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A. Jurewicz : PHOTOPRODUCTION OF PIONS ON NUCLEONS IN THE Δ(1236) RESONANCE REGION.

Lectures on Past and Present Problems.

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PHOTOPRODUCTION OF PIONS ON NUCLEONS IN THE $\Delta(1236)$ RESONANCE REGION

Lectures on Past and Present Problems

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INTRODUCTORY NOTE. -

This text is an expanded English version of the lectures which under the title "Problemi nella fotoproduzione dei pioni alla prima risonanza" were held at Frascati in June, 1971. The lectures were addressed to experimentalists. Their main object was to present, in a comprehensive way, the key points of the theoretical description of pion photoproduction off nucleons in the $\Delta(1236)$ energy region.

The didactic character of the lectures is evident. The author tried not to formalize excessively the presentation but rather to point out some direct consequences of applying a given theoretical model to interpretation of data.

The lectures deal with essentially two problems. The first is determination of the multipoles $M_{1+}^{(3)}$, $E_{1+}^{(3)}$ in the resonance region, which is one of the old classical problems in pion photoproduction. The second is a more recent development, and namely the problem of presence of the isotensor electromagnetic current in photoproduction. This problem raised recently some interest and controversy.

1. - GENERAL REMARKS. -

At the outset it seems essential to ask ourselves what is the real meaning of such statements as : "The theory of photoproduction states this and that", or "From the theory result such and such predictions", and so on. In other words we ask what is the current content of the term "photoproduction theory". We ask what are the foundations and internal consistence of such "theory" and what can we say about its predictive power.

At this point some comment on the term "theory" seems appropriate. The name of "theory" in high energy physics is given to various branches of study which differ substantially in reliability of assumptions, internal consistence and predictive power. Here, we would like to confine the use of the term "theory" to such structures as Quantum Electrodynamics (or, Theory of photons and electrons) which forms a rather self consistent system, deduced from some sound first principles, and being able to give measurable predictions of remarkable quantitative accuracy (e.g. the Lamb shift).

One can hardly say that any theoretical approach to photoproduction forms a system of comparable consistency. What we are actually dealing with are various mixtures of heuristic arguments, deductive reasoning and phenomenology. At the best we could speak about "models", that is, systems essentially based on deduction from some first principles, but leaving undefined some important parts of the system, which then have to be filled with some phenomenological or experimental information. Such an approach lies somewhere between a pure fit to experimental data on one side, and a "theory" in the above mentioned sense on the other. It is known that in particle physics this kind of theoretical description became in the last years one of the most effective tools.

The description of pion photoproduction off nucleons in the threshold and resonance region is usually given in terms of one of the following models :

a) dispersion relations(1-13);
b) isobar model(14);
c) quark model(15-17);
d) current algebra(18).

In these lectures the interest will be fixed almost exclusively on dispersion relations. The first reason for that is simply the author's preference, and the second is that the dispersion model of photoproduction is at present probably the most elaborated description of this reaction at least in a few hundreds MeV energy region.

In the framework of dispersion models we shall further confine ourselves to problems arising in calculating the "resonant" multipoles $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ in the $\Delta(1236)$ region. For obvious reasons the $M_{1+}^{(3)}$ multipole received much attention since the early days of theoretical approach to photoproduction and its accurate determination, together with that of $E_{1+}^{(3)}$ became a problem in itself. This situation has made the case of these two amplitudes particularly suitable for presentation of the details underlying "theoretical predictions" in photoproduction.

In our discussion we will not go into the foundations and generalities of the dispersion approach to photoproduction. These can be found elsewhere, in particular in the detailed Frascati lectures by Weaver[19]. We recall only that the postulate of analyticity of reaction amplitudes, on which (among others) the dispersion approach is based, has never been proved in general, being therefore a mere assumption.

2. - THE DISPERSION EQUATION. -

We recall now the most essential points of the notation. In the centre of mass system of the $\gamma + N \to \pi + N$ channel (s-channel), s is the total energy squared, t is the four-momentum transfer between bosons, $\vec{p}$ is the final three-momentum, m and μ denote nucleon and pion masses, respectively. Adapted will be a convention[12], according to which $\mathcal{M}_j^{(\alpha)}$ denotes a multipole amplitude (possibly multiplied by an appropriate kinematic factor) with isospin index $\alpha = 0, 1, 3$ for isoscalar, isovector $I = 1/2$, isovector $I = 3/2$ transitions, respectively and $j = 4l \pm (-1)^l$ ($j = 4l - 1 \pm (-1)^l$) for electric (magnetic) multipoles of total angular momentum $l \pm 1/2$ and parity $(-1)^l$. Standard multipole amplitudes[1] will be denoted $E_{l\pm}^{(\alpha)}$, $M_{l\pm}^{(\alpha)}$.

Our starting point will be an integral expression for $\mathcal{M}_j^{(\alpha)}$, which stands at the beginning of any effective dispersion calculation. This expression has a general form:

$$
\begin{aligned}
\mathrm{Re}\,\mathcal{M}_j^{(\alpha)}(v) = B_j^{(\alpha)}(v) + \frac{1}{\pi} P \int_{v_o}^{\infty} \frac{\mathrm{Im}\,\mathcal{M}_j^{(\alpha)}(v')}{v' - v}\, dv' \; + \\
+ \sum_{\beta, k} \int_{v_o}^{\infty} K_{jk}^{(\alpha\beta)}(v', v)\, \mathrm{Im}\,\mathcal{M}_k^{(\beta)}(v')\, dv'
\end{aligned}
\tag{2.1}
$$

where v denotes a variable depending on s (e.g.: $W = \sqrt{s}$[2, 7], $\omega = \sqrt{s} - M$[5, 6], p^2[12, 13] etc.) and v_o is its threshold value. Multipole projections of the pole terms (so called Born-terms) are denoted $B_j^{(\alpha)}(v)$. The symbol $K_{jk}^{(\alpha\beta)}(v', v)$ denotes some known, nonsingular functions (kernels) which couple a given multipole $\mathcal{M}_j^{(\alpha)}(v)$ to all the others $\mathcal{M}_k^{(\beta)}(v')$ (including self-coupling) as a consequence of the crossing property of photoproduction amplitudes. The summation over j is infinite with the consequence that formula (2.1) actually represents an infinite system of coupled expressions for $\mathcal{M}_j^{(\alpha)}$. The system (2.1) is usually obtained by performing multipole projections (which means integration over the physical range of the c.m. scattering angle $-1 \leq \cos\theta \leq 1$ with specific weights) of dispersion relations for invariant photoproduction amplitudes[2]. To this aim most commonly have been used "fixed-t dispersion relations", that is integral expressions which exploit analyticity properties of the invariant amplitudes in v along the lines t = const in the complex s, t space[1-10]. Formula (2.1) may also be derived from "fixed angle dispersion relations"[12, 13] which differ from the preceding ones in that the lines $\cos\theta$ = const replace those with t = const. The coupling functions $K_{jk}^{(\alpha\beta)}(v', v)$ are of course in both approaches different.

While performing the multipole projection of fixed-t dispersion relations we encounter the problem of convergence of the multipole expansion in the integrand of the dispersion relation for invariant amplitudes[1]. The point is, that for a given fixed value of t and for v' sufficiently small, $\cos\theta$ goes beyond its physical values and it may happen that the discussed multipole expansion become divergent. There is only one value of t, namely $t = -m\mu^2/(m + \mu)$ for which the scattering angle never goes beyond the physical range $-1 \leq \cos\theta \leq 1$. There were suggestions[20] that just this value should be used to obtain "correct" dispersion relations for photoproduction amplitudes. Integral expressions for multipoles could then be obtained by a differential technique[21]. The question was later studied by Höhler[22] who concluded that the problem of unphysical scattering angles is probably not of great importance. One should however note that this conclusion has a value rather of a practical information than of an exact proof.

Fixed angle dispersion relations[12, 13] were proposed just to avoid troubles with the above mentioned "unphysical angles". In fact no problem with convergence of the multipole series arise, but a price paid for this advantage is the necessity of partial omissions of contributions from higher energy regions in the crossed channel[23].

A direct comparison of the mentioned two types of dispersion relations has so far not been done.

Equation (2. 1) could in principle be something more than a starting point for model calculations. In the good old days of dispersion relations in particle physics (in the early '60s) there were hopes that a complete theory of elementary processes (at least in the elastic region) is imminent. In the particular case of photoproduction the reasoning used to be represented as follows : If we confine ourselves to $\pi\pi$ and πN intermediate states (that is, if we neglect all inelastic processes), the imaginary part of the $\gamma + N \rightarrow \pi + N$ amplitude is connected, through unitarity, with the amplitude of pion nucleon scattering[x] $\pi + N \rightarrow \pi + N$. The imaginary part of the last amplitude is in turn connected with the amplitude of elastic pion-pion scattering $\pi + \pi \rightarrow \pi + \pi$. Starting from the $\pi\pi$ problem we could solve, step by step, the equations for all other amplitudes within the general framework of dispersion relations.

We all know that such a program has never been realized so far, and that even the $\pi\pi$ problem has not been solved to the end.

One may ask therefore why dispersion relations proved to be quite successful in the study of photoproduction? It seems that the reason is essentially twofold - mathematical and phenomenological.

As to mathematics - the point is that integral equations for photoproduction amplitudes are linear, in contradiction to quadratic equations for elastic (e. g. $\pi\pi$ or πN) amplitudes. Linearity of dispersion equations is a common feature of two-body inelastic processes. In virtue of unitarity the imaginary part of the amplitude lineraly depends on the amplitude itself (for elastic processes this dependence is quadratic). This linearity enables us to use standard techniques developed by mathematicians to solve linear integral equations (also singular). Such techniques, generally speaking, do not exist in the case of non-linear eqautions in which case complications dramatically grow. In consequence, the apparently simpler problem of elastic $\pi\pi$ scattering is in fact more complicated than that of photoproduction.

The second, phenomenological reason is the following. Once we gave up to a complete dispersion theory of photoproduction, and we confined ourselves to less ambitious model description we have at our disposal an excellent dynamical information to feed the model. The information is given by the pion-nucleon phase shifts which may be (with some objections, to be discussed later) sufficient to make the dispersion equations calculable. There are of course specific problems arising when pion-nucleon phase shifts (resulting from fits) are practically used in solving photoproduction equations, but it will be more convenient to discuss these problems in connection with a given multipole (partial wave) and with a particular technique of finding the solution of equation (2. 1).

We conclude therefore that our model description of photoproduction will be based on dispersion equations for photoproduction amplitudes, fed with pion-nucleon phase shifts as taken from fitting procedures.

In order to make this model effective one should add the assumption that "the unitarity equation is saturated by πN states". Equivalently one may say that many-particle states and

(x) - More precisely this happens if electromagnetic interactions are included in the first order of e which is the usual approximation made in the study of hadronic interactions induced by photons.

other inelastic effects are either negligible or affect the final result through the inelasticity parameter of $\pi N \to \pi N$ amplitudes. A common but more restrictive expression of this idea is given by the well known Watson theorem[24] which states that in the purely elastic region, i. e. below the $\gamma + N \to 2\pi + N$ threshold holds the relation

$$\mathcal{M}_j^{(\alpha)} = \pm \left| \mathcal{M}_j^{(\alpha)} \right| e^{i\delta_j^{(\alpha)}} \tag{2.2}$$

or alternatively that

$$\operatorname{Im} \mathcal{M}_j^{(\alpha)} = \operatorname{Re} \mathcal{M}_j^{(\alpha)} \operatorname{tg} \delta_j^{(\alpha)} \tag{2.3}$$

where $\delta_j^{(\alpha)}$ is the appropriate (real !) phase shift of πN scattering. Whatever its formulation, the above assumption enables us to transform (and on its effective application we shall comment later) formula (2. 1) into :

$$\begin{aligned} \operatorname{Re} \mathcal{M}_j^{(\alpha)}(v) = {} & B_j^{(\alpha)}(v) + \frac{1}{\pi} P \int_{v_o}^{\infty} \frac{dv'}{v' - v} e^{-i\,\delta_j^{(\alpha)}(v')} \sin \delta_j^{(\alpha)}(v') \mathcal{M}_j^{(\alpha)}(v') + \\ & + \sum_{k,\beta} \int_{v_o}^{\infty} dv' K_{jk}^{(\alpha\beta)}(v', v)\, e^{-i\,\delta_k^{(\beta)}(v')} \sin \delta_k^{(\beta)}(v') \mathcal{M}_k^{(\beta)}(v') , \end{aligned} \tag{2.4}$$

which is just the system of singular integral equations for multipoles. As its parent equation (2. 1), formula (2. 4) represents two (for isovector and isoscalar transitions, respectively), infinite (with respect to the multipole number j) systems. It is of course impossible to solve the system (2. 4) as a whole. Fortunately there is no need for such a complete solution since we know from fits to experimental data that, in the low energy region where (2. 2) is valid only few multipoles give non negligible contributions.

