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A. Bramon: PSEUDOSCALAR MESON DECAYS INTO LEPTON PAIRS AND THE LEE-WICK THEORY. -

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ABSTRACT. -

The branching ratios $R_{P\to l\bar{l}} \equiv \Gamma(P\to l\bar{l})/\Gamma(P\to\gamma\gamma)$ are evaluated using the Lee and Wick theory of QED and assuming that the $f_{P\gamma\gamma}$ couplings are independent of the virtual photon masses. From the experimental value $R_{\eta\to\mu^+\mu^-} = (5.9\pm 2.2)\,10^{-5}$ one obtains $m_B = 32\,^{+40}_{-18}$ GeV and $R_{\pi^0\to e^+e^-} = (2.2\pm 0.4)\,10^{-7}$.

(x) - On leave of absence from Departamento de Física Teórica - Barcelona.

In a recent series of papers Lee and Wick[1,2] have proposed a new theory of QED characterized by the replacement of the usual photon field A_μ by a complex field $A_\mu + iB_\mu$ where B_μ obeys a negative metric and is associated with a heavy mass, m_B, vector boson. The free photon propagator is modified as

(1) $$\frac{1}{k^2} \longrightarrow \frac{1}{k^2} - \frac{1}{k^2 + m_B^2}$$

through which some of the divergence difficulties present in the usual QED are immediately removed. The mass m_B turns out to be the fundamental parameter of the theory and different attempts to estimate its value seem to indicate[1,2,3]

(2) $$9\ \mathrm{GeV} \lesssim m_B \lesssim 100\ \mathrm{GeV}.$$

The purpose of the present comunication consists in applying the Lee and Wick ideas to evaluate the branching ratios

$$R_{P \to l\bar{l}} \equiv \frac{P \to l\bar{l}}{P \to \gamma\gamma}$$

where P indicates a non-strange pseudoscalar meson and l stands for e^+ or μ^+. We show that m_B can be directly obtained from the experimental knowledge of one ratio R. In particular, from the measurement by Hyams et al.[4]

(3) $$R_{\eta \to \mu^+\mu^-} = \frac{\eta \to \mu^+\mu^-}{\eta \to \gamma\gamma} = (5.9 \pm 2.2)\, 10^{-5}$$

we deduce

(4) $$m_B = 32^{+40}_{-18} \text{ GeV}$$

and a prediction is then given for the ratio $R_{\pi^0 \to e^+e^-}$ which does not seem to escape the actual detection possibilities. Its measurement could provide a decisive test for the Lee and Wick theory.

The amplitude for the $P \to \gamma\gamma$ transition is given by

(5) $$A_{\gamma\gamma} = \frac{f_{P\gamma\gamma}}{M} \varepsilon_{\mu\nu\varrho\sigma} k_\mu k'_\nu \varepsilon_\varrho \varepsilon'_\sigma$$

where k, k' and $\varepsilon, \varepsilon'$ are, respectively, the four-momenta and the polarization vectors of the two photons, M stands for the P meson mass and makes the coupling constant $f_{P\gamma\gamma}$ adimensional. The invariant amplitude for the $P \to \mu^+\mu^-$ process, at the lowest order of e.m. interactions, is[5,6]

(6) $$A_{l\bar{l}} = \frac{e^2 f_{P\gamma\gamma}}{M} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{k'^2} \times$$
$$\times \varepsilon_{\mu\nu\varrho\sigma} k_\mu k'_\nu \bar{u}(q^-) \gamma_\varrho \frac{1}{\gamma(k-q^+)-m} \gamma_\sigma v(q^+)$$

and corresponds to the triangle Feynmann diagram shown in Fig. 1. Here $q^+(q^-)$ is the four-momentum of the final $l(\bar{l})$ and m its mass.

Making use of eq. (5) and after summation over the polarizations of the final state we get, in terms of $A_{l\bar{l}}$,

(7) $$R_{P \to l\bar{l}} = 32 \frac{|A_{l\bar{l}}|^2}{f^2_{P\gamma\gamma}} \frac{m^2}{M^2} \left(1 - \frac{4m^2}{M^2}\right)^{1/2}$$

which is independent of the coupling $f_{P\gamma\gamma}$ provided we assume that it is a constant or at least a very slowly varying function of k^2.

