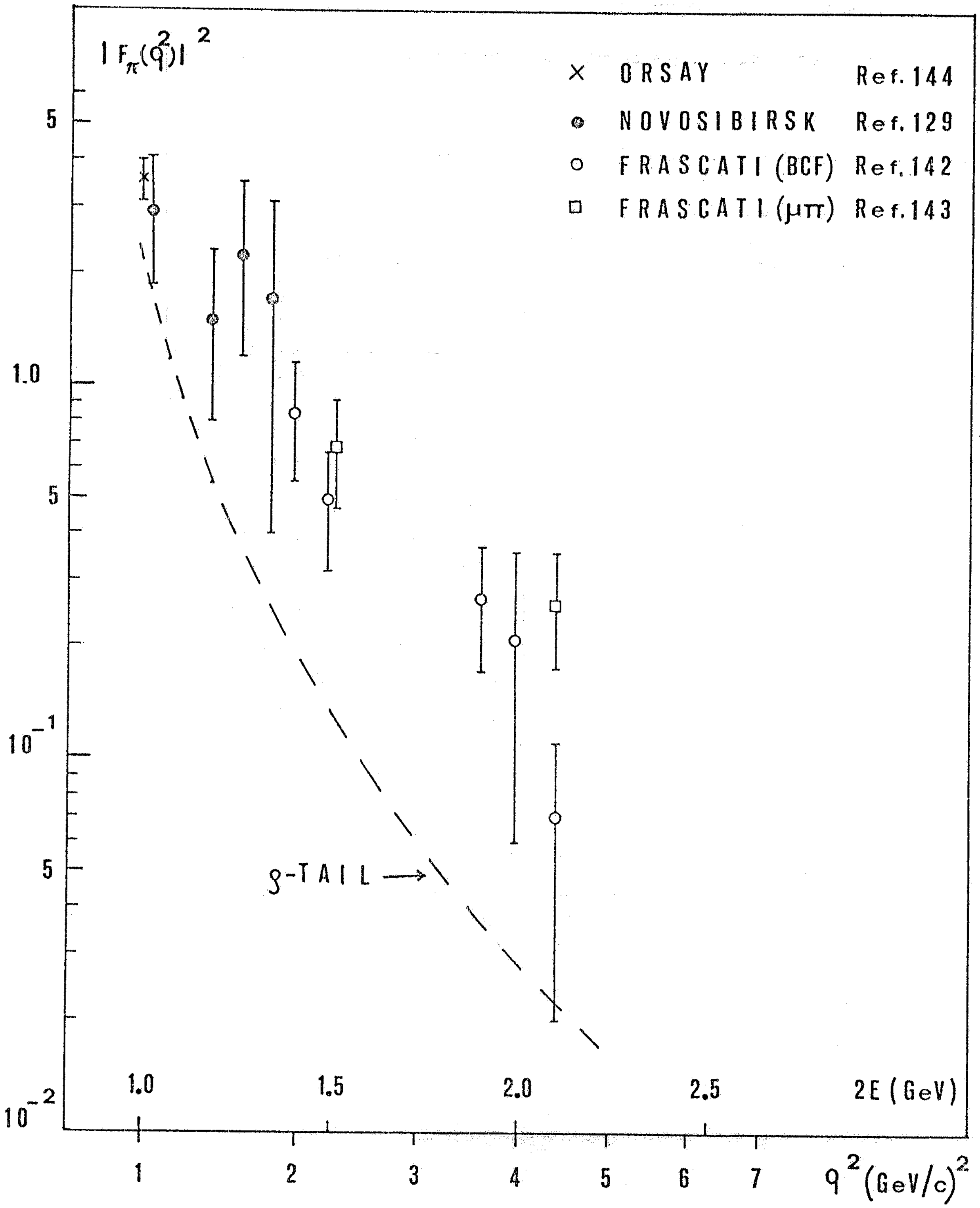


FIG. 32 - $|F_\pi(q^2)|^2$ as determined by measurements of the reaction $e^+e^- \longrightarrow \pi^+\pi^-$ at $q^2 \gtrsim 1$ $(GeV/c)^2$. The expected contribution from the ρ-tail is shown. Corrections for a possible contamination of kaons in the sample of pions are not applied.

II. 4. - THE KAON FORM FACTOR. -

The kaon form factor is related to the cross-section for the process :

(39) $$e^+e^- \longrightarrow k^+k^-$$

by a relation of exactly the same form as equation (38). However, also isoscalar photons can couple to a k^+k^- pair (rather than only isovector photons as in $\pi^+\pi^-$ production).

Around the ϕ - mass, $|F_k|^2$ is dominated by the process $e^+e^- \longrightarrow \phi \longrightarrow k^+k^-$. The production of ϕ-mesons in e^+e^- interactions has been investigated at Orsay[127] and Novosibirsk[128]. The results can be summarized in the following table:

TABLE III

	Orsay	Novosibirsk
$\sigma_{k^+k^-}$ $(q^2 = M_\phi^2)$	$(2.41 \pm 0.13)10^{-30}cm^2$	$(2.13 \pm 0.17)10^{-30}cm^2$
$\sigma_{k^o\bar{k}^o}$ $(q^2 = M_\phi^2)$	$(1.47 \pm 0.21)10^{-30}cm^2$	$(1.01 \pm 0.15)10^{-30}cm^2$
$\sigma_{\pi^+\pi^-\pi^o}$ $(q^2 = M_\phi^2)$	$(1.01 \pm 0.21)10^{-30}cm^2$	$(0.81 \pm 0.21)10^{-30}cm^2$
$\sigma_{\text{all modes}}$ $(q^2 = M_\phi^2)$	$(4.99 \pm 0.40)10^{-30}cm^2$	$(3.96 \pm 0.35)10^{-30}cm^2$
Γ_ϕ	(4.09 ± 0.29) MeV	(4.67 ± 0.42) MeV
$\Gamma_{\phi \to e^+e^-} / \Gamma_{\phi\,all}$	$(3.52 \pm 0.28)10^{-4}$	$(2.81 \pm 0.25)10^{-4}$
$\Gamma_{\phi \to k^+k^-} / \Gamma_{\phi\,all}$	(0.483 ± 0.043)	(0.540 ± 0.034)
$\Gamma_{\phi \to k^o\bar{k}^o} / \Gamma_{\phi\,all}$	(0.295 ± 0.040)	(0.257 ± 0.030)
$\Gamma_{\phi \to \pi^+\pi^-\pi^o} / \Gamma_{\phi\,all}$	(0.202 ± 0.035)	(0.203 ± 0.042)
$\Gamma_{\phi \to e^+e^-}$	(1.44 ± 0.12) KeV	(1.31 ± 0.12) KeV

(x) - However, the simple ϱ, ω and ϕ vector dominance model was already in some trouble due to the behaviour of the isovector form factors F_V of the nucleons as a function of the four-momentum squared $t = -q^2$

$$F_V \propto \frac{1}{t^2}$$

In fact, using the relation

$$F_V(t) \propto \int \frac{\operatorname{Im} F_V(s)}{t - s}\, ds$$

and since $\frac{1}{t-s} = \frac{1}{t} + \frac{1}{t}\frac{s}{t-s}$, we have

$$F_V(t) \propto \frac{1}{t} \int \operatorname{Im} F_V(s)\, ds + \frac{1}{t} \int \frac{s \operatorname{Im} F_V(s)\, ds}{t - s}$$

$F_V(t) \simeq 1/t^2$ requires $\int \operatorname{Im} F_V(s)\, ds = 0$. This however cannot be satisfied if $F_V(s)$ is due to the ϱ contribution only, since the ϱ has a large imaginary part with definite sign.

Above the ϕ mass, only four events have been observed at Novosibirsk at three different values of q^2.

