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HADRONIC CONTRIBUTIONS TO THE MUON ANOMALOUS MAGNETIC MOMENT

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The hadronic contribution to the muon anomalous magnetic moment is calculated using recent results from colliding beam experiments; a value of $(68 \pm 9) \times 10^{-9}$ is obtained.

The importance of a precise measurement of the g-factor of the muon and its comparison with theory is well known [e.g. 1]. A recently proposed high precision experiment on this subject is now under progress [2]. To meet the challenge posed by these refined measurements theoretical effort is now bent towards calculating pure QED corrections from higher and higher order diagrams. A recent calculation by Lautrup [3] of a set of diagrams of the 8th order gives a correction of about 5×10^{-9} indicating clearly that an evaluation of the hadronic contribution $a_\mu(\mathrm{H})$ to the muon anomaly to a similar degree of accuracy is indeed required. Over the years this hadronic correction has been calculated by several authors [4,5] with increasing degree of reliability reflecting the progress made in colliding beam experiments. The most recent and accurate calculation by Gourdin and de Rafael [5] which gives

$$a_\mu(\mathrm{H}) = (65 \pm 5) \times 10^{-9} \qquad (1)$$

was performed when the first series of data from Orsay became available. Since then however several now features, which could substantially change the above result, have been reported. For instance the data from Orsay have on the whole undergone some significant changes since they were first reported [6]. Secondly the two pion production mode indicates a large ρ - ω interference [7] and thirdly the hadron production cross sections at the Frascati [8] and Novosibirsk [9] energies and that of Orsay [6] at 0.99 GeV are very large, an indication that sizeable contributions to $a_\mu(\mathrm{H})$ from these large cross sections are to be expected.

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In this letter we recalculate the hadronic contribution $a_\mu(\mathrm{H})$ to the muon g-factor on the basis of our present knowledge of recent results from colliding beam experiments. The method of calculation of $a_\mu(\mathrm{H})$ is fairly well known; a given hadronic final state f contributes an amount $a_\mu(\mathrm{f})$ given by

$$a_\mu(\mathrm{f}) = \frac{1}{4\pi^2\alpha} \int_{s_{\mathrm{th}}}^{\infty} \mathrm{d}s\, K(s)\, \sigma_{\mathrm{e^+e^-} \to \mathrm{f}}(s) \qquad (2)$$

where $\sigma_{\mathrm{e^+e^-}\to \mathrm{f}}(s)$ is the total cross section for $\mathrm{e^+e^-}$ annihilation into f and s_{th} the threshold of the process. The function $K(s)$ is defined by

$$K(s) = \int_0^1 \mathrm{d}x \frac{x^2(1-x)}{x^2 + (s/m_\mu^2)(1-x)} \qquad (3)$$

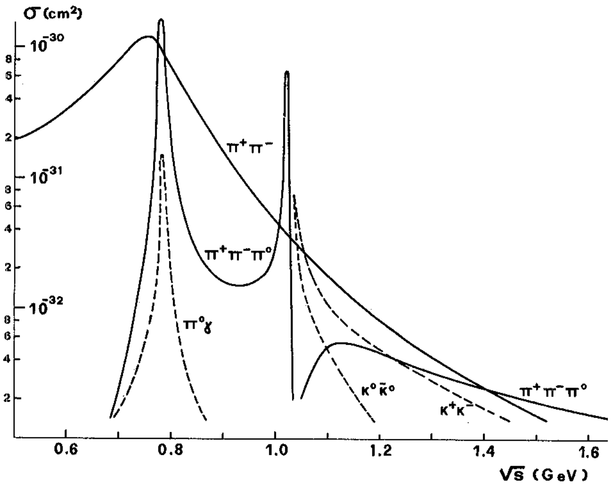

Fig. 1.