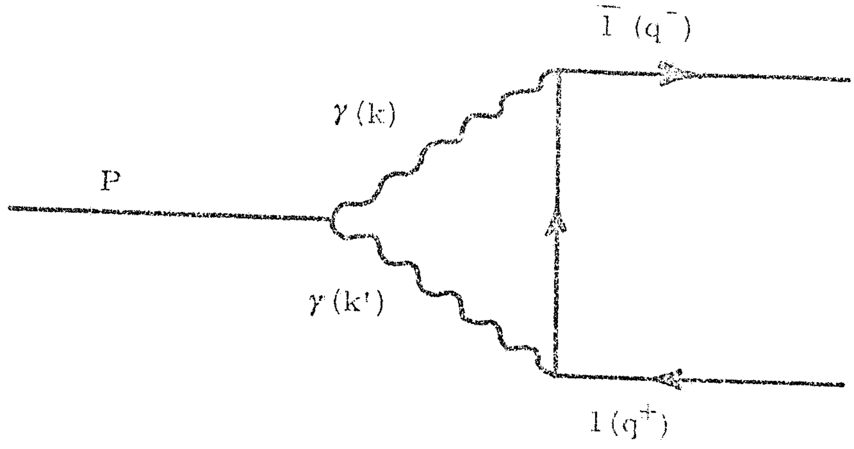

FIG. 1

The imaginary part of eq. (6) is well known[5, 6, 7]

$$\text{Im}\, A_{l\bar{l}} = \frac{\alpha}{8}\, f_{P\gamma\gamma}\, \frac{M}{(M^2-4m^2)^{1/2}}\, \ln \left| \frac{M+(M^2-4m^2)^{1/2}}{M-(M^2-4m^2)^{1/2}} \right| \qquad (8)$$

and provides the lower limit to R corresponding to retaining only the two-photon intermediate state in the unitarity sum for the $P \to l\bar{l}$ transition. Introduction into eqs. (7) and (8) of the masses of the η and μ particles[9] leads to the numerical result

$$R_{\eta \to \mu^+ \mu^-} \gtrsim 1.1 \times 10^{-5} \qquad (9)$$

and it can easily be shown[7] that the inclusion of other real intermediate states, such a 3π, $\pi^+\pi^-\gamma$, etc, imply a negligible modification to the above quoted result.

The real part of $A_{l\bar{l}}$, which according to the preceding discussion should account for the large difference between the experimental result (3) and the lower bound (9), diverges logarithmically if one assumes that it is simply given by eq. (6). Quigg and Jackson[10] have analyzed this problem in the framework of VMD. This implies the introduction in eq. (6) of one (or two) vector meson propagator (s), $(k^2+m_V^2)^{-1}$, which removes the divergence, and the replacement of the coupling constant $f_{P\gamma\gamma}$ by $(e\, m_V^2/f_V)\, f_{PV\gamma}$. The contribution of any single vector meson $V = \rho, \omega$ or φ turns out to be negligible due to the small value of m_V. The only way to get a result in reasonable agreement with eq. (3) implies a choice of values for the three coupling constants $f_{\eta V\gamma}$ which strongly violate SU(3) and clearly disagree with the estimates that one gets from the measured rates[9] for $\omega \to \eta\gamma$, $\varphi \to \eta\gamma$ and $\eta \to \rho^0\gamma \to \pi^+\pi^-\gamma$ decays[11]. We therefore conclude that VMD, at least in its usual form, is far from explaining the above mentioned discrepancy.