These results(129), in terms of $|F_k|^2$, are presented in Fig. 33. The curve B. W. represents the tail of the ϕ Breit Wigner; the other two curves are $|F_k|^2$ as expected in the vector dominance model calculated as follows(129):

$$(40) \qquad F_k(q^2) = \frac{g_{\varrho kk}}{g_\varrho}\frac{m_\varrho^2}{m_\varrho^2 - q^2} + \frac{g_{\omega kk}}{g_\omega}\frac{m_\omega^2}{m_\omega^2 - q^2} + \frac{g_{\phi kk}}{g_\phi}\frac{m_\phi^2}{m_\phi^2 - q^2}$$

If we put $m_\varrho^2 \simeq m_\omega^2$

$$F_k(q^2) = \frac{m_\varrho^2}{m_\varrho^2 - q^2}\left[\frac{g_{\varrho kk}}{g_\varrho} + \frac{g_{\omega kk}}{g_\omega}\right] + \frac{g_{\phi kk}}{g_\phi}\frac{m_\phi^2}{m_\phi^2 - q^2}$$

and use $F_k(0) = 1$, $A = ((g_{\varrho kk}/g_\varrho) + (g_{\omega kk}/g_\omega))$ can be evaluated. Then the curves + and - in Fig. 33 represent respectively what expected if A has the same phase as $g_{\phi kk}/g_\phi$, or a 180° difference in phase.

We see that the extremely poor statistics is still unsufficient to decide wether $|F_k|^2$ is accounted for, in this energy region, by the simple ϱ, ω and ϕ vector dominance model.

II. 5. - MULTIHADRON PRODUCTION IN e^+e^- INTERACTIONS. -

Up to values of $q^2 \sim 1.1$ (GeV/c)^2 multihadron production is essentially limited to the production of $\pi^+\pi^-\pi^0$ via the isoscalar vector mesons ω and ϕ. The contribution from the channel $e^+e^- \to \phi \to \pi^+\pi^-\pi^0$ is already summarized in Table III. The ω production was also investigated at Orsay(130). The results can be summarized as follows:

$$\sigma(e^+e^- \to \pi^+\pi^-\pi^0)(\omega\text{-mass}) = (1.76 \pm 0.13)\ \mu b$$

$$\Gamma_\omega = 12.2 \text{ MeV (from the world average)}$$

$$\Gamma_\omega \to e^+e^- = (1.00 \pm 0.18) \text{ KeV.}$$

Above $q^2 \sim 1.1$ (GeV/c)^2, multihadron production has been investigated at Orsay, Novosibirsk and especially at Frascati.

The first experimental values for multiparticle production cross-sections were presented at the Kiev Conference by the Frascati "Boson"(131) and "$\mu\pi$"(132) groups; the values of the cross-sections ($\gtrsim 30\ 10^{-33}$ cm^2) were at least one order of magnitude larger than expected on the basis of an extrapolation from the lower energy range data.

On the basis of pulse height analysis, shower recognition, and investigation of the interaction properties in the spark chamber plates, it was soon possible to conclude that the produced particles are hadrons (π or k) with a contamination from e and μ which is at most 5 ÷ 10%(133 ÷ 136).

In addition, it was possible to conclude that the production occurs essentially via the annihilation channel, with at most a small contribution (few percent) from graphs of the type (32). In fact,none of the observed multihadron events was detected in coincidence with a small angle electron.

In addition, the distribution of the events as a function of the non-coplanarity angle $\Delta\phi$ appears to be flat(137) (Fig. 34) with no appreciable contribution from the peaked distribution characteristic of the events of the type $e^+e^- \to e^+e^-e^+e^-$, $e^+e^- \to e^+e^-\mu^+\mu^-$, $e^+e^- \to e^+e^-\pi^+\pi^-$.

Assuming that the charged detected hadrons from reaction

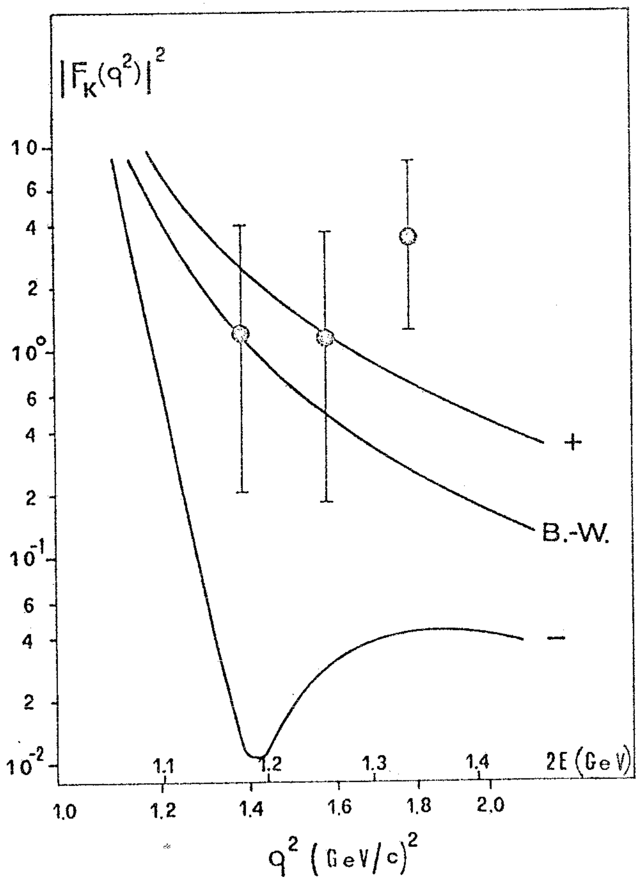


FIG. 33 - $|F_K(q^2)|^2$ as determined by the Novosibirsk group (4 events in total). The meaning of the curves is explained in the text.

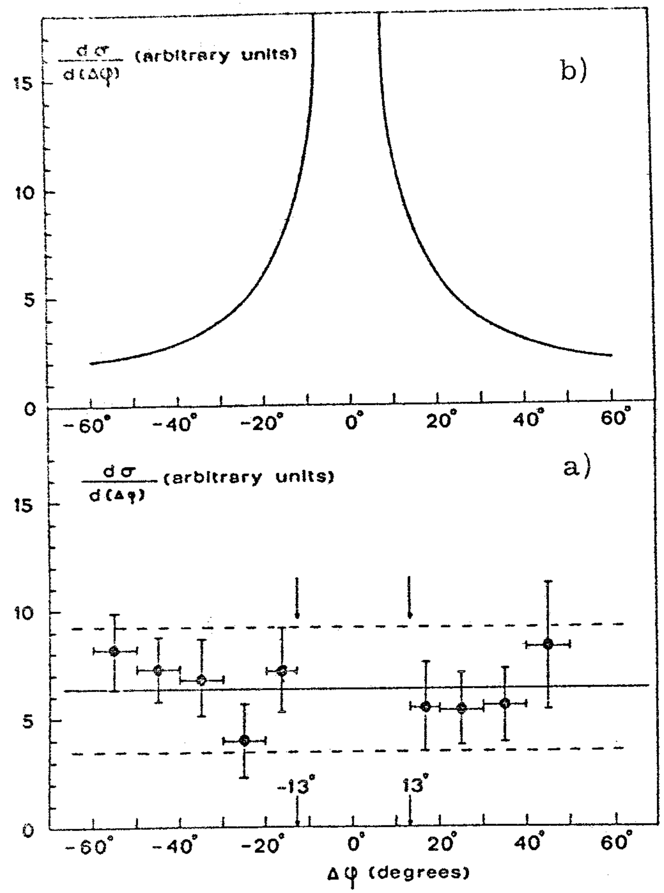


FIG. 34 - a) Distribution of non coplanar (multihadron production) events as a function of $\Delta\varphi$, the non coplanarity angle between pairs of observed charged tracks. The distribution is corrected for the detection efficiency.
b) $\Delta\varphi$ distribution for reaction $e^+e^- \longrightarrow e^+e^-e^+e^-$ as expected according to Baier and Fadin, see ref. (109).

(41) $e^+e^- \longrightarrow$ more than two hadrons

are pions, and that angular distributions are determined by pure phase space, cross-sections have been evaluated for different production channels. The main conclusions would remain unaltered within the errors if the angular and energy distribution are determined by quasi-two-body intermediate states (e. g. $e^+e^- \longrightarrow A_2^+ \pi^- \longrightarrow \pi^+\pi^-\pi^+\pi^-$)(134, 136, 137).

