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Scattering of $n(>2)$ Incident Particles (*).

G. PANCHERI-SRIVASTAVA and Y. SRIVASTAVA (**)

Istituto di Fisica Nucleare dell'Università - Roma
Laboratori Nazionali del CNEN - Frascati

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It is customary to consider the scattering of two incident particles into $N(\geqslant 2)$ final particles. This is but natural since an experimentalist does *only* such an experiment (*i.e.* a beam impinging upon a target). With the advent of colliding beams, one may seriously raise the question about three (incident)-particle scattering, for instance, by having the two beams simultaneously impinge upon a target.

In this note we shall consider the general case of an arbitrary number of incident particles. We develop some simple formalism for the transition rates etc. and towards the end we speculate upon the feasibility of these experiments. For electron machines our estimates indicate that the counting rates are too low, however, for hadronic machines the prospect may not be entirely out of the question if some recent proposals to upgrade the present luminosities materialize.

In the process of obtaining a formula for the transition rate $\mathrm{d}N/\mathrm{d}t$, we shall find that for n_i incident-particle scattering, the invariant quantity has the dimension (of length) $L = 3n_i - 4$. Thus, for a single-particle decay $L = -1$, the « width » of the particle, for 2-particle scattering $L = 2$, the « cross-section », and for 3 incident particles $L = 5$ and so on.

Let us start by considering the S-matrix element for the scattering of $n_i \to n_f$

$$\text{(1)} \qquad \begin{cases} p_1 + p_2 + \dots + p_{ni} \to q_1 + q_2 + \dots + q_{nf}\,, \\ S_{fi} = \delta_{fi} + i(2\pi)^4\,\delta^4(Q_f - P_i)\,\dfrac{T_{fi}(\{q_f\}, \{p_i\})}{\sqrt{V^{n_i+n_f}\prod\limits_{j=1}^{n_i}\varrho(p_j)\prod\limits_{l=1}^{n_f}\varrho(q_l)}}\,, \end{cases}$$

where we have normalized the particles in a box of volume V (1). Here, $\varrho(p) = E/m$ for fermions and $= 2E$ for bosons. The unitarity condition $SS^+ = 1$ yields for the « for-

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(**) On sabbatical leave from Northeastern University, Boston, Mass.
(1) This turns out to be convenient, since in the end its absence bolsters our belief in the correctness of our formulae.

ward » amplitude $T_{ii}(p_i = q_i,\ i = 1, ..., n_i)$

$$\mathrm{Im}\, T_{ii} = \tfrac{1}{2} \sum_n \int \Omega_n (2\pi)^4 \,\delta^4(Q_n - P_i) |T_{in}|^2 \,, \tag{2}$$

where

$$\Omega_n = \prod_{l=1}^{n} \left[\frac{\mathrm{d}^3 q_1}{(2\pi)^3\, \varrho(q_1)} \right].$$

The transition/4-volume from state $i \to f$ is

$$\frac{|S_{fi}|^2}{VT} = (2\pi)^4\, \delta^4(Q_f - P_i) \frac{1}{V^{n_i+n_f}} \frac{1}{\prod\limits_{j=1}^{n_i} \varrho(p_j) \prod\limits_{l=1}^{n_f} \varrho(q_l)} |T_{fi}|^2 \,. \tag{3}$$

The number of events of type $i \to f$, N_{if}, proceeds at the rate

$$\frac{\mathrm{d}N_{i\to f}}{\mathrm{d}t} = \frac{1}{V^{n_i - 1}} \int \Omega_f (2\pi)^4\, \delta^4(Q_f - P_i) \frac{|T_{fi}|^2}{\prod\limits_{j=1}^{n_i} \varrho(p_j)} \,. \tag{4'}$$

The more general expression instead of (4′) is

$$\frac{\mathrm{d}N(i\to f)}{\mathrm{d}t} = \int (\mathrm{d}^3 \boldsymbol{x}) [R_1(\boldsymbol{x}, t) \ldots R_{n_i}(\boldsymbol{x}, t)] \int [\Omega_f (2\pi)^4\, \delta^4(Q_f - P_i)] \frac{|T_{fi}|^2}{\prod\limits_{j=1}^{n_i} \varrho(p_j)} \,, \tag{4}$$

where $R_i(\boldsymbol{x}, t)$ is the *number of particles of type i per unit volume*.