With a small number of equations (and the usual assumption[8, 9, 11] is that $l \leq 3$) we usually try further to simplify the system, by decoupling some equations from the others. Such a procedure would approximately be correct if some coupling kernels $K_{jk}^{(\alpha\beta)}(v', v)$ were much smaller than the others[(x)]. The functions $K_{jk}^{(\alpha\beta)}(v', v)$ resulting from fixed-t dispersion relations were studied under this respect by Schmidt[25] and by Schwela[8, 9]. For example in ref. (8) one finds a table showing the relative importance of various $K_{jk}^{(\alpha\beta)}(v', v)$ with $l \leq 3$. We do not enter into the details of that study and we observe only that the equations for $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ (on which our interest will be focused) to a good approximation decouple from the rest of the system and may be considered separately. This is a quantitative support to an intuitive assumption known from earlier dispersion calculations in photoproduction[4, 5].

The preceding discussion refers to coupling functions in the fixed-t approach. Analogous studies of kernels in the fixed-angle approach have not been so extensive. Preliminary calculations[26] seem to indicate that the $E_{1+}^{(3)}$ amplitude is coupled rather strongly to the $E_{0+}^{(1, 3)}$ multipoles[(o)]. The consequences of this situation have not been exploited so far and numerical results were obtained only for the isolated subsystem of "resonant" multipoles $M_{1+}^{(3)}$, $E_{1+}^{(3)}$ [13].

The present situation can therefore be summarized as follows : As a first step solved are the equations for multipoles $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ (whether they are assumed to be mutually coupled or not, depends on the adopted technique). In the following steps (which are beyond our present discussion) the calculated resonant multipoles are used as input to the remaining

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(x) - More precisely it is the interplay of the magnitude of $K_{jk}^{(\alpha\beta)}(v', v)$ and of the magnitude of the phase shift $\delta_j^{(\beta)}(v')$ (appearing in the imaginary part of $\mathcal{M}_j^{(\alpha)}(v')$) that is ultimately responsible for the strength of a given coupling.

(o) - We remark that just these couplings have also been taken into account in the fixed-t dispersion approach of ref. (11).

equations. These in turn are solved in some specific order depending on the assumed method which may be found in the relevant papers[8, 9, 11].

3. - THE (3, 3) MULTIPOLES AND THE STATIC MODEL. -

According to our general program we will be concerned only with the above mentioned (and probably the most important) first step, which most generally consists in solving a system of two coupled singular integral equations for $\mathcal{M}_j^{(\alpha)}$ (j=2, 3 α =3 in our notation) under the assumption that $\delta_{33} = \delta_2^{(3)} = \delta_3^{(3)}$ (in our notation) is a known function. We shall see later that the last assumption will require further comments.

It became almost customary to quote the work of Chew et al.[1] at the beginning of any theoretical paper on pion photoproduction. The present lectures preserve that custom and by no means for pure sake of tradition. The main points of CGLN paper did not lose their actuality and may serve as instructive examples of quantitatively valid results obtained by simple means.

Let us recall the essential lines of the CGLN procedure. The equations analogous to (2. 1) and based on fixed-t dispersion relations are split into equations for "electric" and "magnetic" parts of the multipoles. This procedure is based on a natural splitting of the inhomogeneous (Born) terms $B_j^{(\alpha)}$ into magnetic and electric parts $B_{j,\mu}^{(\alpha)}$ and $B_{j,e}^{(\alpha)}$ (i. e. terms associated with magnetic moments and electric charge, respectively[1,27]). The resulting equations should then be solved for $\mathcal{M}_{j,\mu}^{(\alpha)}$ and $\mathcal{M}_{j,e}^{(\alpha)}$ which added together form the required solution[x]. The actual calculations of CGLN are performed approximately, namely by keeping only the $l = 0$ and $l = 1$ multipoles (which is a reasonable truncation in the few hundreds MeV energy region) and in the static limit. In our notation this means that $B_j^{(\alpha)}$ and $K_{jk}^{(\alpha\beta)}$ of (2, 1) are expanded in powers of μ/m and calculated in the zeroth (and in some cases in the first) order of μ/m.

The most remarkable is the solution for $M_{1+\mu}^{(3)}$ (i. e. $\mathcal{M}_{2\mu}^{(3)}$). The result is obtained by noting that in the static limit (and with an additional reasonable assumption that the $\Delta(1236)$ resonance region dominates -assumption later called "isobar approximation") the equation for $M_{1+\mu}^{(3)}$ and the analogous (previously solved by Chew et al.[21]) equation for $f_{33} = \frac{1}{p} e^{i\delta_{33}} \sin \delta_{33}$ (which is the (3, 3) amplitude of pion nucleon scattering) - differ only in the inhomogeneous terms and may be written in the following way:

$$\mathrm{Re}A(\omega) = A^{(B)}(\omega) + \frac{1}{\pi} P \int_{\mu}^{\infty} d\omega' \left(\frac{1}{\omega' - \omega} + \frac{1}{9(\omega' + \omega)} \right) \mathrm{Im}\, A(\omega') \tag{3.1}$$

where for A we should substitute f_{33}/p^2 or $M_{1+\mu}^{(3)}/kp$ and $A^{(B)}$ denotes the Born term. The structure of the equation implies that the ratio of Born terms for both processes will be equal to the ratio of total amplitudes, yielding the famous result:

$$M_{1+\mu}^{(3)} = \frac{k}{p} \frac{\mu_p - \mu_n}{2f} f_{33} \tag{3.2}$$

where μ_p (μ_n) denote magnetic moments of the proton (neutron) and f is the πN coupling constant (f $\approx$ 0. 08).

The multipole $E_{1+\mu}^{(3)} = 0$ in this approximation (we get a homogeneous equation for it, since in the static limit $B_{3\mu}^{(3)} \equiv 0$) which reflects the known fact that $\Delta(1236)$ photoexcitation is almost purely dipole magnetic. Incidentally we remark that the vanishing of $E_{1+\mu}^{(3)}$ in the static theory gave rise to some important troubles in more recent developments of dispersion theory. This point will be discussed later in more detail.

(x) - Note, that it is just the linearity of integral equations which makes such a splitting possible at all.

The CGLN solution for the electric parts of both relevant multipoles $\mathcal{M}_{j1}^{(3)}$, j=2, 3, i.e. $M_{1+e}^{(3)}$ and $E_{1+e}^{(3)}$ is rather clumsy and there is no reason to discuss it here at length. The result has the form:

$$\mathcal{M}_{j,e}^{(3)} = B_{j,e}^{(3)} e^{i\delta_j^{(3)}} \cos \delta_j^{(3)} \qquad j = 2, 3 \tag{3.3}$$

and may be called a "unitarized Born approximation".

One quite naturally asks about the quantitative value of CGLN solution, and the answer is quite interesting. Fig. 1 visualizes the remarkable accuracy to which the CGLN solution

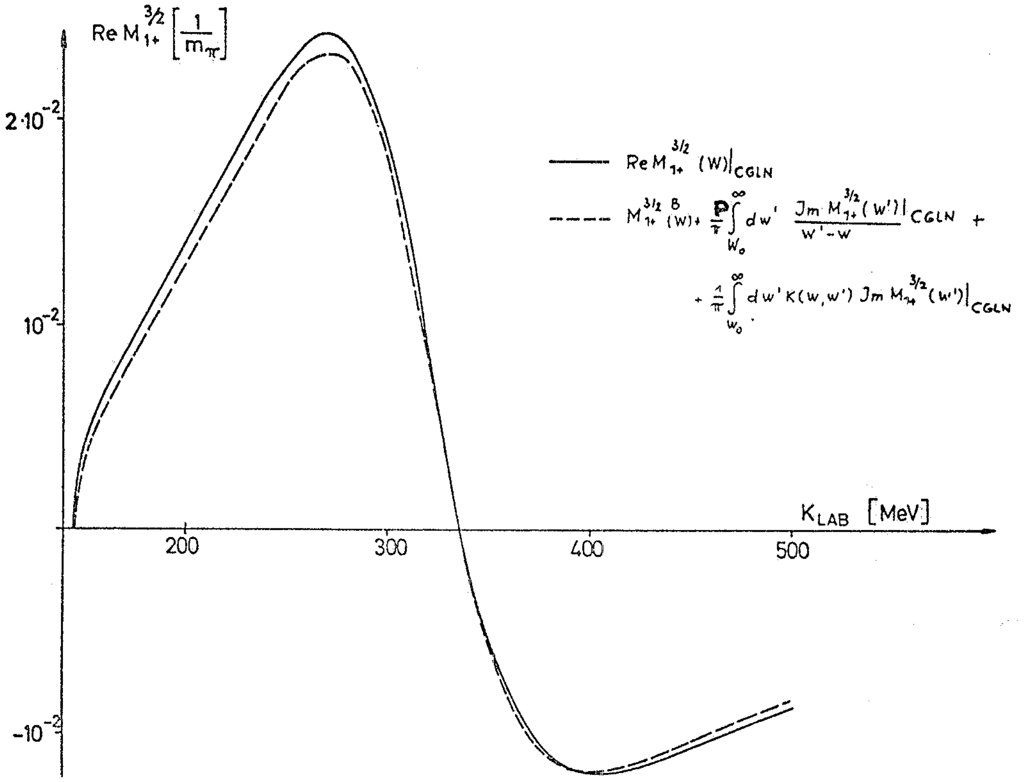

FIG. 1 - Compatibility of the CGLN static solution $M_{1+}^{(3)}|_{CGLN}$ with the relativistic dispersion relation (Ref. (9)).

satisfies the relativistic dispersion equation of the type (2.4). In place of lengthy comments one should only remind that not all recent sophisticated dispersion calculations attained that quantitative value of the static solution. The question remains why static dispersion relations (fed with the assumption that $\Delta(1236)$ dominates in the imaginary parts of amplitudes), give a quantitatively good prediction of photoproduction multipoles. This question should be answered in a broader context of successes achieved by the static theory in describing low energy pion-nucleon interactions. Roughly speaking, the situation looks as follows. Basic conservation laws say us that pseudoscalar mesons and "static" (or "infinitely heavy") nucleons interact in a l = 1 state which in actual calculations[28] turns out to be resonant. This matches very well the experimental data which say us that the low energy pion-nucleon interaction is dominated by a very strong l = 1 resonance.

Later on the original CGLN approach was refined by McKinley[29], Höhler[3], Schmidt[4], Ball[2] and others. There is no need to discuss these calculations in detail since in what concerns the resonant multipoles, the quoted papers brought conceptually little new to the CGLN result. The main progress consisted in the use of fully relativistic kinematics and in attempts to find better solutions for the non resonant amplitudes. The isobar approximation (not to be confused with the "isobar model") based on the CGLN results for $\mathrm{Im}\, M^{(3)}_{1+}$ was generally assumed.

The forthcoming discussion will be focused on these calculations[5-12] which in the last few years attempted to determine the resonant multipoles directly from relativistic dispersion relations (and often this determination was a part of a larger program aiming at a complete theory of low-energy photoproduction of pions, which is however beyond the scope of the present lectures). There is obviously no claim for completeness of this review. As stressed at the begining, the aim will be rather to point out some crucial problems common to different calculations, and to show how these problems are solved in any specific case.

The above mentioned calculations treat differently the problem of transforming the equ. (2. 1) into an integral equation and, subsequently, the problem of finding its solution. We recall that (2. 1) is a Cauchy-type integral formula which connects the real part of an amplitude at a given <u>point</u> of the complex plane with the values of the imaginary part of this amplitude in some <u>regions</u> of that plane. If we want to solve such an integral formula (or, alternatively, formula (2. 4)) for the complex amplitude some additional information about the phase of the amplitude should be fed in. We shall comment on this point later on.

4. - NON SINGULAR INTEGRAL EQUATION AND THE (3, 3) PHASE SHIFT. -

From a purely formal point of view the most natural technique of transforming (2. 4) into a non singular integral equation is the use of the so called Muskhelishvili-Omnès transformation[30](x). One makes the assumption that the functions appearing in (2. 4) fulfil some mathematical conditions of continuity and asymptotic behaviour (which seem plausible) and that the phase of $\mathcal{M}_j^{(\alpha)}$ (entering into the equation through formula (2. 2)) is known on the whole integration interval. We shall see in a moment that the last assumption is questionable and controversial. Taking, however, both assumptions as granted, one transforms formula (2. 4) into a system of non-singular, Fredholm equations :

$$\begin{aligned}\mathcal{M}_k^{(\alpha)}(v) &= \mathcal{B}_k^{(\alpha)}(v)\, e^{i\delta_k^{(\alpha)}(v)} + \sum_{q,\beta}\int_{v_o}^{\infty} dv'\, N_{kq}^{(\alpha\beta)}(v',v)\, e^{i\delta_k^{(\alpha)}(v)}\, \mathcal{M}_q^{(\beta)}(v') + \\ &+ p_{n-1}(v)\, e^{\rho_k^{(\alpha)}(v) + i\delta_k^{(\alpha)}(v)}\end{aligned} \tag{4. 1}$$

where the inhomogeneous term $\mathcal{B}_k^{(\alpha)}(v)$ is connected with the pole term $B_k^{(\alpha)}(v)$ of (2. 4) in the following way :

$$\mathcal{B}_k^{(\alpha)}(v) = B_k^{(\alpha)}(v)\cos\delta_k^{(\alpha)}(v) + \frac{1}{\pi}\, e^{\rho_k^{(\alpha)}(v)}\, P\int_{v_o}^{\infty}\frac{dv'}{v'-v}\, B_k^{(\alpha)}(v')\, e^{-\rho_k^{(\alpha)}(v')}\sin\delta_k^{(\alpha)}(v') \tag{4. 2}$$

The kernel $N_{kq}^{(\alpha\beta)}(v',v)$ is expressed through $K_{kq}^{(\alpha\beta)}(v',v)$ as follows :

$$\begin{aligned}N_{kq}^{(\alpha\beta)}(v',v) &= K_{kq}^{(\alpha\beta)}(v',v)\, e^{-i\delta_q^{(\beta)}(v')}\sin\delta_q^{(\beta)}(v')\cos\delta_k^{(\alpha)}(v) + \\ &+ \frac{1}{\pi}\, e^{\rho_k^{(\alpha)}(v)}\, P\int_{v_o}^{\infty}\frac{dv''}{v''-v}\, e^{-\rho_k^{(\alpha)}(v'')}\sin\delta_k^{(\alpha)}(v'')K_{kq}^{(\alpha\beta)}(v',v'')\, e^{i\delta_q^{(\beta)}(v')}\sin\delta_q^{(\beta)}(v')\end{aligned} \tag{4. 3}$$

(x) - This is the name under which the procedure is known in the physical literature. For mathematicians, this transformation is a standard way of dealing with the so-called Hilbert problem in the theory of analytic functions.