The results are shown in Figs. 35, 36, 37, 38, 39, 40. The total cross-section appears to be very large, larger than the cross-section for production of a pair of point-like fermions. After a steep increase between $\sim$1. 2 and 1. 4 GeV, it shows a slow fall-off consistent with a $1/q^2$ dependence. Different channels show different energy behaviour: for instance, while the channel $e^+e^- \longrightarrow \pi^+\pi^-\pi^+\pi^-$ shows a broad bump at a mass of $\sim$1. 6 GeV (a new vector boson ?), the $e^+e^- \longrightarrow \pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$ shows a slow increase between 1. 5 and 2. 4 GeV.

Some attempts have been made to interpret the above data in terms of conventional vector dominance(140), or in terms of an extended vector dominance including the contribution of a higher mass vector meson ρ'(141). The most general attitude is however to consider these large cross-sections as a different manifestation of the same phenomenon which is observed in deep inelastic electron proton-scattering.

II. 6. - COMMENTS AND CONCLUSIONS. -

The scattering of charged leptons on hadron targets, as well as the production of hadronic systems in e^+e^- interaction, has been an active field of investigation.

Due to the validity of three experimentally confirmed hypotheses - one photon exchange approximation, point-like fermions, and $1/q^2$ photon propagator - the experimental results have a straightforward phenomenological interpretation, in terms of a small number of form factors or structure functions closely connected with the electromagnetic structure of the concerned hadrons, and of the e. m. current generated by hadrons. The experimental information is quite abundant for nucleons in the space-like region. On the contrary, the experimental knowledge of the e. m. structure of unstable particles, as well as the data in the time-like region above $q^2 \simeq 1(\mathrm{GeV}/c)^2$, is still scarce or lacking.

In spite of the experimental difficulties - connected with the extremely small cross-sections and, in the time-like region, with the need of using the technique of beam-beam interaction - a general effort to overcome our present lack of experimental knowledge is to be expected - and stimulated - in the near future. Actually, it appears unprobable to us that a real understanding of the interactions of the elementary particles will be achieved until their e. m. structure will be known.

From the point of view of theory, we are still far from being able to predict the behaviour of form factors and structure functions, of connecting them with one another, of understanding the time-like region in terms of the data in the space like-region; that is to say, as yet, we do not have any theory. The experimental information on any particle, in any q^2 region, is therefore extremely useful and will complement the already available phenomenological knowledge.

One of the most exciting and elegant ideas in elementary particle physics was first suggested by the behaviour of e.m. formfactors: the idea that a gauge-field is generated by each conserved quantum number. The experimental data on hadron production from e^+e^- collisions below 1 GeV have nicely confirmed this idea, suggesting that nature was applying it in its simplest form - the vector dominance model. New data at higher energy, as well as results in the space-like region, show that this simple model is not adequate; but do not destroy the validity of the idea. As a compensation for this complication, the new data have suggested another simple idea - the parton models. Also this idea is certainly too simplified, and we already know that their simplest versions are in trouble: in the parton-quark identification for example, we know that the simple 3-quark structure of the nucleons is not adequate; a sea of virtual quarks is added to the valence quarks, etc.

But we already knew that it is unlikely that the extremely complicated phenomeno-

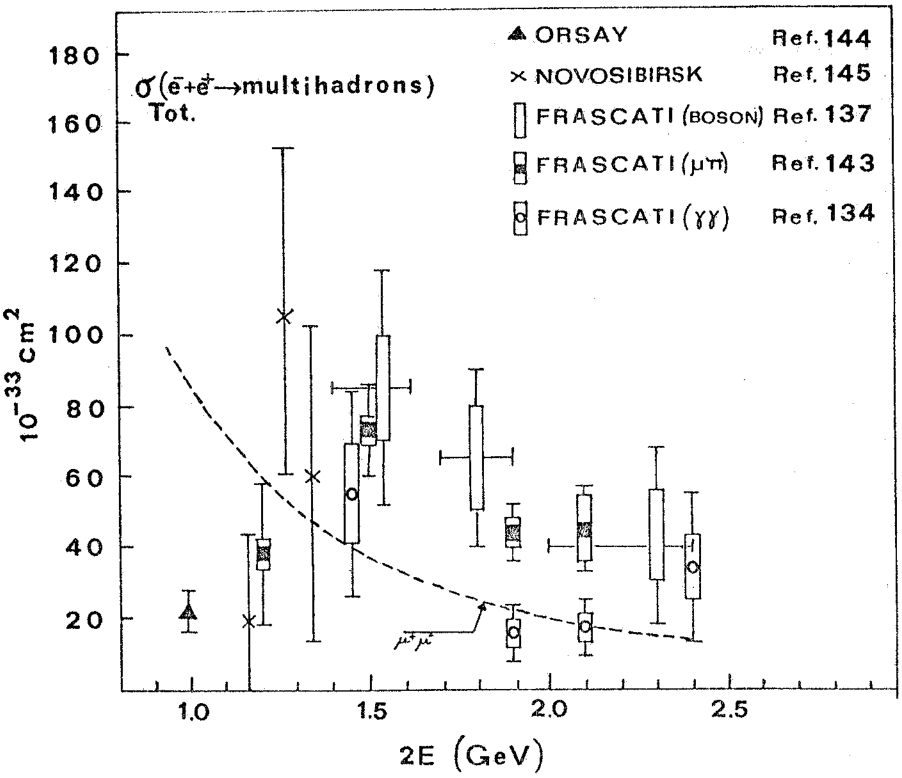


FIG. 35 - The total cross section σ_{total} ($e^+e^- \rightarrow$ more than two hadrons) as a function of the total c. m. energy $2E = E^+ + E^- = \sqrt{q^2}$. For reference, the dashed line shows the calculated total cross-section for production of $\mu^+\mu^-$ pairs.

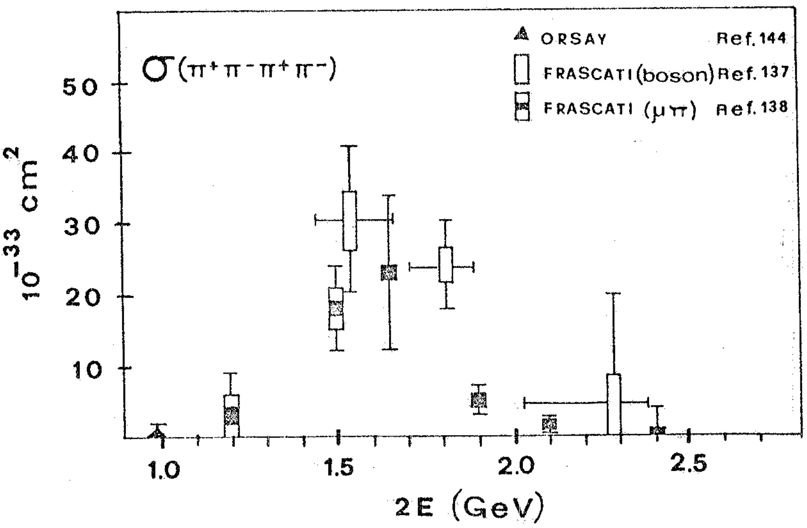


FIG. 36 - The cross-section to produce four charged pions $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-)$ as a function of the total c. m. energy 2E. In this and in the following figures the symbols used to indicate the experimental points from different experiments are the same as in Fig. 35.

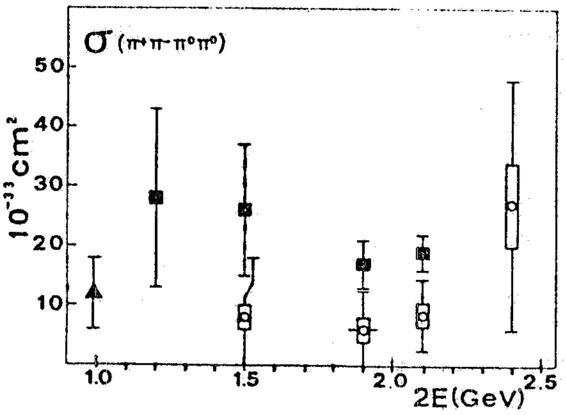


FIG. 37 - The cross-section $\sigma(e^+e^- \longrightarrow \pi^+\pi^-\pi^0\pi^0)$ as a function of the total c. m. energy 2E.