Table 1

f	$a_\mu(\mathrm{f}) \times 10^9$
$\pi^+\pi^-$	49.4 ± 5.7
K^+K^-	2.2 ± 0.2
K^oK^o	1.5 ± 0.2
$\pi^+\pi^-\pi^o$	6.5 ± 0.9
$\pi^o\gamma, \eta\gamma$	0.5 ± 0.1
$\pi^+\pi^-\pi^o\pi^o$	0.9 ± 0.2
ρ', ω', φ'	5.3 ± 1.5
$s > 4\ \mathrm{GeV}^2$	~1.5
Total	68 ± 9

where m_μ is the muon mass. The explicit form of $K(s)$ is known both for $s \leqslant 4m_\mu^2$ and $s \geqslant 4m_\mu^2$ [10].

The calculation of a_μ(H) by Gourdin and de Rafael [5] from the above set of equations consists in splitting it up into an isoscalar and isovector contributions. The isoscalar part $a_\mu(I=0)$ is calculated taking into account the ω and ϕ contributions in the narrow width approximation. The isovector part $a_\mu(I=1)$ on the other hand has been calculated assuming that the $I = 1$ hadronic system is dominated by the $\pi\pi$ P-wave and using the expression proposed by Gounaris and Sakurai for the pion form factor.

We have carried out our calculation first in this approximation and found that the overall value of a_μ (H) has decreased by about 10% with respect to eq. (1). As an improvement of the above scheme we have considered the important hadronic final states individually and evaluated their contributions from eq. (2). The cross sections for these final states are plotted in fig. 1 and in table 1 we give their contributions to a_μ (H) obtained in the manner discussed below.

a) $\pi^+\pi^-$: For this final state we used the pion form factor given by

$$F_\pi(s) = \frac{M_\rho^2(1+\delta\Gamma_\rho/M_\rho)}{M_\rho^2 - s - \mathrm{i}\,(M_\rho^2\Gamma_\rho/\sqrt{s})(P/P_o)^3} + \epsilon\,\exp(\mathrm{i}\phi)M_\omega^2/(M_\omega^2 - s - \mathrm{i}M_\omega\Gamma_\omega) \tag{4}$$

which includes ρ - ω mixing and fits the Orsay data with the following choice of parameters [7]

$$M_\rho = (778.5 \pm 6.2)\ \mathrm{MeV}; \qquad M_\omega = 784\ \mathrm{MeV}$$

$$\Gamma_\rho = (160.7 \pm 16.4)\ \mathrm{MeV}; \qquad \Gamma_\omega = (9.2 \pm 1.0\ \mathrm{MeV}$$

$$\delta = 0.83 \pm 0.34; \qquad \phi = (83.6 \pm 13.4)^{\mathrm{o}}$$

$$\epsilon = (1.89 \pm 0.5) \times 10^{-2} \tag{5}$$

$$P = (\tfrac{1}{4}s - m_\pi^2)^{1/2}; \qquad P_\mathrm{o} = (\tfrac{1}{4}M_\rho^2 - m_\pi^2)^{1/2}.$$

Experimentally this fit extends only up to $\sqrt{s}$ = 0.99 GeV but we have extrapolated it to $\sqrt{s}$ = 2 GeV and find in the range $4m_\pi^2 \leqslant s \leqslant 1$ a contribution to a_μ(H) of $(48.7 \pm 5.7) \times 10^{-9}$ and in the range $1 \leqslant s \leqslant 4$, 0.7×10^{-9}. The extrapolated cross section is a little bit more than an overall factor of 2 below the Frascati data [8]. Note however that in the Frascati data no attempt was made to separate $K^\pm$ from $\pi^\pm$.

b) $K\overline{K}$: The kaon form factor is given by

$$F_K(s) = \sum_{V=\rho,\omega,\phi} \frac{g_{VK\overline{K}}}{f_V} \frac{M_V^2}{M_V^2 - s - \mathrm{i}M_V\Gamma_V} \tag{6}$$

where using SU(3) one gets

$$F_K(s) = \frac{g_{\phi K\overline{K}}}{f_\phi}\left[\frac{\frac{3}{2}M_\rho^2}{M_\rho^2 - s} \pm \frac{\frac{1}{2}M_\omega^2}{M_\omega^2 - s} \pm \frac{M_\phi^2}{M_\phi^2 - s - \mathrm{i}\,M_\phi\Gamma_\phi}\right] \tag{7}$$

with the upper sign giving the K^+K^- form factor and the lower sign that for $K^o\overline{K^o}$. From the experimental value [11] of the cross section $\sigma(e^+e^- \to K^+K^-) = (2.27 \pm 0.15)\ \mu$b at the ϕ peak one deduces the value of the coupling constant $(g_{\phi K^+K^-}/f_\phi)^2 = 0.11$. The coupling constant $(g_{\phi K^o\overline{K^o}}/f_\phi)$ is determined from a knowledge of the branching ratio $(\phi \to K^+K^-)/(\phi \to K^o\overline{K^o})$ = 1.31 ± 0.17 [12]. We have used the values m_ϕ = 1020 MeV and Γ_ϕ = 4 MeV.