In the foregoing expressions $\rho_k^{(\alpha)}(v)$ has the meaning:

(4.4) $$\rho_k^{(\alpha)}(v) = P \int_{v_o}^{\infty} \frac{\delta_k^{(\alpha)}(v')}{v' - v} \, dv'$$

and the notation $e^{-\rho_k^{(\alpha)}(v)} = D_k^{(\alpha)}(v)$ is often used[(x)]. The symbol $p_{n-1}(v)$ denotes an arbitrary polynomial of order $n-1$ in v, which appears when $\delta_j^{(\alpha)} \to n\pi$ for $v \to \infty$. The system (4.1) may be integrated by standard methods known from the rigorous theory of integral equations (and in practice, the integration will be numerical).

Assumptions that can be made about phases $\delta_j^{(\alpha)}$ are by no means unique and, as will be seen below, these assumptions bear essentially on the results. The only rigorous information as to the phase $\delta_j^{(\alpha)}$ of a photoproduction multipole $\mathcal{M}_j^{(\alpha)}$ comes from Watson's theorem (2.2) which is however valid only for energies up to the first inelastic threshold. Above that limit we can make only guesses since the multipole phase referred to in (2.2), (2.4), and continued above the inelastic threshold equals neither the respective pion nucleon phase shift nor its real part. Since these phase-shifts are intended to be an essential dynamical input to our model, quite natural a question arises how should this problem be handled.

In the spirit of our program the discussion of phase-shifts will be confined to the (3,3) phase shift pertinent to the multipoles $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$. We leave apart the treatment of the so called "small phases" for which the reader should refer to original papers[(6, 8, 10, 11)].

Recent partial wave analysis of pion-nucleon scattering[(31)] tell us that the region of "effective elasticity" of δ_{33} (or - in other words - the region where the respective inelasticity parameter equals 1 to a very good approximation) can be extended far beyond the two pion production threshold, probably as far as to about 1 GeV. Moreover we note that δ_{33} approaches 180° in the upper end of that region. This information is quite important for us. Firstly, it means that Watson theorem may in practice be applied over a large energy interval which contains the bulk of contributions to the integrands in (4.2) and (4.3). This is due to a rather rapid decrease of $K_{kq}^{(\alpha\beta)}(v', v)$ and $N_{kq}^{(\alpha\beta)}(v', v)$ at large energies. Secondly, if we infer from extrapolation of data that δ_{33} indeed asymptotically approaches π as shown on Fig. 2 (which assumption may not be true for genuine asymptotics, but seems at the moment

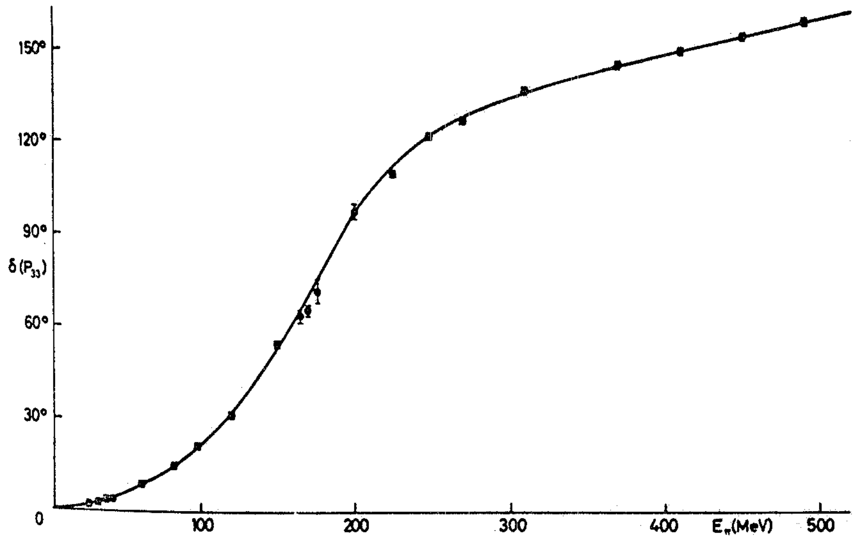

FIG. 2 - Pion-nucleon scattering phase shift $\delta_{33} = \delta_2^{(3)} = \delta_3^{(3)}$ according to the fit of Donnachie et al.[(31)] (Ref. (11)).

(x) - Depending on the asymptotic behaviour of $\delta_k^{(\alpha)}(v)$ subtractions in formula (4.4) may be necessary.

the most natural guess), we are left with an arbitrary constant (zero-order polynomial in v) in the equation (4. 1). The constant can not be determined within the so-far defined formalism. Formally, its presence reflects the possibility of adding to the final result a solution of the homogeneous equation derived from (4. 1). From our point of view, the model as we defined it above, turns out to be incomplete and open are possibilities of inserting some new assumptions. As discussed below, the possible assumptions turn out to be crucial for the final result.

5. - THE MULTIPOLE $M_{1+}^{(3)}$. -

There were several attempts to solve equation (4. 1) for the multipole $M_{1+}^{(3)}$ (and, possibly also for $E_{1+}^{(3)}$)(5, 7, 8, 9, 10, 13, 32) under the assumption that the equation for $M_{1+}^{(3)}$ decouples from the rest of the system, being therefore of the form :

$$(5.1)\qquad \mathcal{M}_2^{(3)}(v) = \mathcal{E}_2^{(3)}(v)\, e^{i\delta_2^{(3)}(v)} + \int_{v_0}^{\infty} dv'\, N_{22}^{(33)}(v',v)\, e^{i\delta_2^{(3)}(v)}\, \mathcal{M}_2^{(3)}(v') + \frac{C_2^{(3)}}{D_2^{(3)}(v)}\, e^{i\delta_2^{(3)}(v)}$$

Since the value of $C_2^{(3)}$ can not be a priori fixed, one faces a tremendous problem of solving a one-parameter family of integral equations. This of course can hardly be done and simplified solutions should rather be considered. Our further discussion of this point will not always follow the chronological sequence of events but will rather exhibit links existing between calculations published in different times and by different groups.

The very first calculations based on formula (5. 1) and on the assumption that the shape of the (3, 3) phase shift was that shown on Fig. 3 have demonstrated that it is not easy to get an $M_{1+}^{(3)}$ as reliable as that of CGLN. The amusing point was here that the considered equation was much more sophisticated (and apparently more exact) than that of Ref. (1). In particular, a naïve hope that the constant $C_2^{(3)}$ may equal zero (what would simplify to a large extent the calculations) turned out to be quite misleading. Korth et al. (33) who solved a simplified (by putting $C_2^{(3)} = 0$ and $K_{22}^{(33)}(v', v) = 0$) version of the equation (5. 1) have shown that their procedure yields values of $\mathrm{Im}\, M_{1+}^{(3)}$ (which they named "particular solution") by far too low when compared to those of CGLN and to those supposed to fit the data. This can be seen on Fig. 4 on which we also note that the imaginary part (and by unitarity - also the real part) of $M_{1+}^{(3)}$ vanishes at 380 MeV photon momentum in lab, contradicting the known properties of photoproduction in this region. The situation hardly improves if self-coupling of $M_{1+}^{(3)}$ (through $K_{22}^{(33)}(v', v) \neq 0$) is included. One may also assert that other possible couplings, much weaker than the just mentioned self-coupling, do not help much. In the quoted paper of Korth et al.(33) the authors proposed to find a value of $C_2^{(3)}$ by matching the obtained solution at threshold to the respective values of CGLN amplitude. This line has been followed in later calculations of the Bonn group. Their result shown in Fig. 4 as "our solution" is seen to follow rather well the expected values. The fact itself is not quite trivial since the de facto performed one parameter fit could a priori be insufficient to get satisfactory results. Unfortunately, at this stage of calculations the meaning of the constant remains rather obscure. The situation changes little, both conceptually and numerically if we try to determine $C_2^{(3)}$ by fitting "our solution" to the value of $\mathrm{Im}\, M_{1+}^{(3)}$ at resonance, as discussed in Refs. (8, 9).

An ambitious program of solving the integral equations (5. 1) and (4. 1) for $M_{1+}^{(3)}$ (and other multipoles) with a simultaneous fit to experimental data in order to determine the open constants, has been attempted by Schwela et al. (7-10). A discussion of the whole program unfortunately goes far beyond the scope of these lectures. In what concerns $M_{1+}^{(3)}$ shown in Fig. 5 the program follows the line elaborated earlier at Bonn(33, 34). The important points to note are the following :

Firstly, in the discussed calculation while solving the equation (5. 1) for $M_{1+}^{(3)}$ we actually couple it (through the procedure of determining the open constant) to the solutions of the remaining equations of the system (4. 1) and finally we rely on some information extracted from data.

Secondly, the details of the high energy behaviour of the (3, 3) phase shift are of little

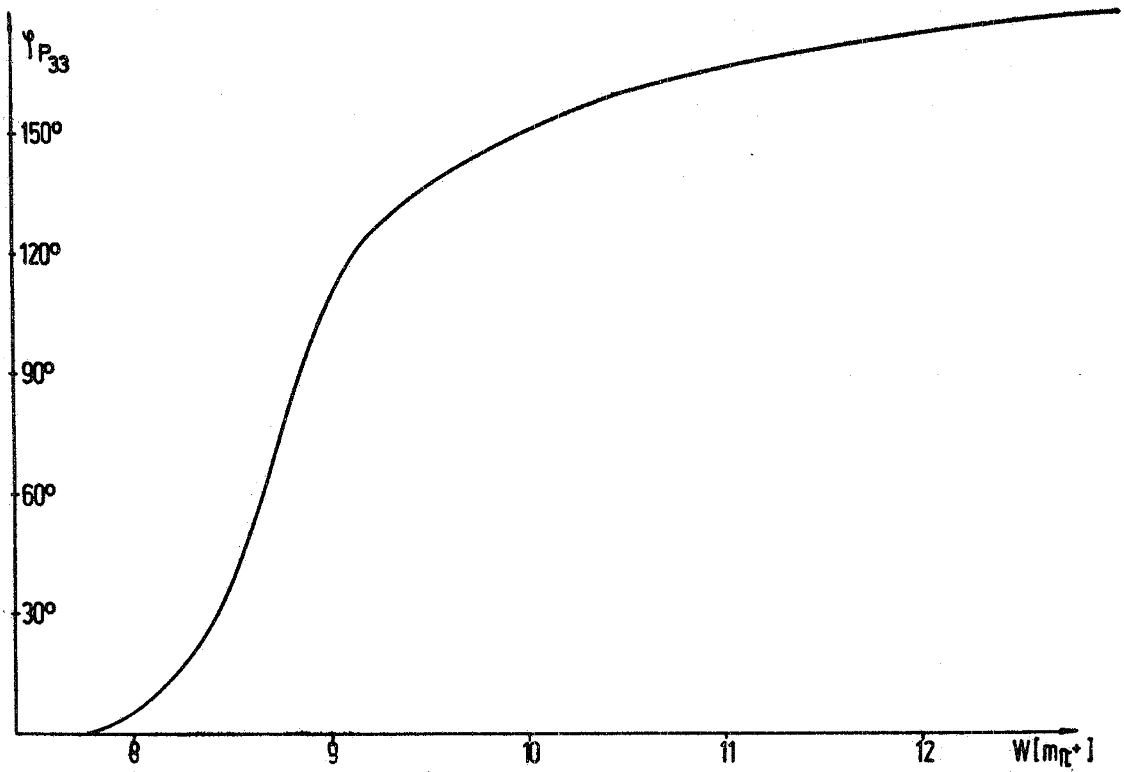

FIG. 3 - Multipole phase $\varphi_{P_{33}} = \delta_2^{(3)} = \delta_3^{(3)}$ as used in a numerical calculation (Ref. (9)).