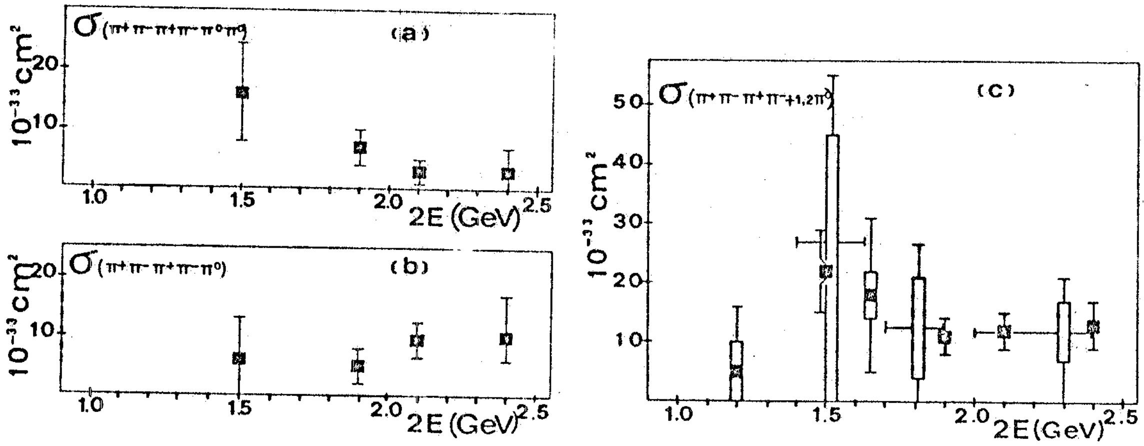


FIG. 38 - a) The cross-section $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0)$ as a function of 2E.
b) The cross-section $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0)$ as a function of 2E.
c) - $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0) + \sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0)$ as a function of 2E.

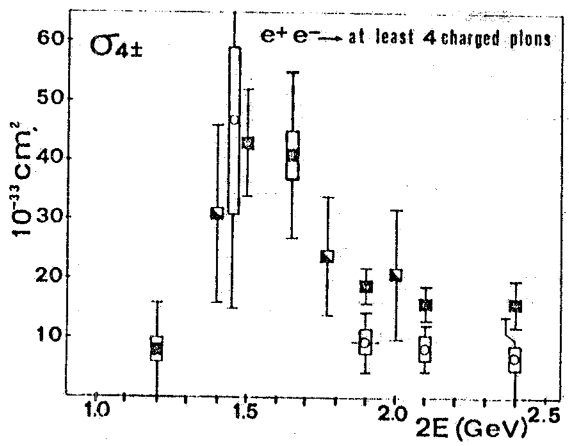


FIG. 39 - The cross-section $\sigma(e^+e^- \longrightarrow$ at least four charged pions) as a function of 2E. The points indicated by the symbol are taken from ref. (139).

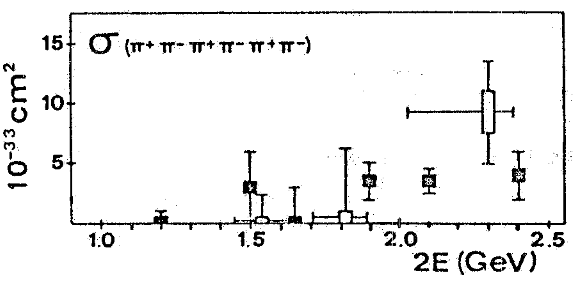


FIG. 40 - The cross-section $\sigma(e^+e^- \longrightarrow \pi^+\pi^-\pi^+\pi^-\pi^+\pi^-)$ as a function of 2E.

logy of elementary particle interactions can be completely understood in the frame of a simple model.

However, the close connection between the experimental numbers and fundamental quantities - current commutators, spectral functions, Wigner terms, etc - tells us that these difficult experiments will certainly give important results since they will provide the foundation on which to build all future theories.

REFERENCES. -

(1) - See, for instance, S. D. Drell and F. Zachariasen, Electromagnetic Structure of Nucleons, Oxford University Press, 1961.
(2) - A. M. Bincer, Phys. Rev. 118, 855 (1960).
(3) - M. Rosenbluth, Phys. Rev. 79, 615 (1950).
(4) - P. Tsai, Phys. Rev. 122, 1898 (1961); N. Meister and D. R. Yennie, Phys. Rev. 130, 1210 (1963).
(5) - U. Günther and R. Rodenberg, Nuovo Cimento 2A, 25 (1971); S. D. Drell and M. A. Ruderman, Phys. Rev. 106, 561 (1957); S. D. Drell and S. Fubini, Phys. Rev. 113, 741 (1959); N. R. Werthamer and M. A. Ruderman, Phys. Rev. 123, 1005 (1961); D. Flamm and W. Kummer, Nuovo Cimento 28, 33 (1963); S. D. Drell and J. D. Sullivan, Phys. Rev. Letters 19, 516 (1965); G. K. Greenhut, Phys. Rev. 184, 1860 (1969).
(6) - M. Gourdin, Diffusion des Electron de haute Energie (Manon et Cie, Paris 1966).
(7) - See, for instance, T. Janssen, R. Hofstadter, E. B. Hughes and M. R. Yearian, Phys. Rev. 142, 922 (1966); K. Berkelman in Nuclear Structure, edited by R. Hofstadter and L. Schiff, Stanford University press, (1964); R. Wilson in Particle Interactions at High Energies, edited by T. W. Preist and L. L. J. Vick, Edinburg and London 1967; H. J. Behrend, F. W. Brass, J. Engler, H. Hultshig, S. Galster, G. Hartwig, H. Schopper and E. Gansssauge, Nuovo Cimento 48A, 140 (1967); W. Bartel, B. Dudelzak, H. Krehbiel, J. M. McElroy, U. Meyer-Berkhout, R. J. Morrison, H. Nguyen-Ngoc, W. Schmidt and G. Weber, Phys. Rev. Letters 17, 608 (1966); W. Albrecht, H. J. Behrend, F. W. Brasse, W. Flauger, H. Hultshig and K. G. Steffen, Phys. Rev. Letters 17, 1192 (1966).
(8) - L. Camilleri, J. H. Christenson, M. Kramer, L. M. Lederman, Y. Nagashima and T. Yamanouchi, Phys. Rev. Letters 23, 149 (1969).
(9) - J. Mar, B. C. Barish, J. Pine, D. H. Coward, H. Destaebler, J. Litt, A. Minten, R. E. Taylor and M. Breidenbach, Phys. Rev. Letters 21, 482 (1968).
(10) - D. Yount and J. Pine, Phys. Rev. 128, 1842 (1962).
(11) - A. Browman, F. Liu and C. Schaerf, Phys. Rev. 139B, 1079 (1965).
(12) - W. Bartel, B. Dudelzak, H. Krehbiel, J. M. McElroy, R. J. Morrison, W. Schmidt, V. Walther and G. Weber, Phys. Letters 25B, 242 (1967).
(13) - B. Bouquet, D. Benaksas, B. Grossetête, B. Jean-Marie, G. Parrour, J. P. Poux and R. Tchapoutian, Phys. Letters 26B, 178 (1968).
(14) - R. L. Anderson, B. Borgia, G. L. Cassiday, J. W. DeWire, A. S. Ito and E. C. Loh, Phys. Rev. Letters 17, 407 (1966); Phys. Rev. 166, 1336 (1968).
(15) - G. Cassiday, J. WeWire, H. Fischer, A. Ito, E. Loh and J. Rutherfoord, Phys. Rev. Letters 19, 1191 (1967).
(16) - Although $\bar{A}$ and P are zero if $\mathrm{Im}\, A_{2\gamma}$ is zero, they cannot be expressed in terms of $\mathrm{Im}\, A_{2\gamma}$, since $A_{2\gamma}$ of eq. (4) was already summed and averaged over the final and initial spin states. For explicit calculations of polarizations see, for instance, M. L. Goldberger and K. M. Watson, Collision Theory, Wiley, New York (1965).
(17) - G. V. Di Giorgio, E. Ganssauge, R. Gomez, G. Gorini, S. Penner, S. Serbassi, M. L. Vincelli, E. Amaldi and G. Stoppini, Nuovo Cimento 39, 474 (1965).
(18) - J. C. Bizot, J. M. Buon, J. LeFrancois, J. Perez-y-Jorba and P. Roy, Phys. Rev. 140B, 1387 (1965).
(19) - D. E. Lundquist, R. L. Anderson, J. V. Allaby and D. M. Ritson, Phys. Rev. 168, 1527 (1968).