The analogue of the 2-particle « optical theorem » is easily obtained using the « forward » unitarity condition (2) in conjunction with (4). The number of events/time for $i \to$ anything is given by

$$\frac{\mathrm{d}N\,(i\to \text{anything})}{\mathrm{d}t} = \int \mathrm{d}^3 \boldsymbol{x} \left[\prod_{j=1}^{n_i} R_j(\boldsymbol{x}, t) \right] \left[\frac{2\, \mathrm{Im}\, T_{ii}}{\prod\limits_{j=1}^{n_i} \varrho(p_j)} \right]. \tag{5}$$

Equations (4) and (5) are our basic formulae connecting the « differential » and « total » transition rates which an experimentalist would (hopefully!) measure, to the matrix elements of T. If we denote by $\mathscr{F}_{n_i}$ the incident flux, we may *define* the invariant quantity (²)

$$\mathrm{d}\Sigma_{fi} \equiv \frac{1}{\mathscr{F}_{n_i}} \Omega_f (2\pi)^4\, \delta^4(Q_f - P_f) \frac{|T_{fi}|^2}{\prod\limits_{j=1}^{n_i} \varrho(p_j)} \,. \tag{6}$$

From (1) and (6) it is clear that dim. (Σ_{fi}) is $L = 3n_i - 4$ and is *independent* of f (*i.e.* the number of particles in the final state) as it should be. As mentioned in the beginning, this recovers for us the known results for $n_i = 1$ and 2 (width and cross-section, respectively) and obtains for $n_i = 3$, an invariant of dimensionality 5.

(²) It is Lorentz invariant because $[\mathscr{F}_{n_i} \prod_{j=1} \varrho(p_j)]$ is an invariant and so is Σ_f.

From (5) and (6) we may also obtain the « total » Σ_i:

$$\Sigma_i(i \to \text{anything}) = \frac{1}{\mathscr{F}_{n_i}} \frac{2\,\mathrm{Im}\, T_{ii}}{\prod\limits_{j=1}^{n_i} \varrho(p_j)} . \tag{7}$$

For two-particle, boson-boson scattering (7) reduces to the well known formula [3]

$$\sigma_{12}^{\text{tot}}(S) = \frac{1}{\sqrt{\lambda(S_{12}, m^2, m_2^2)}} \,\mathrm{Im}\, T_{22}(p_1 p_2; p_1 p_2) .$$

(In the c.m. frame $\sqrt{\lambda} = 2q_{\text{c.m.}}\sqrt{S}$).

So far we have considered the general case. Let us now particularize to the interesting case of $n_i = 3$. According to (7), Σ_3 is proportional to the absorptive part of the 3-to-3 « forward » amplitude, which may be taken to be a function of the three invariants $S_{ij} = (p_i + p_j)^2$, $i, j = 1, 2, 3$. A possible experiment may be to imagine that two colliding beams (4-mom. p_2 and p_3) impinge upon a target (4-mom. p_1). It is straightforward to obtain the various Regge limits depending upon the momentum configurations. Here we only recorded the result for one case of particular interest for the colliding beam set ups where the two beams are roughly collinear and of almost equal energies ($E_2 \simeq E_3$, $p_2 \simeq -p_3$). For this case

$$\Sigma_3 \xrightarrow[E_2, E_3 \to \infty]{} \left(\frac{S_{12}}{S_0}\right)^{\alpha_{\mathrm{P}}(0)-1} \left(\frac{S_{23}}{S_0}\right)^{\alpha_{\mathrm{P}}(0)-1} \left(\frac{S_{31}}{S_0}\right)^{\alpha_{\mathrm{P}}(0)-1} G(p_\perp) , \tag{8}$$

where α_{P} is the pomeron [4]. Thus, if $\alpha_{\mathrm{P}}(0) = 1$, Σ_3 approaches a *limit* at infinite energies [5].

Apart from the totally inclusive experiments (*i.e.* measurement of Σ_3) we may also consider the partially inclusive (*i.e.* detection of a given number of final particles and sum over the rest) or the exclusive processes. The formulae for such may easily be obtained from (6).

The main trouble with 3 incident particles is, of course, with the semi-disconnected parts, *i.e.* where particles 1 and 2 interact, while the third particle goes undisturbed. For this reason, one may choose those final states which require all to participate [6]. At any rate the final state produced by real 3-particle scattering is, in general, kinematically different. We give a simple example for e^+e^- case. Let us have e^+ and e^- beams (of equal energy but oppositely directed) hit a proton at rest and suppose we observe and $\mathcal{N}^*$ resonance in the final state (which later decays into $\mathcal{N}$ and π). To lowest order in α, there are two types of scattering (see Fig. 1). If the incident energy of the $e^\pm$ beams were appropriately low, for a given $\mathcal{N}^*$ mass, the reaction proceeds only via Fig. 1 *b*). Even apart from these threshold type constraints, a careful recording of the $\mathcal{N}^*$ momentum (that is to say of its decay products $\mathcal{N}+\pi$) can resolve between processes 1 *a*) and 1 *b*): $\mathcal{N}^*$ is produced at rest in the process 1 *b*)) whereas in process 1 *a*) it is produced in flight in general. More elaborate tests can clearly be designed.