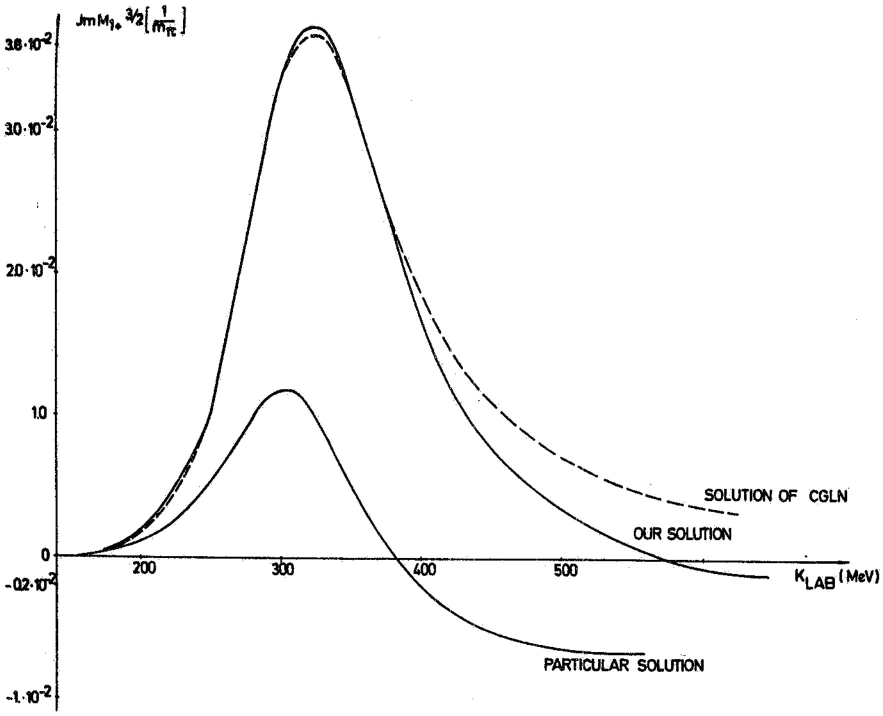

FIG. 4 - Comparison of Im $M_{1+}^{(3)}$ as resulting from the CGLN static equation with solutions of a simplified dispersion equation of Korth et al.[33]. "Particular solution" was obtained when the open constant C = 0. "Our solution" was obtained by using C to match the static solution at threshold (Ref. (33)).

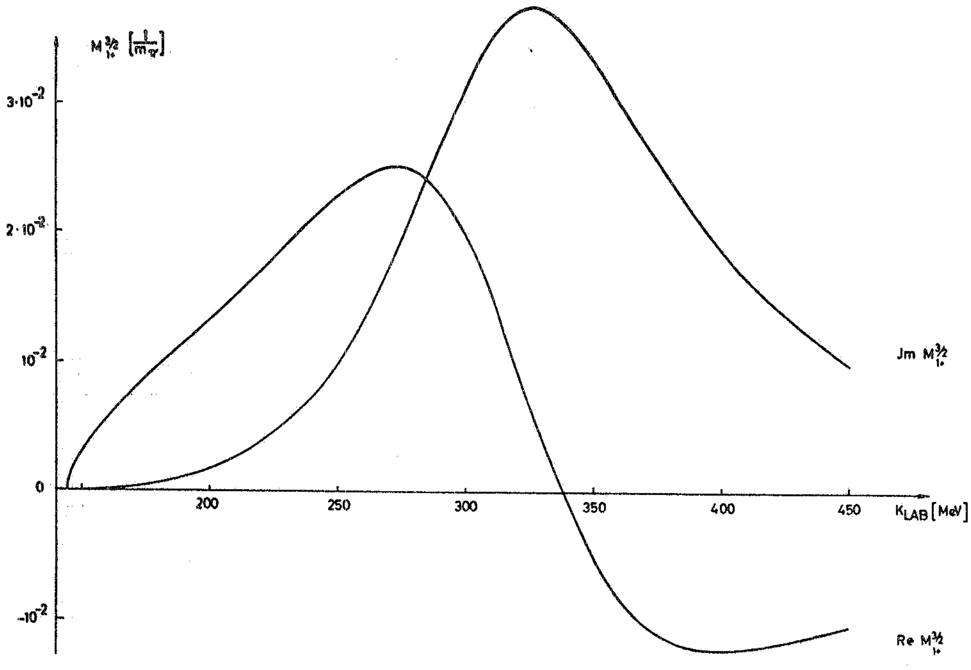

FIG. 5 - The multipole $M_{1+}^{(3)}$ obtained from a totally decoupled equation resulting from the fixed-t dispersion approach of the Bonn group[9]. The multipole phase of Fig. 3 was used and the open constant served to fit the multipole to experimental data (Ref. (9)).

importance for the solution, as long as the assumption about the asymptotic limit $\delta_2^{(3)} \rightarrow \pi$ is maintained.

More details on the approach of Refs. (7-10) we postpone to our discussion of the multipole $E_{1+}^{(3)}$.

Since our prediction of the value of Im $M_{1+}^{(3)}$ (e. g. around the resonance) is not absolute, and relies on a fit, the question immediately arises what are the actual limits imposed by the experimental and other errors on the accuracy of this prediction. This problem will be discussed a little later. Here we remark, that the basic role of $M_{1+}^{(3)}$ in describing low energy photoproduction implies that any ambiguity in the determination of this amplitude[x] will bear upon eventual results concerning the other multipoles. We have just mentioned an example of such dependence in connection with the calculation of Refs. (8, 9, 10), and later we shall see how much is this dependence important in the case of $E_{1+}^{(3)}$.

Concluding this discussion we see that for advantages got by assuming an elastic (3, 3) phase shift with asymptotic behaviour extrapolated from low-energy data we have paid the price of having equations depending on arbitrary parameters.

An approach at first sight different from the above discussed was proposed by Finkler[5]. Just as Bonn's approach, also Finkler's calculation [o] is based on equation (2.4) transformed by the Omnè's method, but with a peculiar assumption about the phase. Finkler shows that reasonable values of $M_{1+}^{(3)}$ can be obtained if we assume that the (3, 3) phase shift at higher energies is discontinous, namely undergoes a jump from about π down to zero (see Fig. 6). This assumption looks strange at first sight but as will be seen in a moment, works surprisingly well, what also has been confirmed by more recent and refined calculations[32, 39]. The question why such an artificial proposal proved itself so effective clearly deserves some comment.

(x) - Which, as we remember, is usually determined in the first step of calculation.
(o) - We leave apart some details irrelevant to the present discussion.

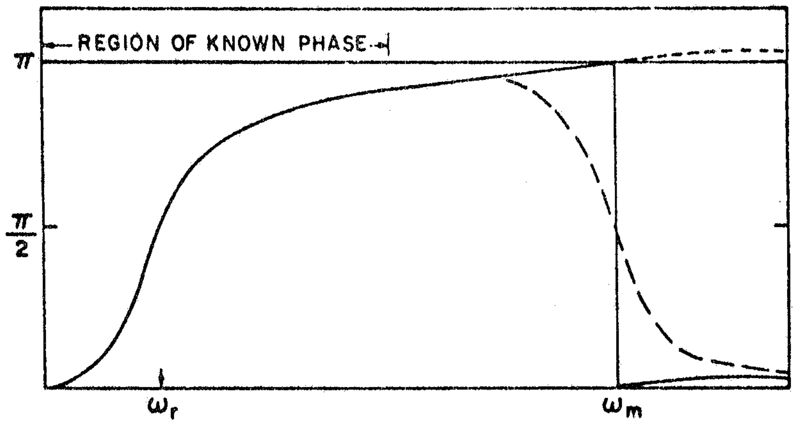

FIG. 6 - Presumed behaviour of a multipole phase above the region of known phase passing through $\pi/2$ at ω_r. The short dashed curve represents the extension of the phase shift under the assumption that it passes through π at ω_m. The solid curve represents the phase shift in the case where it drops rapidly toward zero (Ref. (5)).

There is no need to discuss here all arguments which led Finkler to his assumption. We fix our attention on formal and physical consequences of the postulated discontinuity.

One should note that from a purely formal point of view by making such an assumption we introduce in the multipole amplitude a zero (so called CDD zero[35, 36]) at some arbitrary value of energy. We have therefore an open parameter - the zero's position - to fix it by fitting the solution to data. The analogy of this and Bonn's procedure suggests that both approaches may not be so different from each other. In fact, both methods of introducing a constant are mathematically equivalent. Preferences given therefore to one or other method are rather a matter of taste[(x)] and have nothing to do with underlying mathematical properties, which in both cases are identical.

One may quote here an amusing coincidence between Finkler's assumption and a recent phenomenological multipole analysis of Noelle et al.[37]. These authors note that their Argand diagram for $M_{1+}^{(3)}$ shows a zero in $M_{1+}^{(3)}$ at some high energy. This can be seen on Fig. 7. Not entering into a slippery subject of whether a genuine CDD zero shows up on the diagram, we may nevertheless infer from Fig. 7 that an (elastic !) extrapolation of $M_{1+}^{(3)}$ to higher energies naturally leads to a zero in this amplitude. The diagram of Fig. 7 may therefore serve as an a posteriori justification of Finkler's assumption.

Engels and Schmidt[32] of Karlsruhe studied the problem of evaluating $M_{1+}^{(3)}$ (and also $E_{1+}^{(3)}$) along the general line of Finkler's formalism, but they tried to separate "in a lucid way" the well established theoretical and phenomenological knowledge from our ignorance and hypothetical assumptions. The aim was to make clear the influence of the latter assumptions on final results. The starting point in Ref. (32) is the equation (4.1) written (in our notation) in the following form

$$\text{Re}\,\mathcal{M}_k^{(\alpha)}(v) = \bar{\mathcal{M}}_k^{(\alpha)}(v) + \frac{1}{\pi} P \int_{v_o}^{v_c} \frac{dv'}{v' - v} \, \text{Im}\,\mathcal{M}_k^{(\alpha)}(v') \tag{5.2}$$

where $\bar{\mathcal{M}}_k^{(\alpha)}(v)$ denotes the "inhomogeneous term" (i.e. the Born term plus other contributions as they result from (4.1)).

(x) - The author of these lectures prefers Finkler's method since this procedure seems to exhibit more directly the role of the constant in taking account of the high energy region (this point will be discussed below).

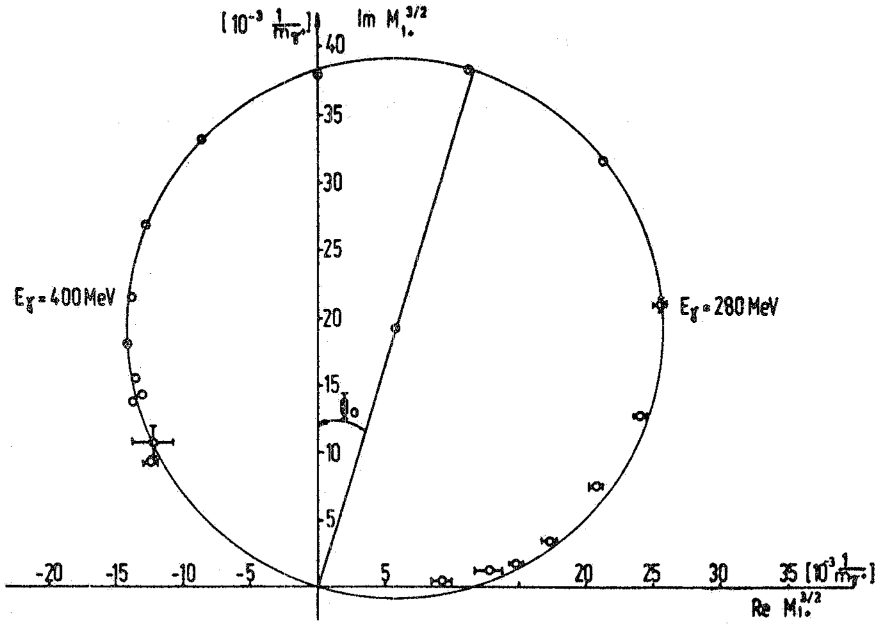

FIG. 7 - Argand diagram resulting from the phenomenological fit to photoproduction data of Noelle et al.[37]. A smooth extrapolation of the circle above the region of available data shows up a zero in $|M_{1+}^{(3)}|$ (Ref. (37)).

The form (5.2) derives its origin from the assumption that the phase $\delta_2^{(3)}$ is actually known only on a finite interval $v_o < v < v_c$ where it is given by the values of the (3,3) pion nucleon phase shift, in accordance with Watson's theorem (possibly extrapolated to higher energies). The inhomogeneous term $\bar{\mathcal{M}}_k^{(\alpha)}$ incorporates therefore not only the usual Born term but also high energy contributions for $v > v_c$ and, at least in principle, also contributions due to coupling with other multipoles. A sophisticated procedure (which will not be described here) is applied in order to estimate the importance of those contributions (and resulting errors) while evaluating the resonant multipoles $M_{1+}^{(3)}$ (and $E_{1+}^{(3)}$) at low energies. Numerical estimates obtained in the sharp resonance approximation are often used as guide in the discussion. A strong decrease of $\mathrm{Im}M_{1+}^{(3)}$ (and $\mathrm{Im}E_{1+}^{(3)}$) at energies far above the $\Delta(1236)$ resonance is an essential experimental information used in the estimates.

The upper limit of integration v_c is fixed in such a way as to make the influence of high energy contributions as small as possible. It turns out that v_c should be placed above the "second resonance" $N^*(1518)$. On the other hand errors due to a substantial violation of Watson's theorem in that region puts an upper limit on v_c. According to the estimates of Ref. (32) the limit should be fixed at about $W_c \approx 12\ \mu$.