(20) - Stanford, unpublished. Referred to in ref. (21).
(21) - T. Powell, M. Borghini, O. Chamberlain, R. Z. Fuzesy, C. G. Morehouse, S. Rock, G. Shapiro, H. Weisberg, R. L. A. Cottrell, J. Litt, L. W. Mo and R. E. Taylor, Phys. Rev. Letters 24, 753 (1970).
(22) - See for instance: J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics, McGraw-Hill Book Company, 1964.
(23) - See, for instance, references 1, 2, 6 and N. Kroll, Nuovo Cimento 45A, 65 (1966).
(24) - J. C. Wesley and A. Rich, Phys. Rev. Letters 24, 1320 (1970); J. Bailey, W. Bartl, G. von Bochman, R. C. A. Brown, F. J. M. Farley, H. Jöstlein, E. Picasso and R. W. Williams, Phys. Letters 28B, 287 (1968).
(25) - C. Møller, Ann. Physik 14, 531 (1932).
(26) - H. J. Bhabha, Proc. Roy. Soc. (London) A154, 195 (1935).
(27) - W. C. Barber, B. Gittelman, G. K. O'Neill and B. Richter, Phys. Rev. Letters 16, 1127 (1966).
(28) - W. C. Barber, G. K. O'Neill, B. Gittelman and B. Richter, SLAC, Report, CLNS-139 (February 1971), to be published.
(29) - V. Alles-Borelli, M. Bernardini, D. Bollini, P. L. Brunini, E. Fiorentino, T. Massam, L. Monari, F. Palmonari and A. Zichichi, Direct check of QED in e^+e^- interactions at high q^2 values, CERN Report (June 1971), to be published in Nuovo Cimento.
(30) - B. Borgia, F. Ceradini, M. Conversi, L. Paoluzi, W. Scandale, G. Barbiellini, M. Grilli, P. Spillantini, R. Visentin and A. Mulachié, Phys. Letters 35B, 340 (1971).
(31) - B. Bartoli, F. Felicetti, H. Ogren, V. Silvestrini, G. Marini, A. Nigro, N. Spinelli and F. Vanoli, Phys. Letters 36B, 593 (1971); B. Bartoli, B. Coluzzi, F. Felicetti, G. Goggi, G. Marini, F. Massa, D. Scannicchio, V. Silvestrini and F. Vanoli, Nuovo Cimento 70A, 615 (1970).
(32) - J. E. Augustin, J. Buon, B. Delcourt, J. Jeanjean, D. Lalanne, H. Nguyen Ngoc, J. Perez-y-Jorba, P. Petroff, F. Richard, F. Rumpf and D. Treille, Phys. Letters 31B, 673 (1970).
(33) - R. W. Ellsworth, A. C. Melissinos, J. H. Tinlot, H. von Briesen jr., T. Yamanouchi, L. M. Lederman, M. J. Tannenbaum, R. L. Cool and A. Maschke, Phys. Rev. 165, 1449 (1968).
(34) - L. Camilleri, J. H. Christenson, M. Kramer, L. M. Lederman, Y. Nagashima and T. Yamanouchi, Phys. Rev. Letters 23, 153 (1969).
(35) - B. D. Dieterle, T. Braunstein, J. Cox, F. Martin, W. T. Toner, M. L. Perl, T. F. Zipf, W. L. Lakin and H. C. Bryant, Phys. Rev. Letters 23, 1187 (1969).
(36) - W. T. Toner, T. J. Braunstein, W. L. Lakin, F. Martin, M. L. Perl, T. F. Zipf, H. C. Bryant and B. D. Dieterle, Phys. Letters 36B, 251 (1971).
(37) - See, for instance, R. Wilson in "Particle Interactions at High Energies", edited by T. W. Preist and L. L. J. Vick, Oliver & Boyd, Edinburg & London 1966.
(38) - W. Bartl, B. Dudelzak, H. Krehbiel, J. M. McElroy, U. Meyer-Berkhout, R. J. Morrison, H. Nguyen-Ngoc, W. Schmidt and G. Weber, Phys. Rev. Letters 17, 608 (1966).
(39) - W. Bartl, B. Dudelzak, H. Krehbiel, J. M. McElroy, U. Meyer-Berkhout, R. J. Morrison, H. Nguyen-Ngoc, W. Schmdt and G. Weber, Phys. Letters 25B, 236 (1967).
(40) - T. Janssens, R. Hofstadter, E. B. Hughes and M. R. Yearian, Phys. Rev. 142, 922 (1966).
(41) - H. J. Behrend, F. W. Brasse, J. Engler, H. Hultshig, S. Galster, G. Hartwig, H. Schopper, E. Ganssauge, Proc. of the XIII Intern. Conf. on High Energy Physics, Berkeley 1966, and Nuovo Cimento 48A, 140 (1967).
(42) - W. Albrecht, H. J. Behrend, F. W. Brasse, W. Flauger, H. Hultshig and K. G. Steffen, Phys. Rev. Letters 17, 1192 (1966).
(43) - K. Berkelman, M. Feldman, R. M. Littauer, G. Rousse and R. R. Wilson, Phys. Rev. 130, 2061 (1963).
(44) - K. W. Chen, J. R. Dunning, jr., A. A. Cone, N. F. Ramsey, J. K. Walker and R. Wilson, Phys. Rev. 141, 1267 (1966).
(45) - L. E. Price, J. R. Dunning jr., M. Goitein, K. Hanson, T. Kirk and R. Wilson, Phys. Rev. D4, 45 (1971).
(46) - D. J. Drickey and L. N. Hand, Phys. Rev. Letters 9, 521 (1962).
(47) - B. Dudelzak, G. Sauvage and P. Lehman, Nuovo Cimento 28, 18 (1963).
(48) - Chr. Berger, E. Gersing, G. Knop, B. Langenbeck, K. Rith and F. Schumacher, Phys. Letters 28B, 276 (1968).

(49) - W. Bartel, F. N. Büsser, W. R. Dix, R. Felst, D. Harms, H. Krehbiel, P. E. Kuglman, J. McElroy and G. Weber, Phys. Letters 33B, 245 (1970).

(50) - D. H. Coward, H. De Staebler, R. A. Early, J. Litt, A. Minten, L. W. Mo, W. K. H. Panofsky, R. E. Taylor, M. Breidenbach, J. I. Friedman, H. W. Kendall, P. N. Kirk, B. C. Barish, J. Mar and J. Pine, Phys. Rev. Letters 20, 292 (1968).

(51) - J. Litt, G. Buschhorn, D. H. Coward, H. De Staebler, L. W. Mo, R. E. Taylor, B. C. Barish, S. C. Loken, J. Pine, J. I. Friedman, G. C. Hartman and H. W. Kendall, Phys. Letters 31B, 40 (1970).