c) $\pi^+\pi^-\pi^o$: For this final state we have added coherently the ϕ and ω contributions and fit the experimental cross sections [6] $\sigma_{3\pi}(M_\omega^2) = (1.75 \pm 0.20)\ \mu$b and $\sigma_{3\pi}(M_\phi^2) = (0.66 \pm 0.10)\ \mu$b at the ω and ϕ peaks respectively. The interference is assumed constructive for $M_\omega^2 \leqslant s \leqslant M_\phi^2$ as suggested by the Orsay data point at 0.99 GeV. This choice in the relative sign of the ω and ϕ couplings gives rise to a dip in the cross section immediately after the ϕ mass as indicated in fig. 1. The value of a_μ (H) is however insensitive to this choice.

d) $\pi^o\gamma$: As in the 3π final state we have summed the ρ and ω contributions coherently and deduced the cross section $\sigma_{\pi^o\gamma}$ from the known branching ratio $(\omega \to \pi^o\gamma)/(\omega \to 3\pi) = 0.103 \pm 0.014$ [12]. The contribution of the $\eta\gamma$ final state can also be estimated from the branching ratio $(\phi \to \eta\gamma)/(\phi \to \mathrm{all})$ [6,13] with the result 0.2×10^{-9}.

e) $\omega\,\pi^o$: Our calculation of the cross section for this model is similar to that of Layssac and Renard [14] and agrees with the Orsay data point at 0.99 GeV.

The final states considered above give a satisfactory picture of the experimental situation in colliding beam production of hadrons up to an energy of about 1.2 GeV. Above this energy a large multi-hadron cross section of the order of about 60 nb, much larger than the tail of the cross section from the region $\sqrt{s} < 1.2$ GeV has been reported from ADONE [8]. Although the experimental situation at present is not completely settled, the four charged pion mode shows evidence [15] of a bump structure at about 1.6 GeV which can be explained with the ρ' meson [16]. Using the fit parameters $M_{\rho'} = 1.6$ GeV, $\Gamma_{\rho'} = 350$ MeV and the cross section [15] $\sigma_{2(\pi^+\pi^-)}(M^2_{\rho'}) \approx 25$ nb one gets $a_\mu(2(\pi^+\pi^-)) \approx 1.4 \times 10^{-9}$. The branching ratio of $\rho' \to 2(\pi^+\pi^-)$ to ($\rho' \to$ all) has been estimated [16] to be about $\frac{1}{3}$; using this one gets $a_\mu(\rho') = 4.0 \times 10^{-9}$ which is comparable with the ϕ contribution. From the knowledge of $a_\mu(\rho')$ it is possible to estimate the contributions of the $I = 0$ SU(3) partners of ρ'; one finds approximately 1.3×10^{-9}.

Finally we estimate the contribution above 2 GeV using the cross section

$$\sigma_{tot}(e^+e^- \to \text{hadrons}) \approx \sigma(e^+e^- \to \mu^+\mu^-) \qquad (8)$$

which is theoretically supported by parton-like models and find 1.5×10^{-9}.

The total hadronic contribution to the muon g-factor is

$$a_\mu(\text{H}) = (68 \pm 9) \times 10^{-9} \qquad (9)$$

where the error only reflects the uncertainties in the experimental cross sections. From a comparison of eqs. (1) and (9) one finds that, numerically our value of $a_\mu(\text{H})$ agrees with that of Gourdin and de Rafael [5]. It should be noted however that theoretically there has been an improvement due to the inclusion of threshold effects and new experimental information over a wider range of energies.

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