Apart from the pole term, contributions to $\bar{\mathcal{M}}_k^{(\alpha)}$ are assumed to be sensible only to gross features of the imaginary parts of multipoles coupled to $M_{1+}^{(3)}$ and may be hence expressed through a few paramaters - "coupling constants" in the language of Ref. (32). One should also note in this respect that the coupling kernels $K_{kq}^{(\alpha\beta)}(v', v)$ strongly decrease with energy so that the high energy region is suppressed in the coupling integrands.

The multipole $M_{1+}^{(3)}$ is finally obtained in such a way that firstly we fix the cutoff v_c in energy (in Ref. (32) it corresponds to $E_\gamma \approx 800$ MeV lab. photon energy). Then some of the "coupling constants", and namely those which are connected with the $\Delta(1236)$ resonance, are chosen in such a way that $\mathrm{Im}M_{1+}^{(3)}$ (and $\mathrm{Im}E_{1+}^{(3)}$) at a point close to resonance (precisely, at E_γ = 320 MeV) lies within certain limits given by phenomenological fits to π^o photoproduction data. In the case of $\mathrm{Im}M_{1+}^{(3)}$ one used data on $d\sigma/d\Omega\,|_{90^o}$ as this quantity strongly depends on $M_{1+}^{(3)}$. The quoted limits, as given by Ref. (32) are shown in Fig. 8 which indicates that at the time when the paper was published, the prediction concerning $\mathrm{Im}M_{1+}^{(3)}$ at resonance could be made only within an error of 10 %. The authors point out however, that once the multipoles $M_{1+}^{(3)}$ (and $E_{1+}^{(3)}$) are fixed at resonance their energy dependence within the resonance region can safely be predicted[38]. The result is shown in Fig. 9. From what has been told so far it

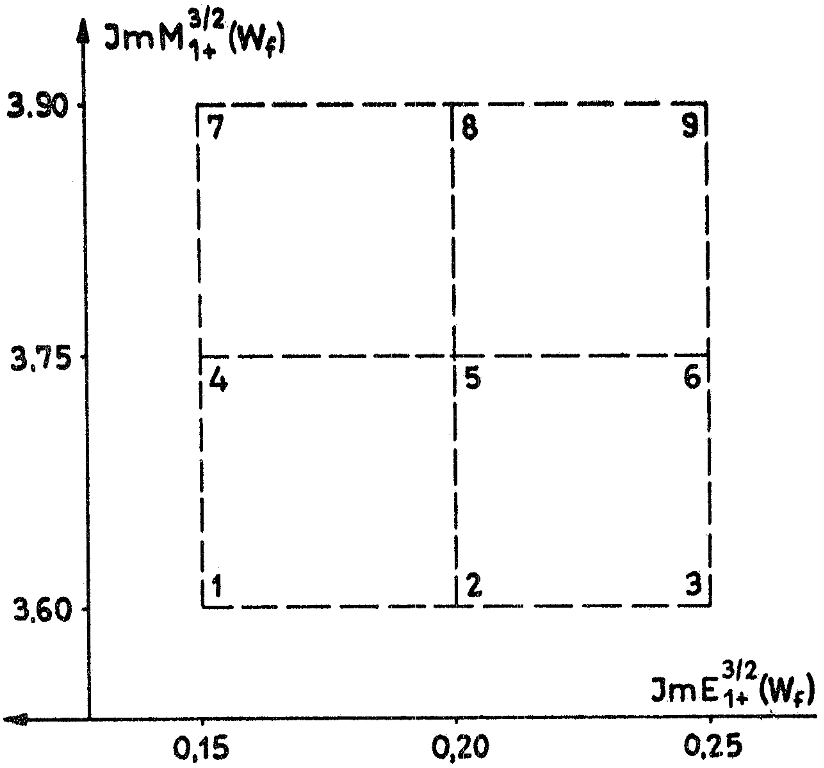

FIG. 8 - The range of uncertainty of $ImM_{1+}^{(3)}$ and $ImE_{1+}^{(3)}$ at energy value W_f below the (3, 3) resonance, as is estimated by Engels and Schmidt[32]. The numbers 1...7 refer to various solutions discussed by the authors (Ref. (32)).

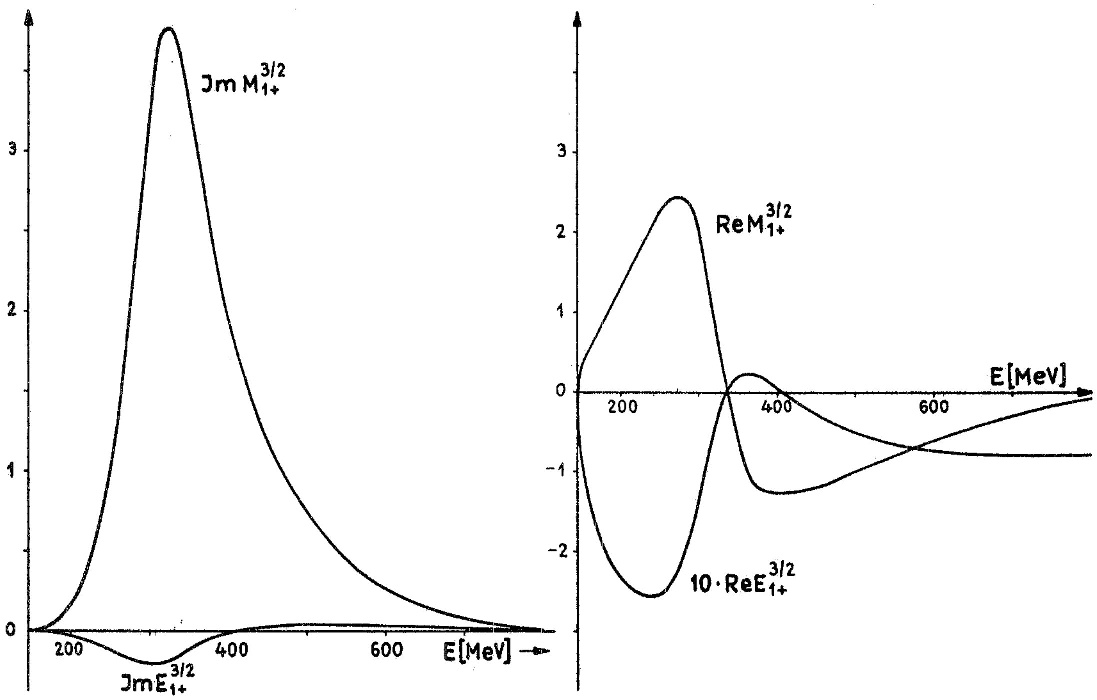

FIG. 9 - Values of $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ as resulting from the approach of Engels and Schmidt (Ref. (32)).

becomes clear that the error is essentially due to uncertainties in estimating the high energy contributions. Terms due to coupling between $M_{1+}^{(3)}$ and other multipoles (or more precisely uncertainties in determining these terms) seem to produce less important errors.

The just presented discussion of the "Finkler type approach" clearly indicates that the open constant (reflecting the presence of a CDD zero, no matter how it was introduced to the formalism) plays a role of a bag where contributions of the high energy region may be hidden. One may find lucky a situation when the mathematical structure of the model itself requires the presence of such a constant which fact is a priori independent of the role we assign to it.

Calculations of the Karlsruhe group[32, 39] indicate moreover that one open constant - the cutoff in energy (or in other words - the position of a CDD zero) presumably is not sufficient to fit the data so that some additional "coupling constants" should also give account of our ignorance.

At any case, however, the dispersion model in the form presented here (i.e. with pion-nucleon phase shifts fed in) is not sufficient to describe photoproduction of pions on nucleons in the $\Delta(1236)$ region. Some direct recourse to photoproduction data is necessary (if one does not like comparison with CGLN result).

A natural question hence arises about possible ways of reducing the above mentioned 10 % error, or in other words, about narrowing the limits as shown in Fig. 8. An effort was made towards that aim by Engels et al.[39] who fitted to data with polarized γ some quantities which strongly depended on the values of the resonant multipoles. Since this problem will be discussed below, in connection with the $E_{1+}^{(3)}$ multipole, we mention only that according to Ref. (39) the ambiguity in $M_{1+}^{(3)}$ at resonance can be reduced to less than 5 %.

Needless to say, since our discussion deals with the resonant multipoles only, a serious problem of fitting simultaneously all available data with all calculated (up to some constants) multipoles and of errors resulting in such a fit can not be treated.

6. - THE MULTIPOLE $M_{1+}^{(3)}$ AND CONFORMAL MAPPING TECHNIQUE. -

A different approach to dispersion equations has been proposed by Donnachie and Shaw[6]. This point of view may be summarized as follows : Since we can hardly say anything about the multipole's phase at high energies and in particular about its asymptotic behaviour, the procedure of Muskhelishvili - Omnès is of little value. Instead of imposing some asymptotic conditions on phases, it is more reliable to impose some conditions on the asymptotic decrease of multipole amplitudes and to look for a method of solution which "does not depend appreciably on the details of the high energy behaviour of the multipole amplitudes and which allows the inclusion of elastic unitarity whenever it is applicable"[9]. Donnachie and Shaw propose to solve the integral equation (2.4) directly by an iterative method. In this approach it is essential what has been assumed about the decrease of multipole amplitudes at infinity. As to the iteration method itself one should remark that the equation to be solved by iteration is a singular one, in which case the final result may depend crucially on the form of a zero-order approximation used to start the iteration procedure. The situation is different from that of a Fredholm-type equation in which case we know that (at least in some domain) an iteration series really approximates, step by step, the true solution. Fortunately for the specific case of $M_{1+}^{(3)}$ we have an excellent zeroth approximation - the static CGLN solution of (3.2) which is just used as a starting point in the calculation of Ref. (9). The obtained result is quite reasonable, claimed stable to about 0.5 % from threshold to E_γ = 700 MeV. It is shown in Fig. 10 where it is also compared with the solution of Schwela et al.[8, 9]. The solutions differ most sensibly around resonance, which seems however due to differences in parametrization of the (3, 3) phase shift used in the calculations.

From a formal point of view it is clear that the solution of Ref. (6) is mathematically different from those discussed in Sec. 5. It would be probably fruitless to look for close affinities between both types of solutions. In particular, it would be misleading to say that the approach of Donnachie and Shaw[6] is less ambiguous since it does not contain arbitrary constants which we discussed in Sec. 5. Actually, if those constants are interpreted as bags containing

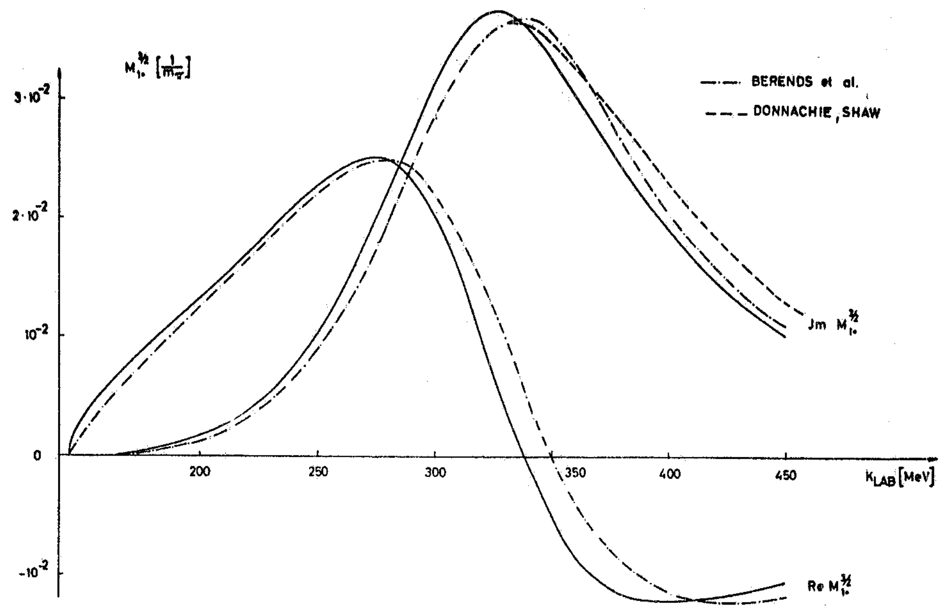

FIG. 10 - Comparison of $M_{1+}^{(3)}$ resulting from various calculations. Solid curve: Schwela and Weizel[9]; dashed curve: Donnachie and Shaw[6]; dash-dot curve: Berends et al.[11] (Ref. (9)).

our ignorance on the high energy region, a similar (though not necessarily identical) information is contained e.g. in the assumptions made in Ref. (6) about the high energy behaviour of multipole amplitudes and subsequent smooth joining of the "high energy multipole tail" to low energy solutions[6]. One should also bear in mind that in the particular case of $M_{1+}^{(3)}$ an appreciable amount of dynamical information has been put into the equation by using the static solution of CGLN as zeroth approximation of the iteration procedure. We shall see later that in the case of $E_{1+}^{(3)}$ when a similar zeroth order approximation is missing it is much more difficult to get a reasonable solution.

The approach exposed in Ref. (6) and summarized just above has been refined and applied to an extensive and sophisticated study of pion photoproduction by Berends et al.[11].

With its appearance the paper aroused many discussions and also some controversy. A brief outline of the method seems therefore of interest. For details the reader should refer to the original papers[11, 40].