(52) - The data points from some old experiments are not shown in the figures. However, most of the corresponding cross-section measurements have been used to improve the Rosenbluth plots of later experiments. The references of these experiments are: P. Lehman, R. Taylor and R. Wilson, Phys. Rev. 126, 1183 (1962); R. W. Chen, A. A. Cone, J. R. Dunning, jr., S. F. G. Frank, N. F. Ramsey, J. K. Walker and R. Wilson, Phys. Rev. Letters 11, 561 (1963); B. Dudelzak, A. Isakov, P. Lehman, R. Tchapoutian, Proc. of the XII Intern. Conf. on High Energy Physics, Dubna (1964), vol. I, 916; W. Albrecht, H. J. Behrend, F. W. Brasse, W. Flauger, H. Hultshig, Proc. of the XIII Intern. Conf. on High Energy Physics, Berkeley (1966); E. E. Chambers and R. Hofstadter, Phys. Rev. 103, 1454 (1956); F. Bumiller, M. Croissaux, E. Dally and R. Hofstadter, Phys. Rev. 124, 1623 (1961); D. Yount and G. Pine, Phys. Rev. 128, 1842 (1962); D. N. Olson, H. F. Schopper and R. R. Wilson, Phys. Rev. Letters 6, 286 (1961).

(53) - M. Goitein, R. J. Budnitz, L. Carroll, J. R. Chen, J. R. Dunning, jr., K. Hanson, D. C. Imrie, C. Mistretta and R. Wilson, Phys. Rev. 1D, 2449 (1970) give results on electron scattering cross section and comparison with the dipole fit. In terms of form factors their data are included under ref. (45).

(54) - See for instance: A. L. Licht and A. Pagnamenta, Phys. Rev. 4, 2810 (1971) and references quoted there; T. Narita, Progr. Theoret. Phys. 42, 1336 (1969).

(55) - E. Fermi and L. Marshall, Phys. Rev. 72, 1139 (1947).

(56) - V. E. Krohn and G. R. Ringo, Phys. Letters 18, 297 (1965) and Phys. Rev. 148, 1303 (1966).

(57) - E. Melkonian, B. M. Rustad and W. W. Havens, Phys. Rev. 114, 1571 (1959).

(58) - D. Hughes, J. A. Harvey, M. D. Goldberger and M. J. Stafne, Phys. Rev. 90, 497 (1953).

(59) - T. Hamada and J. D. Johnston, Nuclear Phys. 34, 382 (1962).

(60) - I. Mc Gee, Phys. Rev. 151, 772 (1966).

(61) - H. Feshbach and E. Lomon, Revs. Modern Phys. 36, 611 (1967).

(62) - F. Gross, Phys. Rev. 142, 1025 (1966); 152, 1516(E) (1966).

(63) - S. Galster, H. Klein, J. Moritz, K. H. Schmidt, D. Wegener and J. Bleckwenn, Desy Preprint 71/7 (1971).

(64) - Ch. Berger, V. Burkhart, G. Knop, B. Langenbeck and K. Rith, Physikalisches Institut, Universität Bonn, Preprint 1-075 (1969).

(65) - D. J. Drickey and L. N. Hand, Phys. Rev. Letters 9, 521 (1962).

(66) - B. Grossetête, D. Drickey and P. Lehman, Phys. Rev. 141, 1425 (1966).

(67) - D. Benaksas, D. Drickey and D. Frèrejacque, Phys. Rev. 148, 1327 (1966); Phys. Rev. Letters 13, 353 (1964).

(68) - C. D. Buchanan and M. R. Yearian, Phys. Rev. Letters 15, 303 (1965).

(69) - J. I. Friedman, H. W. Kendall and P. A. Gram, Phys. Rev. 120, 992 (1960).

(70) - A. O. Barut, D. Corrigan and H. Kleinert, Phys. Rev. Letters 20, 167 (1968).

(71) - R. Budnitz, J. Appel, L. Carrol, J. Chen, J. R. Dunning jr., Mr. Goitein, K. Hanson, D. Imrie, C. Mistretta, J. K. Walker and R. Wilson, Phys. Rev. Letters 19, 809 (1967).

(72) - K. M. Hanson, J. R. Dunning, jr., M. Goitein, T. Kirk, L. E. Price and R. Wilson, Presented at the 1971 International Symposium on Electron and Photon Interactions, Cornell (1971).

(73) - W. Bartel, F. W. Busser, W. R. Dix, R. Felst, D. Harms, H. Krehbiel, P. E. Kuhlman, J. Mc Elroy, J. Mayer and G. Weber, Presented at the 1971 International Symposium on Electron and Photon Interactions, Cornell (1971).

(74) - D. Braess, D. Hasselmann and G. Kramer, Z. Physik 198, 527 (1966); D. Braess and G. Kramer, Z. Physik 189, 242 (1966); Analysis of data from ref. (75).

(75) - E. B. Hughes, T. A. Griffy, M. R. Yearian and R. Hofstadter, Phys. Rev. 139, B 458 (1968).

(76) - R. J. Budnitz, J. Appel, L. Carrol, J. Chen, J. R. Dunning, jr., M. Goitein, K. Hanson, D. Imrie, C. Mistretta, J. K. Walker and R. Wilson, Phys. Rev. 173, 1357 (1968).

(77) - J. R. Dunning, jr., K. W. Chen, A. A. Cone, G. Hartwig, N. F. Ramsey, J. K. Walker and R. Wilson, Phys. Rev. 141, 1286 (1966).
(78) - W. Bartel, F. W. Büsser, W. R. Dix, R. Felst, D. Harms, H. Krehbiel, P. E. Kuhlman, J. Mc Elroy, W. Schmidt, V. Walther and G. Weber, Phys. Letters 30B, 285 (1969).
(79) - C. W. Akerlof, K. Berkelman, G. Rouse and M. Tigner, Phys. Rev. 135, B810 (1964).
(80) - E. Amaldi, Frascati Internal Report, LNF-71/41 (1971). is an excellent review of the experiments and methods in electroproduction of pions.
(81) - S. L. Adler, Ann. Phys. (N. Y.) 50, 189 (1968).
(82) - N. Zagury, Phys. Rev. 145, 1112 (1966).
(83) - B. De Tollis and F. Nicolò, Nuovo Cimento 48A, 281 (1967).
(84) - F. A. Berends, Phys. Rev. D1, 2590 (1970).
(85) - R. C. E. Devenish and D. H. Lyth, University of Lancaster preprint, April 1971, to be published in Phys. Rev. (1972).
(86) - G. Von Gehlen, Nuclear Phys. B9, 17 (1969);
G. Von Gehlen and M. G. Schmidt, Nuclear Phys. B20, 173 (1970); G. Von Gehlen, Bonn University, PI-2-80 (1970).
(87) - See for instance: Ph. Salin, Nuovo Cimento 32, 521 (1964); M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193 and 309 (1963); J. P. Loubaton, Nuovo Cimento 39, 591 (1965); P. Kessler, College de France, Report PAM N. 6505 (1965); J. D. Walecka, Phys. Rev. 162, 1462 (1967); J. D. Bjorken and J. D. Walecka, Ann. Phys. 38, 35 (1966); P. L. Pritchett and J. D. Walecka, Phys. Rev. 168, 1638 (1968).
(88) - See for instance: S. Fubini and G. Furlan, Ann. Phys. 48, 322 (1968); A. M. Gleeson, M. G. Gundzik and J. G. Kuriyan, Phys. Rev. 173, 1708 (1968); G. Furlan, N. Paver and C. Verzegnassi, Nuovo Cimento 62A, 519 (1969) and Nuovo Cimento 70A, 247 (1970); Y. Nambu and M. Yoshimura, Phys. Rev. Letters 24, 25 (1970).
(89) - N. Dombey and B. J. Read, paper contributed to the 1971 International Symposium on Electron and Photon Interactions at High Energy, Cornell (1971).
(90) - C. Driver, K. Heinloth, K. Höhne, G. Hofmann, P. Karow, D. Schmidt, G. Specht and J. Rathje, Phys. Letters 35B, 77 (1971); Phys. Letters 35B, 81 (1971).
(91) - C. Mistretta, J. A. Appel, R. J. Budnitz, L. Carroll, J. Chen, J. R. Dunning, jr., M. Goitein, K. Hanson, D. C. Imrie and R. Wilson, Phys. Rev. 184, 1487 (1969).
(92) - E. Amaldi, B. Borgia, P. Pistilli, M. Balla, G. V. Di Giorgio, A. Giazotto, S. Serbassi and G. Stoppini, Nuovo Cimento 65A, 377 (1970).
(93) - C. W. Akerlof, W. W. Ash, K. Berkelman, A. C. Lichtenstein, A. Ramanauskas and R. H. Siemann, Phys. Rev. 163, 1482 (1967).
(94) - A. Sofair, J. Allison, B. Dickinson, E. Evangelides, M. Ibbotson, R. Lawson, R. S. Meaburn, H. E. Montgomery, W. J. Shuttleworth, A. B. Clegg, F. Foster, G. Hughes P. Kummer and R. Siddle, Daresbury Preprint DNPL/P 97 (December 1971).
(95) - C. N. Brown, C. R. Canizares, W. E. Cooper, A. M. Eisner, G. J. Feldman, C. A. Lichtenstein, L. Litt, W. Lockeretz, V. B. Montana and F. M. Pipkin, Phys. Rev. Letters 26, 987 (1971); Phys. Rev. Letters 26, 991 (1971).
(96) - R. Wilson, Rapporteur talk at the 1971 International Symposium on Electron and Photon Interactions at High Energies, Cornell (1971). Proceedings pag. 98.
(97) - W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas and R. H. Siemann, Phys. Letters 24B, 165 (1967).
(98) - H. Harari and H. J. Lipkin, Phys. Rev. 140, B1617 (1965); C. Becchi and G. Morpurgo, Phys. Letters 17, 352 (1965).
(99) - M. Gourdin and P. Salin, Nuovo Cimento 27, 193 (1963); D. J. Drickey and R. F. Mozley, Phys. Rev. Letters 8, 291 (1962).
(100) - W. Bartel, B. Dudelzak, H. Krehbiel, J. Mc Elroy, U. Meyer-Berkhout, W. Schmidt, V. Walter and G. Weber, Phys. Letters 28B, 148 (1968).
(101) - C. Mistretta, D. Imrie, J. A. Appel, R. Budnitz, L. Carrol, J. Chen, J. Dunning, jr., M. Goitein, K. Hanson, A. Litke and R. Wilson, Phys. Rev. Letters 20, 1070 (1968); D. Imrie, C. Mistretta and R. Wilson, Phys. Rev. Letters 20, 1074 (1968).
(102) - M. Breidenbach, J. I. Friedman, H. W. Kendall, E. D. Bloom, D. H. Coward, H. De Staebler, J. Drees, L. W. Mo and R. E. Taylor, Phys. Rev. Letters 23, 930 (1969); Phys. Rev. Letters 23, 935 (1969) and SLAC-PUB 815 (To be published in Phys. Rev.); SLAC-PUB 796 (Presented to the XV International Conference on High Energy Physics, Kiev, USSR (September 1970); J. I. Friedman, Invited talk presented at the Topical Se-