The theory of Lee and Wick, on the other hand, provides us with an attractive and very immediate way to evaluate $\mathrm{Re}\, A_{l\bar{l}}$. The two photon propagators appearing in eq. (6) must be replaced according to (1) leading, after the use of well known techniques, to the finite expression

$$\mathrm{Re}\, A_{l\bar{l}} = \frac{\alpha}{8\pi} f_{P\gamma\gamma} \int_0^1 x\,dx \int_0^{1-x} (1-x-y)\,dy \int_0^{m_B^2} dt \int_0^{m_B^2} du \times \tag{10}$$
$$\times \left\{ \frac{2(M^2-4m^2)\,y^2}{\left[m^2y^2+ux+(t-xM^2)(1-x-y)\right]^3} - \frac{6}{\left[m^2y^2+ux+(t-xM^2)(1-x-y)\right]} \right\}$$

The integrals of this last equation are easily evaluated when the mass ratios m/M and M/m_B both tend to zero, as it is the case when the final leptons are an e^+e^- pair. One obtains

$$\mathrm{Re}\, A_{e^+e^-} = \frac{\alpha}{8\pi} f_{P\gamma\gamma}\left(6 \ln \frac{m m_B}{M^2} + \frac{\pi^2}{6} + 4 + \frac{1}{4}\right) \tag{11}$$

Note however that such approximation is too crude in the $\eta \to \mu^+\mu^-$ case and a numerical integration, for some parts of the expression, has to be performed. One gets

(12) $$\mathrm{Re}\,A_{\eta \to \mu^+\mu^-} = \frac{\alpha}{8\pi} f_{\eta\gamma\gamma} \left(6 \ln \frac{m_B}{M_\eta} + 1.4\right)$$

The final value for m_B quoted in eq. (4) in then obtained introducing eqs. (8) and (12) into eq. (7) and making use of the experimental result (3).

It is worthwhile to observe that our result is in the expected range of values (2) and follows from the theory in a rather model independent way. Unfortunately, Re $A_{l\bar{l}}$ has a logarithmic dependence on m_B implying a large error on the determined value of this mass which will be hard to reduce considerably even if more accurate data on $R_{\eta \to \mu^+\mu^-}$ were available.

An interesting check of all these ideas will be possible when a second ratio of the type $R_{P \to l\bar{l}}$ will be measured. The $\pi^0 \to e^+e^-$ transition seems to be the most adequate one for such a purpose. In fact, making use of the value of m_B given in eq. (4) and introducing eqs. (8) and (11) into expression (7) one obtains

(13) $$R_{\pi^0 \to e^+e^-} = (22 \pm 4)\, 10^{-8}$$

which does not seem to be prohibitively low for the present detection techniques. The corresponding unitarity lower limit, $R_{\pi^0 \to e^+e^-} \geqslant 4.8 \times 10^{-8}$, indicates that the main contribution to the value (13) comes from Re $A_{e^+e^-}$ which crucially depends on the validity of the model. In this sense it is also instructive to compute the same ratio under the assumption that the removal of the divergences present in the real part of eq. (6) is achieved by means of a prescription different from (1). If, for instance, we introduce into eq. (6) the expression proposed by Berman and Geffen[6], dependent of the cut-off Λ, the experimental data (3) can be used to estimate $\Lambda \simeq$ 13 GeV and, finally, one obtains $R_{\pi^o \to e^+e^-} \simeq$ $\simeq 5 \times 10^{-8}$, to be compared with the result quoted in eq. (13).

Our last comment concerns the $K_L^o \to \mu^+ \mu^-$ decay. If one neglects the effects due to the weak interactions and, quite naivelу, assumes that this case can be treated along the same lines, one obtains

$$\text{(14)} \qquad \mathrm{Re}\, A_{K_L^o \to \mu^+\mu^-} = \frac{\alpha}{8\pi} f_{K^o\gamma\gamma} \left(6 \ln \frac{m_B}{M_{K^o}} + 1.7\right)$$

With $m_B = 32$ GeV the contribution of $\mathrm{Re}\, A_{K_L^o \to \mu^+\mu^-}$ to $R_{K_L^o \to \mu^+\mu^-}$ is found to be 7×10^{-5} and turns out to be a factor $4.5 \div 6$ higher than the contribution coming from the absorptive part[8]. In this case, the well known difficulties associated with the $K_L^o \to \mu^+\mu^-$ will also be extended to the real part of the amplitude and, therefore, considerably enlarged.

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