The starting point is formula (2.1) with W taken as independent variable. We expand $\mathrm{Im}\,\mathcal{M}_q(W)$ (suppressing isospin indices for simplicity) in a series

$$\mathrm{Im}\,\mathcal{M}_q(W) = \sum_{k=1}^{\infty} (a_q)_k\, h_k(W) \tag{6.1}$$

where $h_k(W)$ are some functions which ensure the correct threshold behaviour of $\mathrm{Im}\,\mathcal{M}_q(W)$, and sufficiently rapidly vanish asymptotically. They will be specified below. The series (6.1) converges under very general mathematical conditions.

We also make use of the unitarity condition

$$\mathrm{Im}\,\mathcal{M}_q(W) = \mathrm{Re}\,\mathcal{M}_q(W)\,\mathrm{tg}\,\delta_q(W) \tag{6.2}$$

extended, as usually, to the energy range where Watson's theorem is satisfied to a good approximation.

Introduced is moreover the expression

$$(6.3) \qquad F_q(W) = B_q(W) + \sum_{q \neq q'} \int_{m+\mu}^{\infty} dW' \, K_{qq'}(W',W) \, \mathrm{Im}\, \mathcal{M}_q(W') \, .$$

We note that non-singular self-coupling of $\mathcal{M}_q(W)$ is not included into $F_q(W)$. Substituting (6.1) - (6.3) into the dispersion equation (2.1) we obtain

$$(6.4) \qquad \sum_{k=1}^{\infty} (a_q)_k \left\{ h_k(W) - (g_k(W) + \tilde{g}_k(W)) \, \mathrm{tg}\, \delta_q(W) \right\} = F_q(W) \, \mathrm{tg}\, \delta_q(W)$$

where $g_k(W)$ is given by the following formula (known in mathematics as the Hilbert transform of $h_k(W)$) :

$$(6.5) \qquad g_k(W) = \frac{1}{\pi} P \int_{m+\mu}^{\infty} dW' \, \frac{h_k(W')}{W' - W}$$

and

$$(6.6) \qquad \tilde{g}_k(W) = \int_{m+\mu}^{\infty} dW' K_{kk}(W', W) \, h_k(W')$$

i.e. $\tilde{g}_k(W)$ arises from nonsingular self-coupling of $\mathcal{M}_k(W)$ (cf. (6.3)).

The functions $F_k(W)$ are assumed to be known either by assumption or from earlier calculations (and one should remember that in dispersion theory such terms are interpreted as long-range forces).

In the interesting case of $M_{1+}^{(3)}$ Berends et al.[11] assume that $F_2(W) = B_2(W)$, hence they neglect the coupling of $M_{1+}^{(3)}$ to all other multipoles. The equation for $M_{1+}^{(3)}$ decouples therefore from the rest of the system, as is often the case in such calculations.

The infinite series in (6.1) and (6.4) is now truncated at some value K, which in fact means that $\mathrm{Im}\,\mathcal{M}_q(W)$ is now approximately expressed through a combination of a few terms. We should bear in mind that nothing can be told at the moment about errors due to this truncation.

By requiring now that W in formula (6.4) assumes discrete values W_i $(i=1,\ldots,N)$ at which phase shifts $\delta_q(W_i)$ are known from fits, we get a system of N linear equations for K coefficients $(a_q)_k$:

$$(6.7) \qquad \sum_{k=1}^{K} (a_q)_k \left\{ h_k(W_i) - (g_k(W_i) + \tilde{g}_k(W_i)) \, \mathrm{tg}\, \delta_q(W_i) \right\} = F_q(W_i) \, \mathrm{tg}\, \delta_q(W_i) \qquad (i=1,\ldots,N)$$

We recall that all the remaining terms in eq. (6.7) are assumed to be known. Since $K \ll N$ in practical calculations, the system (6.7) is highly overdetermined and by a fitting procedure makes possible a choice of the best set of coefficients $(a_q)_k$. Actually, while solving the system (6.7), in place of the variable W, Berends et al.[11] introduce a new variable $x = \dfrac{\overline{W} - (W - m - \mu)}{\overline{W} - (W + m + \mu)}$. The variable x defines a conformal transformation (which gave the name of "conformal mapping method" to the whole approach) of the complex W plane on the complex x plane, The open parameter $\overline{W}$ (a so called "conformal zero") is also varied in the fitting procedure. The transformation maps the integration domain $m+\mu < W < \infty$ on the range $-1 < x < 1$. The functions $h_q(W)$, which as yet, have not been specified, are in Ref. (11) chosen to be Gegenbauer polynomials[41] in x. This choice ensures a correct threshold behaviour required by (6.1) but is by no means unique.

An important point is now that the fitting procedure aiming at the best set of $(a_g)_k$ is performed for x_i corresponding to W_i in the medium energy region. The resulting expansion

(6.1) is nevertheless claimed to describe the imaginary part of a multipole in the whole energy region. Such a continuation to high energies (or - equivalently - to x close to 1) clearly depends on the choice of a set $h_q(W)$. In particular one should not be misled by a naïve argument that the continuation of the expansion(6.1) in x proceeds from the bulk of the (-1, 1) range to its tiny fraction close to x = 1 (corresponding, as we have told, to a domain of large W up to infinity), what makes the continuation immune to ambiguities. Actually the relative weight of the points close to x = 1 is enormous, and negligible differences in x may produce large effects in W. One therefore should be cautious with procedures of this kind.

A freedom in specifying a set h_q and of choosing a particular conformal transformation is probably analogous to the arbitrariness in fixing the open constant in the approach described in Sec. 5. It is of course difficult to establish any closer correspondence between both methods so that more quantitative conclusions concerning their comparison do not seem possible at the moment.

The conclusion should be however, that an information about the high energy region is to be inserted to the model in this of other way. Comparison with data can only discriminate between different calculations.

The result of Ref. (11) for $M_{1+}^{(3)}$ is shown in Fig. 11. A comparison with the CGLN solution shows differences of about 10 % especially in the region below resonance. Larger dif-

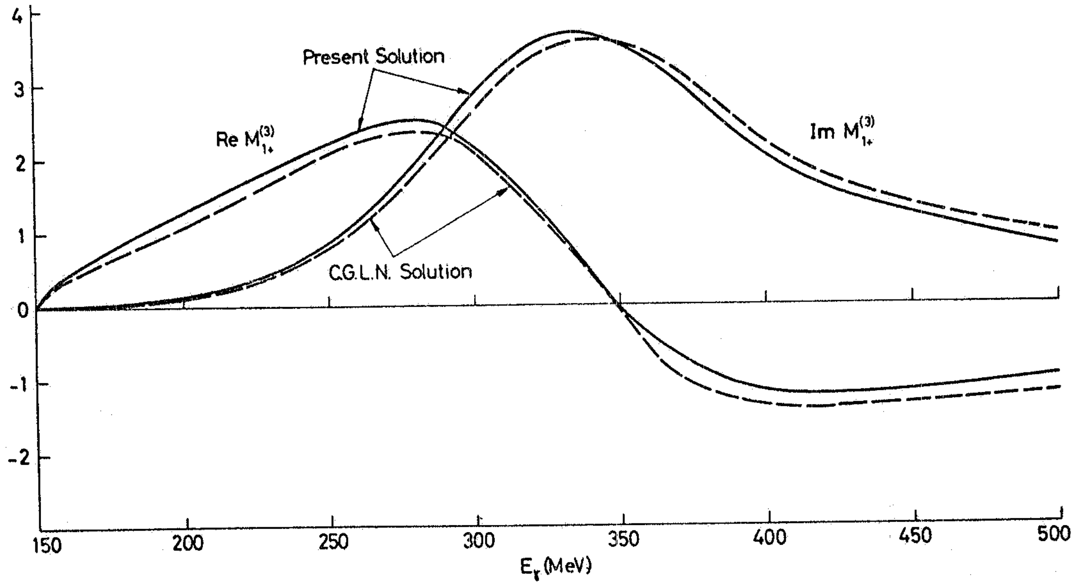

FIG. 11 - Comparison of the $M_{1+}^{(3)}$ static solution of CGLN with the solution of Berends et al. (denoted "Present solution") (Ref. (11)).

ferences far above the resonance are probably less significant. When compared to the results of the Bonn group[(8, 9)] the solution of Berends et al.[(11)] shows large differences around resonance. In particular the zero in Re $M_{1+}^{(3)}$ is shifted. This is probably due to differences in parametrizing the (3, 3) phase-shift. The solution of Ref. (11) shows little change with respect to the earlier result of Donnachie and Shaw[(6)]. According to Engels et al.[(39)] differences between the conformal mapping approach and the isobar approximation are practically negligible in the case of $M_{1+}^{(3)}$. Incidentally one may remark that these calculations differ for "small" multipoles. In particular Ref. (39) points out that if we use multipoles of Ref. (11) at 400 MeV as input to the isobar approximation, the results calculated in this approximation at 200 MeV do not coincide with the results of the conformal mapping approach at the same energy. This discrepancy seems independent of variations in Im$M_{1+}^{(3)}$ shown in Fig. 8.

The conclusion therefore should be that dispersion models for $M_{1+}^{(3)}$ as defined in the beginning are not complete. Some information about the high energy region should be someway inserted. This complementary information ultimately rests on phenomenology e.g. on fitting the obtained solutions to data. The resulting $M_{1+}^{(3)}$ obtained by various methods do not differ appreciably from each other (differences are of the order of 5-10 %) indicating that most probably different authors put different labels on essentially the same complementary dynamical input to the dispersion equation. A wider margin of freedom in this respect is probably restrained by our phenomenological knowledge about the multipole $M_{1+}^{(3)}$.

We shall see below that the situation is much more confused in the case of a "small multipole" such as the electric quadrupole $E_{1+}^{(3)}$.

7. - THE MULTIPOLE $E_{1+}^{(3)}$. -

Since the earliest studies of photoproduction it has been conjectured that in the resonance region $E_{1+}^{(3)}$ (i. e. the electric quadrupole excitation of $\Delta(1236)$) should be small. The angular distribution of π^o measured around resonance[42] behaves, to a good accuracy, as $5 - 3 \cos^2\theta$ which is typical for a pure M_{1+} (and, of course for E_{2-}, which is however excluded by parity assignments). The static approximation of CGLN[1] yields $E_{1+,\mu}^{(3)} = 0$ (in contrast, we remind, to a resonant $M_{1+,\mu}^{(3)}$). A vanishing $E_{1+}^{(3)}$ results also from calculations based on a non relativistic quark model[15]. All these indications have of course rather qualitative value and more refined experimental measurements and theoretical calculations will be discussed in a while. One should however bear in mind that the just mentioned estimates qualify $E_{1+}^{(3)}$ as a "small multipole" with the obvious consequence that its correct determination is much more difficult than a determination of $M_{1+}^{(3)}$. The presumed smallness of $E_{1+}^{(3)}$ also motivates a different treatment of $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ in the present lectures.

In our discussion of $M_{1+}^{(3)}$ we emphasized rather those points related to theoretical problems. We were not paying much attention to the analysis of data. The circumstance that $M_{1+}^{(3)}$ dominates pion-photoproduction in the resonance region makes life rather easy and allows for an almost direct comparison of model calculations with data. For $E_{1+}^{(3)}$ it is no more the case. One has to rely on shaky calculations compared with indirect information extracted from measurements of small effects. It seems therefore necessary to put more emphasis on problems connected with phenomenological analysis of data.

In Fig. 12 are shown the helicity amplitudes[(x)] of π^o photoproduction close to resonance as obtained from a phenomenological fit of Walker[43]. They indicate that near 90° the differential cross section comes mainly from H_1 and H_3, whereas at forward and backward direc

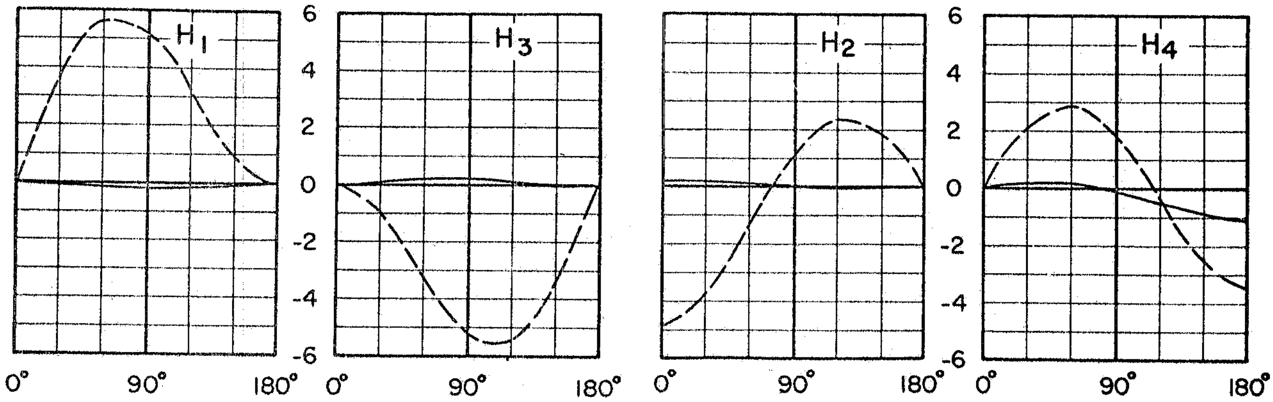

FIG. 12 - Helicity amplitudes, $H_n(\theta)$ for $\gamma p \to \pi^o p$ at $k_{Lab} = 350$ MeV. The real parts are shown by solid curves, the imaginary parts by dashed curves (Ref. (43)).