minar on Electromagnetic Interactions, ICTP Trieste, 21-26 June 1971.
(103) - D. E. Andrews, K. Berkelman, D. G. Cassel, D. L. Hartill, J. Hartman, R. Kerchner, E. Lazarus, R. M. Littauer, R. L. Loveless, R. Rohlfs, D. H. White and A. J. Sadoff, Phys. Rev. Letters 26, 864 (1971).
(104) - In addition to the data of ref. (102), only a few more data are available from a Desy experiment: W. Albrecht, F. W. Brasse, H. Dorner, W. Flauger, K. H. Frank, J. Gayler, H. Hultshig, V. Korbel, J. May and E. Ganssauge, Desy Report 69/46 (1969).
(105) - R. Feynman, Phys. Rev. Letters 23, 1415 (1969).
(106) - J. D. Bjorken, Phys. Rev. 179, 1547 (1969); R. Jackiw, R. Van Royen and G. B. West, Phys. Rev. D2, 2473 (1970); R. A. Brandt, Phys. Rev. D1, 2808 (1970); H. Leutwyler and J. Stern, Nuclear Phys. B20, 77 (1970); B. L. Ioffe, Phys. Letters 30B, 123 (1969); Y. Frishman, Phys. Rev. Letters 25, 966 (1970).
(107) - E. Bloom and F. Gilman, Phys. Rev. Letters 25, 1140 (1970).
(108) - See, for instance: S. D. Drell, rapporteur talk in the Proceedings of the Amsterdam International Conference on Elementary Particles, Amsterdam (1971), p. 307; See also G. Preparata, Rivista del Nuovo Cimento 1, 423 (1971).
(109) - L. Landau and E. Lifshitz, Soviet Phys. 6, 244 (1934); V. E. Balakin, V. M. Budnev and I. F. Ginzburg, JETP Letters 11, 388 (1970); JETP Letters 11, 559 (1970); S. J. Brodsky, T. Kinoshita and H. Terazawa, Phys. Rev. Letters 25, 972 (1970); CLNS-152 (1971); V. N. Baier and V. S. Fadin, Zhur. Eksp. i Teoret. Fiz. 13, 293 (1971); Lett. Nuovo Cimento 1, 481 (1971); Novosibirsk preprint (1971); N. Arteaga-Romero, A. Jaccarini, P. Kessler and I. Parisi, PAM 71-Q2 (1971).
(110) - V. E. Balakin, G. I. Budker, E. V. Pakhtusova, V. A. Sidorov, A. N. Skrinsky, G. M. Tumaikin and A. G. Khabakhpashev, Phys. Letters 34B, 661 (1971).
(111) - C. Bacci, R. Baldini-Celio, G. Capon, C. Mencuccini, G. P. Murtas, G. Penso, A. Reale, G. Salvini, M. Spinetti and B. Stella, Lett. Nuovo Cimento 4, 709 (1972)
(112) - G. Parisi, Private communication.
(113) - G. Barbiellini, Private communication.
(114) - D. C. Cheng, Private communication.
(115) - V. Silvestrini, A review of high energy work in Quantum Electrodynamics, Invited talk at the 3rd Rencontre de Moriond sur les Interactions Electromagnetique, 1968; LNF-68/17, Average of the data from the following experiments: J. K. De Pagter, J. I. Friedman, G. Glass, R. C. Chase, M. Gettner, E. Von Goeler, R. Weinstein and A. M. Boyarski, Phys. Rev. Letters 16, 35 (1966); B. D. Hyams, W. Koch, D. Potter, L. Von Lindern, E. Lorentz, G. Lütjens, V. Stierlin and P. Weilhammer, Phys. Letters 24B, 634 (1967); A. Wehman, E. Engels, jr., C. M. Hoffman, P. G. Innocenti, R. Wilson, W. A. Blampied, D. J. Drichey, L. N. Hand and G. D. Stairs, Phys. Rev. Letters 18, 929 (1967); P. L. Rothwell, A. Boyarski, R. C. Chase, J. De Pagter, J. I. Friedman, M. Gettner, G. Glass, E. Von Goeler and R. Weinstein, International Symposium on Electron and Photon Interactions at High Energies, Stanford (1967); J. G. Ashbury, U. Becker, W. K. Bertram, P. Joos, M. Rohde, A. J. S. Smith, C. L. Jordan and S. C. C. Ting, Phys. Rev. Letters 19, 869 (1967); S. S. Hertzbach, R. W. Kraemer, L. Madansky, R. A. Zdanis and R. Strand, Phys. Rev. 155, 1461 (1967); M. N. Khachaturyan, M. A. Azimov, A. M. Baldin, A. S. Belousov, I. V. Chuvilo, R. Firkowski, J. Hladky, M. S. Khvastunov, J. Manca, A. T. Mathyushin, V. H. Mathyushin, G. A. Ososkov, L. N. Shtarkov and L. I. Zhuravleva, Phys. Letters 24B, 349 (1967); V. L. Auslander, G. I. Budker, N. Pestov, V. A. Sidorov, A. N. Skrinskij and A. G. Khabakhpashev, Phys. Letters 25B, 433 (1967); J. E. Augustin, J. C. Bizot, J. Buon, J. Haissinski, D. Lalanne, P. C. Marin, J. Perez-y--Jorba, F. Rumpf, E. Silva and S. Tavernier, Orsay Report LAL-1181 (1967); Phys. Rev. Letters 20, 126 (1968).
(116) - V. E. Balakin, G. I. Budker, L. M. Kurdadze, A. P. Onuchin, E. V. Pakhtusova, S. I. Serednyakov, V. A. Sidorov, A. N. Skrinsky and A. G. Khabakhpashev, Novosibirsk preprint 53-71.
(117) - B. Borgia, F. Ceradini, M. Conversi, L. Paoluzi, R. Santonico, G. Barbiellini, M. Grilli, P. Spillantini, R. Visentin and F. Grianti, Lett. Nuovo Cimento 3, 115 (1972).
(118) - V. Alles-Borelli, M. Bernardini, D. Bollini, P. L. Brunini, E. Fiorentino, T. Massam, L. Monari, F. Palmonari and A. Zichichi, Lett. Nuovo Cimento 2, 376 (1971).
(119) - Results of the Orsay group: J. E. Augustin, A. Couran, B. Dudelzak, F. Fulda, G. Gros-