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(3) $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$.

(4) Equation (8) is, of course, for the strong process.

(5) This is analogous to the pionization limit of the single particle production initiated by 2 particles, *i.e.*, $a+b \to c+x$. See, for instance, A. MUELLER: *Phys. Rev. D*, **2**, 2963 (1970).

(6) A trivial example is provided by $p+p+p \to \mathcal{N}^{*++}+d$.

Regarding the calculatoin of the rate for the process 1 *b*), one may use time-reversal invariance to connect this matrix element to that of the decay of $\mathcal{N}^* \to p + e^+ + e^-$ (7). With the present luminosity of Adone (Frascati) e^+e^- machine, and a liquid-hydrogen target, the observation of this process is completely out of the question, since the # of events $\approx 10^{-17}$/s.

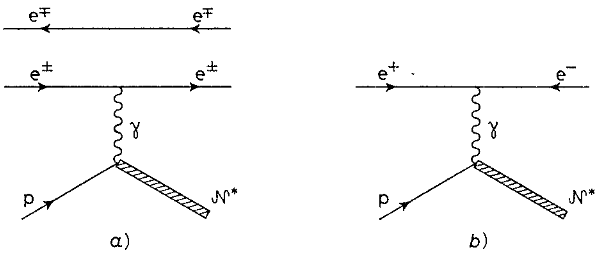

Fig. 1. – *a*) Semi-disconnected diagram. Here the photon is space like. *b*) True 3-body scattering. Photon is timelike.

For CERN ISR, if one considers the two colliding beam protons to interact with a proton at rest (provided by a liquid-hydrogen target) then the rate for 3-protons→ → anything, is $\approx 10^{-14}$ events/s. (A factor of 10^2 or 10^3 may be gained by using a denser target). We have obtained this number using the present IRS luminosity $L \approx 2 \cdot 10^{28}/\text{cm}^2 \cdot \text{s}$ (8). There have been proposals (9) to upgrade this number to $\approx 10^{36}/\text{cm}^2 \cdot \text{s}$ for $E = 25$ GeV and as much as $\approx 3.3 \cdot 10^{38}/\text{cm}^2 \cdot \text{s}$ at $E = 250$ GeV. If these light luminosities can be attained, then our estimate leads us to hope that out the tremendous background provided by the semi-disconnected (2-particle) scatterings.

It should be obvious that were experiments of the type proposed here to be performed, a completely new and rich field of study shall be developed. We are confident that the ingenuity of the experimentalists shall allow for these measurements in the near future.

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(7) In principle, processes of this type can provide for direct tests of *T* invariance. For instance, comparison of the rates for $n + p \to d + e^+ + e^-$ and the inverse process $e^+ + e^- + d \to n + p$, allow one to test *T* invariance for off-shell photon as well. We mention this example since recently in the literature conflicting experimental results regarding *T* invariance (for physical photons) have been presented through comparison of rates for $\gamma + d \to p + n$ and $p + n \to \gamma + d$. See, D. BARTLETT, C. FRIEDBERG, K. GOULIANOS, I. HAMMERMAN and D. HUTCINSON: *Phys. Rev. Lett.*, **23**, 893 (1969); B. SCHROCK, R. HADDOCK, J. HELLAND, M. LONGON, S. WILSON, K. YOUNG, D. CHENG and V. PEREZ-MENDEZ: *Phys. Rev. Lett.*, **26**, 1659 (1971). Moreover, A. SANDA and G. SHAW: *Phys. Rev. Lett.*, **26**, 1057 (1971), assert that *T*-violation in EM processes is due to an isotensor part of the EM current. Again, this can, in principle, be directly checked.

(8) L. RATNER, R. ELLIS, G. VANNINI, B. BABCOCK, A. KRISCH and J. ROBERTS: *Phys. Rev. Lett.*, **27**, 68 (1971).

(9) E. KEIL and A. SESSLER: *Proceedings of the VI International Conference on High-Energy Accelerators* (1967).