(x) - The notation is $H_i = H_{\mu\lambda}$ where the helicity subscripts are $\mu = -\lambda_{N+}$, $\lambda = \lambda_\gamma - \lambda_{Ni}$. In particular $H_1 = H_{1/2\,3/2}$, $H_2 = H_{1/2\,1/2}$, $H_3 = H_{-1/2\,3/2}$, $H_4 = H_{-1/2\,1/2}$. The unpolarized differential cross section is $d\sigma/d\Omega \sim \sum_i |H_i|^2$. For details the reader may refer to rewiev papers[25, 43, 44].

tion - from H_2 and H_4 (due to angular momentum conservation). Since H_1 and H_3 are, roughly speaking, proportional to the sum of $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ whereas H_2 and H_4 are proportional to their difference[25, 43] a comparison of cross sections at 90° with forward and backward scattering cross sections could in principle be used to determine the admixture of $E_{1+}^{(3)}$ to $M_{1+}^{(3)}$ around resonance. A simultaneous use of forward and backward data is important in order to eliminate contributions due to interference with E_{0+}. Unfortunately data in the forward direction[44, 45, 46] are still missing what practically makes cumbersome this most direct way of determining the multipole $E_{1+}^{(3)}$.

It seems that at the moment the electric quadrupole $E_{1+}^{(3)}$ may be determined most effectively through measurements of angular distributions of π^0 produced by polarized photons. This is an indirect determination based on a possibility of extracting the ratio $E_{1+}^{(3)}/M_{1+}^{(3)}$ from these distributions. The respective reasoning deserves some comment. The angular distribution of π^0 produced by linearly polarized photons may be written in the form:

(7.1) $$\frac{d\sigma}{d\Omega}(W, \theta, \varphi) = \frac{d\bar{\sigma}}{d\Omega}(W, \theta) + \sin^2\theta \cos 2\varphi \, I(W, \theta)$$

where θ denotes the scattering angle and φ the azimuthal angle between the polarization plane and the production plane. The cross section for unpolarized photons $d\bar{\sigma}/d\Omega$ may be represented as

(7.2) $$\frac{d\bar{\sigma}}{d\Omega}(W, \theta) = A + B\cos\theta + C\cos^2\theta + \cdots$$

and $I(W, \theta)$ connected with the polarized photon asymetry has the form

(7.3) $$I(W, \theta) = I_o + I_1 \cos\theta + I_2 \cos^2\theta + \cdots \ .$$

Measurements of $d\sigma/d\Omega$ and $d\bar{\sigma}/d\Omega$ at $\theta = \pi/2$ provide us with values of the ratio I_o/A which may be then combined with information on angular distributions $d\bar{\sigma}/d\Omega\,(W, \theta)$ to yield the ratio I_o/C. Our interest in the latter derives from the fact that I_o/C strongly depends on $E_{1+}^{(3)}/M_{1+}^{(3)}$[47]. In terms of partial waves we have

(7.4) $$\frac{I_o}{C} = \frac{\mathrm{Re}\left[M_{1+}^{*}(M_{1+}+2M_{1-})\right] - 3\mathrm{Re}\left[E_{1+}^{*}(2M_{1+}-2M_{1-}+E_{1+})\right] + (1>1 \text{ terms})}{\mathrm{Re}\left[M_{1+}^{*}(M_{1+}+2M_{1-})\right] - \mathrm{Re}\left[E_{1+}^{*}(2M_{1-}-2M_{1+}+3E_{1+})\right] + (1>1 \text{ terms})} \ .$$

A total M_{1+} dominance would imply $I_o/C = 1$. Data say us[48] that I_o/C is indeed close to 1 around resonance but clearly depends on energy and differs from 1 at lower energies. Formula (7.4) suggests that a non-negligible E_{1+} could indeed produce the expected energy dependence of I_o/C and this point of view is now generally accepted.

One should however realize that this interpretation might in principle be not unique, unless we had other possibilities of determining $E_{1+}^{(3)}$, which at present are slim. For example there were attempts[49] to explain the energy dependence of I_o/C by including ρ and ω exchange to the CGLN static model. Even if we do not accept it today, this interpretation is worth mentioning just to show that information on $E_{1+}^{(3)}$ extracted indirectly from available data is far from being unambiguous. Such a situation makes a theoretician's life difficult the more that model calculations of $E_{1+}^{(3)}$ (as will be seen in a while) are themselves subject to many more ambiguities than in the case of $M_{1+}^{(3)}$.

As mentioned in Sec. 3 the static model of CGLN[1] gives $E_{1+,\mu}^{(3)} = 0$. This can be easily understood if we note that in the static lmit the equation for $E_{1+,\mu}^{(3)}$, analogous to (3.1) contains only self-coupling terms (no isobar approximation!) and is homogeneous since in the static limit $B_{3\mu}^{(3)} = 0$. A null $E_{1+,\mu}^{(3)}$ is obviously consistent with such an equation. By taking $E_{1+,e}^{(3)} = B_{3,e}^{(3)} \sin\delta_3^{(3)} e^{i\delta_3^{(3)}}$ (to be compared with the analogous result for $M_{1+,e}^{(3)}$) we obtain a small $E_{1+}^{(3)}$ which is consistent with the above mentioned indications[42, 15]. Such an $E_{1+}^{(3)}$ produces a strongly energy dependent ratio $E^{(3)}_{1+}/M^{(3)}_{1+}$ (as it should be) which however changes sign around resonance and in this point seems to contradict the data[32, 47]. One should also bear in mind that the smallness of $E^{(3)}_{1+}$ in the static approximation does not imply that relativistic corrections to it are small. The example of $M^{(3)}_{1+}$ (which to a good approximation is given by its static value) is by no means indicative. One says (in theoretician's language) that forces - crossed channel contributions - may a priori be in both cases different. The influence of these contributions on a small $E^{(3)}_{1+}$ may produce essential alterations.

The vanishing of the static $E^{(3)}_{1+}$ merely indicates a qualitative smallness of the electric quadrupole excitation. However, it caused a number of problems in some later calculations. The reason was the absence of a guide similar to the static $M^{(3)}_{1+\mu}$ which we have seen to be very useful in solving many practical problems in dispersion calculations.

In the already mentioned paper of Korth et al.[33] the authors presented an evaluation of $E^{(3)}_{1+}$. They solved equation (4.1) with $K^{(\alpha\beta)}_{ij} = 0$. Such a total decoupling from the rest of the system and in particular from $\mathrm{Im}M^{(3)}_{1+}$ is in agreement with the already quoted table of Schwela and Weizel[8]. To fix the open constant the value of the Born approximation at threshold was fitted[x]. The results are shown in Fig. 13 where a non resonant character of the solution is clearly seen. It is also interesting that the solution of Ref. (33) is completely different from the isobar approximation of Ref. (3). A preliminary character of the paper (33) explains why no comparison with the data has been attempted there. One should only remark that the calculated $E^{(3)}_{1+}/M^{(3)}_{1+}$ ratio of Ref. (33) shows a strong energy dependence and a change of sign at 300 MeV lab photon energy what seems inconsistent with data (cf. Ref. (47)).

The ideas suggested in Ref. (33) have been later applied (what we have already discussed in Sec. 5) to calculate $E^{(3)}_{1+}$ within the framework of an ambitious study of pion photoproduction by the Bonn group[8, 9, 10]. The result of calculations is shown in Fig. 14. The calculated $E^{(3)}_{1+}$ is clearly non resonant. The procedure is essentially similar to that of Ref. (33). Equation (4.1) with self-coupling only is solved for $E^{(3)}_{1+}$ and the open constant is determined by a least-squares fit to data[50] on π^o differential cross sections in the lab. energy range 200-400 MeV and for angles 70^o-160^o. One should remark that in the whole calculation appear four free parameters which are fitted simultaneously to the just mentioned Bonn data[50]. The multipole $E^{(3)}_{1+}$ results coupled in this way to the remaining multipoles. It is clear that the values of the constants depend on the adopted fit. Schwela and Weizel[8] observe that by fitting $M^{(3)}_{1+}$ to the CGLN solution at resonance one does not achieve agreement with π^o data. Moreover a good fit to those data seems impossible if we insist on two constants only - precisely those appearing in $M^{(3)}_{1+}$ and $E^{(3)}_{1+}$ equations[o].

We note once more that in order to determine the multipole $E^{(3)}_{1+}$ an ultimate recall to data was necessary. Unfortunately in Bonn papers[8, 9] we do not find error limits on the calculated values of $E^{(3)}_{1+}$. One could ask for example to what extent the choice of Bonn data[50] as a fitting pattern determines the non resonant character of $E^{(3)}_{1+}$. In this connection we observe that Orsay data[51] on cross sections at backward direction (which are quite sensible to $E^{(3)}_{1+}$ variations) is not reproduced by the calculated cross sections of Ref. (8, 9). A point which seems important to be emphasized is that we cannot totally separate the determination of multipoles $M^{(3)}_{1+}$ and (especially) $E^{(3)}_{1+}$ from solving the rest of the system what becomes evident when we look, as in Ref. (8, 9) for an overall fit to photoproduction data.

The already mentioned (Sec. 5) calculations of Finkler[5] do not go that far. In addition to $M^{(3)}_{1+}$ Finkler analyses only the multipole $E^{(3)}_{1+}$ (or more precisely, he introduces the sum and difference of the two amplitudes) and in terms of these two tries to fit the then available data. These fits are of little interest today and need not be discussed. More interesting seem instead some general features of the amplitude $E^{(3)}_{1+}$ obtained in Ref. (5). The multipole is small but resonant. Its magnitude is a direct consequence of the relative smallness of respective pole terms (as compared to pole terms of $M^{(3)}_{1+}$).The resonant character of the amplitude,

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(x) - We remind here that the CGLN solution at threshold was used in Ref. (33) to fix the constant corresponding to $M^{(3)}_{1+}$.
(o) - As to the number of open constants appearing in photoproduction calculations cf. also the discussion of Engels and Schmidt[32] summarized in our Sec. 5.

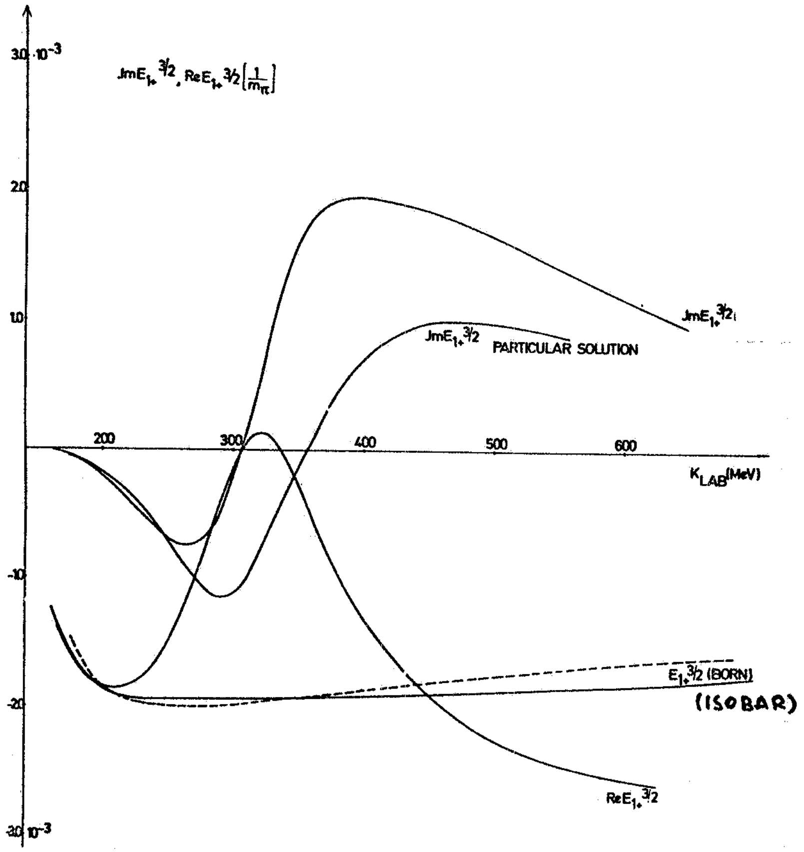

FIG. 13 - The values of $E_{1+}^{(3)}$ as resulting from the simplified relativistic dispersion equation of Korth et al.(33). "Particular solution" denotes the situation when the open constant $c = 0$. Solid curve $ImE_{1+}^{(3)}$ and $Re\,E_{1+}^{(3)}$ were obtained by using c to match the pole term at threshold. Dashed curve $E_{1+}^{(3)}$ (Born) denotes the pole term (purely real), solid curve $E_{1+}^{(3)}$ (ISOBAR) results from the isobar approximation of Refs. (3, 4) (Ref. (33)).