didier, J. Haissinski, J. L. Masnou, R. Riskalla, F. Rumpf and E. Silva, Presented to the 1971 International Symposium on Electron and Photon Interactions at High Energies, Cornell (1971).
(120) - G. Di Giugno, J. W. Humphrey, E. Sassi, G. Troise, U. Troya, S. Vitale and M. Castellano, Lett. Nuovo Cimento 2, 873 (1971); Latest results presented at the Informal Meeting on e^+e^- Interactions, May-2-3, Frascati (1972).
(121) - N. Cabibbo and R. Gatto, Phys. Rev. 124, 1577 (1961).
(122) - M. Conversi, T. Massam, Th. Muller and A. Zichichi, Nuovo Cimento 40A, 690 (1965).
(123) - D. L. Hartill, B. C. Barish, D. G. Fong, R. Gomez, J. Pine and A. V. Tollestrup, Phys. Rev. 184, 1485 (1969).
(124) - V. L. Auslander, G. I. Budker, Ju. N. Pestov, V. A. Sidorov, A. N. Skrinsky and A. G. Khabakpashev, Phys. Letters 25B, 433 (1967); V. L. Auslander, G. I. Budker, E. V. Pakhtusova, Ju. N. Pestov, V. A. Sidorov, A. N. Skrinsky and A. G. Khabakhpashev, Soviet J. Nuclear Phys. 9, 144 (1969).
(125) - J. E. Augustin, J. C. Bizot, J. Buon, J. Haissinski, D. Lalanne, P. Marin, J. Perez-y-Jorba, F. Rumpf, E. Silva, S. Tavernier, Phys. Rev. Letters 20, 126 (1968); J. E. Augustin, J. C. Bizot, T. Buon, J. Hassinski, D. Lalanne, P. Marin, H. Nguyen Ngoc, J. Perez-y-Jorba, F. Rumpf, E. Silva and S. Tavernier, Phys. Letters 28B, 508 (1969). D. Benaksas, G. Cosme, B. Jean-Marie, S. Jullian, F. Laplanche, J. Lefrancois, A. D. Liberman, G. Parrour, J. P. Repellin, G. Sauvage, G. Szklarz, presented to the International Symposium on Electron and Photon Interactions at High Energies, Cornell (1971).
(126) - See, for instance, N. Kroll, T. D. Lee and B. Zumino, Phys. Rev. 157, 1376 (1967).
(127) - J. C. Bizot, J. Buon, Y. Chatelus, J. Jeanjean, D. Lalanne, H. Nhuyen Ngoc, J. P. Perez-y-Jorba, P. Petroff, F. Richard, F. Rumpf and D. Treille, Phys. Letters 32B, 416 (1970).
(128) - V. E. Balakin, G. I. Budker, E. V. Pakhtusova, V. A. Sidorov, A. N. Skrinsky, G. M. Tumaikin, A. G. Khabakhpashev, Phys. Letters 34B, 328 (1971).
(129) - V. A. Sidorov, Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Cornell 1971, pag. 66, and private communication.
(130) - J. E. Augustin, D. Benaksas, J. Buon, F. Fulda, V. Gracco, J. Haissinski, D. Lalanne, F. Laplanche, J. Lefrançois, P. Lehmann, P. C. Marin, J. Perez-y-Jorba, F. Rumpf and E. Silva, Lett. Nuovo Cimento 2, 214 (1969).
(131) - B. Bartoli, B. Coluzzi, F. Felicetti, V. Silvestrini, G. Goggi, D. Scannicchio, G. Marini, F. Massa and F. Vanoli, Intern. Conf. on High Energy Physics, Kiev (1970); Nuovo Cimento 70A, 615 (1970).
(132) - G. Barbiellini, M. Conversi, M. Grilli, A. Mulachiè, M. Nigro, L. Paoluzi, P. Spillantini, R. Visentin and G. T. Zorn, Frascati Report LNF-70/38 (1970).
(133) - B. Bartoli, F. Felicetti, H. Ogren, V. Silvestrini, G. Marini, A. Nigro, N. Spinelli and F. Vanoli, Phys. Letters 36B, 598 (1971).
(134) - C. Bacci, R. Baldini-Celio, G. Capon, C. Mencuccini, G. P. Murtas, G. Penso, A. Reale, G. Salvini, M. Spinetti and B. Stella, Phys. Letters 38B, 551 (1972).
(135) - V. Alles-Borelli, M. Bernardini, D. Bollini, P. L. Brunini, E. Fiorentino, T. Massam, L. Monari, F. Palmonari and A. Zichichi, Proc. of the EPS Conf. on Meson Resonances and Related Electromagnetic Phenomena, Bologna (1971).
(136) - B. Borgia, M. Conversi, M. Grilli, E. Iarocci, M. Nigro, L. Paoluzi, P. Spillantini, L. Trasatti, V. Valente, R. Visentin and G. T. Zorn, Intern. Symp. on Electron and Photon Interactions at High Energies, Cornell (1971); LNF-71/62 (1971).
(137) - B. Bartoli, F. Felicetti, G. Marini, A. Nigro, H. Ogren, V. Silvestrini and F. Vanoli, LNF-71/91 (1971), to be published in Phys. Rev.
(138) - G. Barbarino, F. Ceradini, M. Conversi, M. Grilli, E. Iarocci, M. Nigro, L. Paoluzi, R. Santonico, P. Spillantini, L. Trasatti, V. Valente, R. Visentin and G. T. Zorn, LNF-71/96 (1971); Lett. Nuovo Cimento 3, 689 (1972); B. Borgia, F. Ceradini, M. Conversi, M. Grilli, E. Iarocci, M. Nigro, L. Paoluzi, R. Santonico, P. Spillantini, L. Trasatti, V. Valente, R. Visentin and G. T. Zorn, LNF-72/43, to be published.
(139) - Results of the $\mu\pi$ group. (Same authors of ref. (132)) presented by G. Barbiellini to the Informal Frascati Meeting (September, 1970).
(140) - J. Layssac and F. M. Renard, Lett. Nuovo Cimento 1, 197 (1971), Montpellier Preprint P. M. 71/2 (1971).
(141) - A. Bramon and M. T. Greco, Lett. Nuovo Cimento 1, 739 (1971); 3, 693 (1972).

(142) - V. Alles-Borelli, M. Bernardini, D. Bollini, P. L. Brunini, E. Fiorentino, T. Massam, L. Monari, F. Palmonari and A. Zichichi, Presented to the Informal Frascati Meeting, (May, 1972).
(143) - Results of the μ - π group (same authors as ref. (132)) presented to the Informal Frascati Meeting, (May, 1972).
(144) - J. Lefrançois, Proc. of the Intern. Symp. on Electron and Photon Interactions at High Energies, Cornell (1971), pag. 59.
(145) - V. A. Sidorov, presented to the Informal Frascati Meeting, (May, 1972).