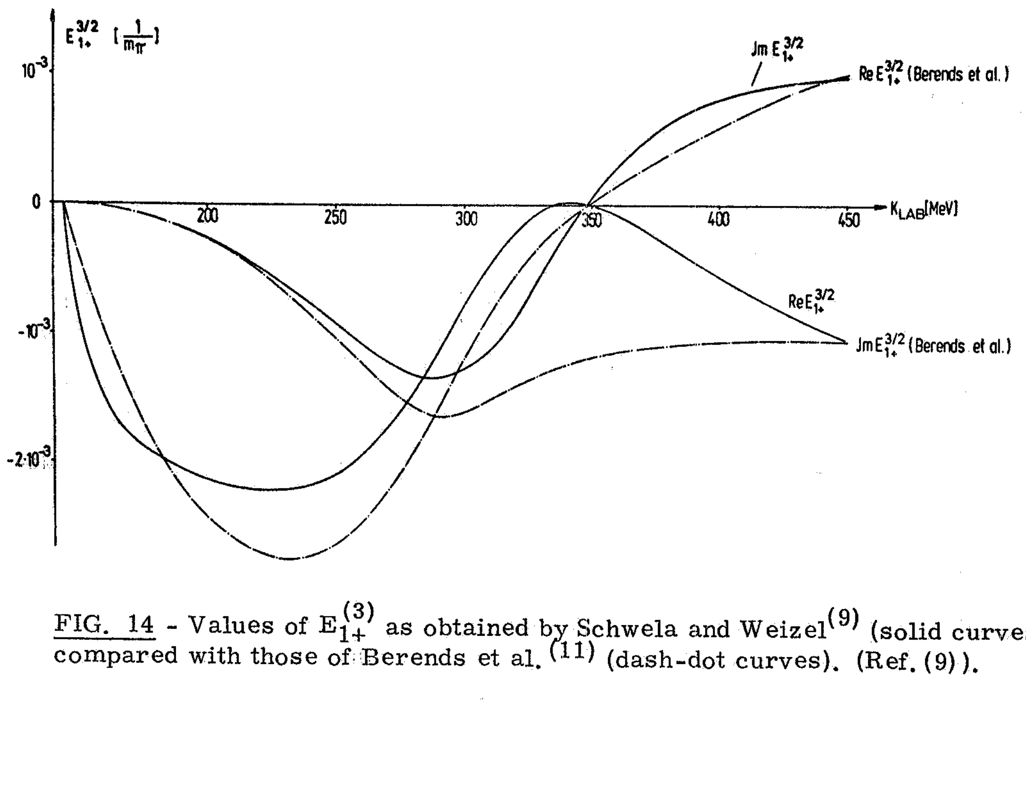

FIG. 14 - Values of $E_{1+}^{(3)}$ as obtained by Schwela and Weizel(9) (solid curves) compared with those of Berends et al.(11) (dash-dot curves). (Ref. (9)).

clearly seen in Fig. 15 suggests a qualitative difference between Finkler's solution and those

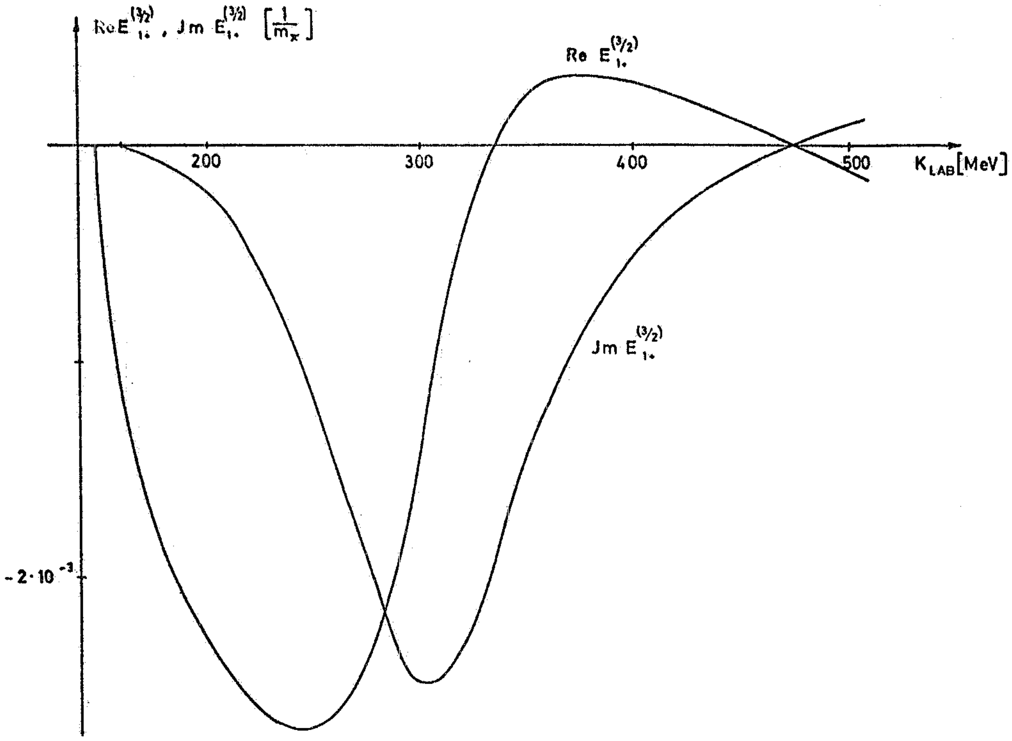

FIG. 15 - Values of $E_{1+}^{(3)}$ as resulting from Finkler's calculation[5] (Ref. (9)).

of Ref. (1), Ref. (33) and Ref. (8, 9) (cf. also Fig. 16). One should however note that $E_{1+}^{(3)}$ = $E_{1+,e}^{(3)}$ as calculated by CGLN may be obtained from the "charge part" (i. e. part proportional to $B_{3,e}^{(3)}$) of Finkler's approximate solution as a limiting case when the position of phase's discontinuity goes to infinity. This observation shows that both solutions are in fact closely related - their resonant or non resonant character being a property which may be realized within the same class of solutions. Schwela and Weizel[8] argue that their solution also shows similar properties which in this case depend on the value of the open constant of (4. 1).

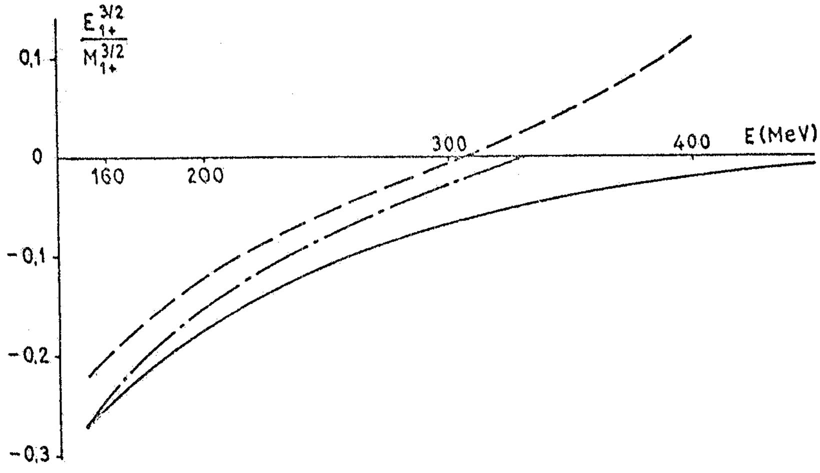

FIG. 16 - The ratio $E_{1+}^{(3)}/M_{1+}^{(3)}$ vs $E = k_{Lab}$. Solid curve : Finkler[5], dashed curve : CGLN[1], dash-dot curve : Korth et al.[33]. (Ref. (47)).

The positions of phase's discontinuities (or in other words - of the cutoffs) are in Finkler's approach equal for both multipoles $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$. This not only means that the number of degrees of freedom is reduced in all fitting procedures but also has an important effect on the behaviour of $E_{1+}^{(3)}/M_{1+}^{(3)}$ around the resonance. In Finkler's model this ratio strongly decreases for energies from threshold to resonance but does not change sign around resonance contrary to the previously discussed results of Refs. (1, 8, 33). There were suggestions(47) that an $E_{1+}^{(3)}/M_{1+}^{(3)}$ nonvanishing in the resonance region better fits data on polarized photon assymetry. Recent phenomenological fits(37, 52) do not confirm that point of view and confirm a change of sign of the ratio $E_{1+}^{(3)}/M_{1+}^{(3)}$ in the 300-400 MeV range. Unfortunately it is not clear from Refs. (37, 52) how closely the obtained multipoles reproduce the polarized photon assymetry data.

The procedure of Engels and Schmidt(32) of calculating the resonant multipoles has been outlined in Sec. 5. As to the case of $E_{1+}^{(3)}$ we note that a prediction of this multipole is especially sensible to the performed approximations, and in particular to the treatment of high energy contributions. In Fig. 9 is shown a possible $E_{1+}^{(3)}$. Its resonant character is evident. However, within the adapted approximation scheme the solution is not unique. By comparing Fig. 8 and Fig. 17 we note that the model itself is not able to predict even the sign of $\mathrm{Im}E_{1+}^{(3)}$ at resonance since the uncertainties in $E_{1+}^{(3)}$ and $M_{1+}^{(3)}$ (fitted to data at some point below resonance) as shown in Fig. 8 allow variations of the zero $E_{1+}^{(3)}/M_{1+}^{(3)}$ within the interval 300-400 MeV. Possibilities of reducing these ambiguities on purely theoretical grounds are, according to Ref. (32) rather slim. Improvements in the prediction according to this point of view require more numerous and more precise data. Data on π^0 photoproduction in the forward and backward direction are for this aim mostly wanted.

We pass now to another school of thinking namely that of Donnachie and Shaw(6). We note that calculations of Ref. (6) failed in the case of $E_{1+}^{(3)}$. We recall here that they were based on an iterative solution of the singular equation (2.4) (See Sec. 6). Difficulties with finding a reliable zeroth order approximation to $E_{1+}^{(3)}$ (and the static model was of little help) made the whole iterative procedure useless in this case. As a result the Born approximation to $E_{1+}^{(3)}$ was adopted in Ref. (6) and this assumption certainly could be a source of some errors.

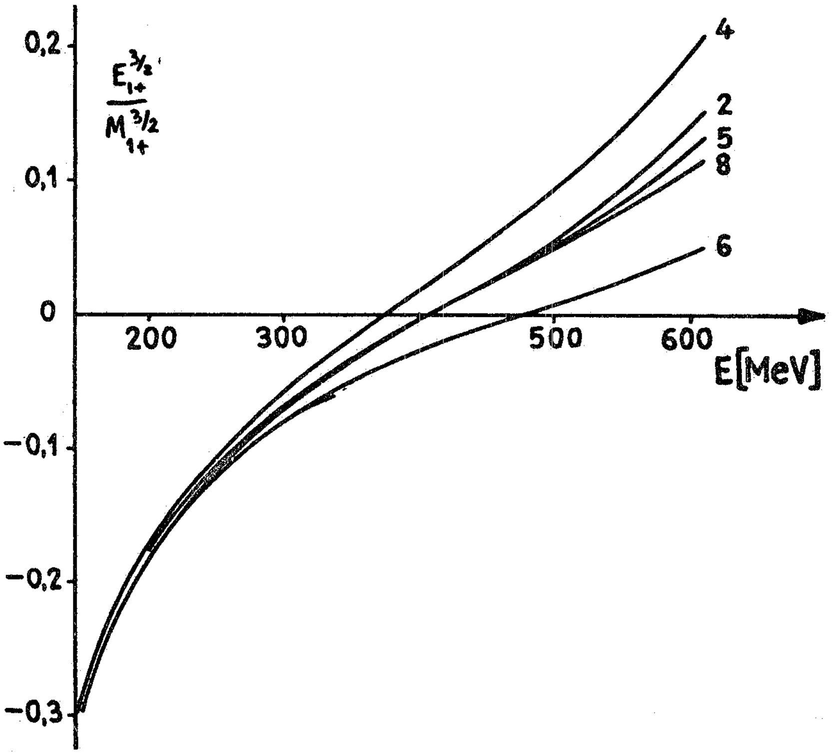

FIG. 17 - The ratio $E_{1+}^{(3)}/M_{1+}^{(3)}$ vs $E = k_{Lab}$ as calculated by Engels and Schmidt. The numbers correspond to those of Fig. 8 (Ref. (32)).

A more refined attempt on the way of calculating $E_{1+}^{(3)}$ is given by Berends et al. [11]. The spirit of this approach has been outlined in Sec. 6. Here we summarize only some peculiarities of the $E_{1+}^{(3)}$ case. The authors solve the equation (6.7) for $E_{1+}^{(3)}$ taking into account non negligible coupling to $M_{1+}^{(3)}$, $E_{o+}^{(1)}$ and $E_{o+}^{(3)}$ (which enter the equation through $F_3^{(3)}$). These amplitudes are assumed to be known from earlier steps of the calculation. The obtained values of $E_{1+}^{(3)}$ are compared in Fig. 14 with those of the Bonn group[8, 9]. In addition to some differences in the region below resonance we observe a striking discrepancy between the two solutions in the energy region around 350 MeV. Contrary to the $E_{1+}^{(3)}$ of Schwela[8, 9] the solution of Berends[11] may be characterized as resonant. In addition, the latter solution exhibits no zero in the calculated energy region, which distinguishes the multipole $E_{1+}^{(3)}$ of Ref. (11) from those discussed before. It is of course difficult to answer whether $\mathrm{Im}E_{1+}^{(3)}$ should show up a zero or not. Arguments which on the basis of phenomenological multipole analysis may be advanced[37, 52] in favour of such a zero are not fully decisive. Whatever the situation the energy dependence of the ratio $E_{1+}^{(3)}/M_{1+}^{(3)}$ as calculated in Ref. (11) clearly deviates from other theoretical predictions as can be seen in Fig. 18.

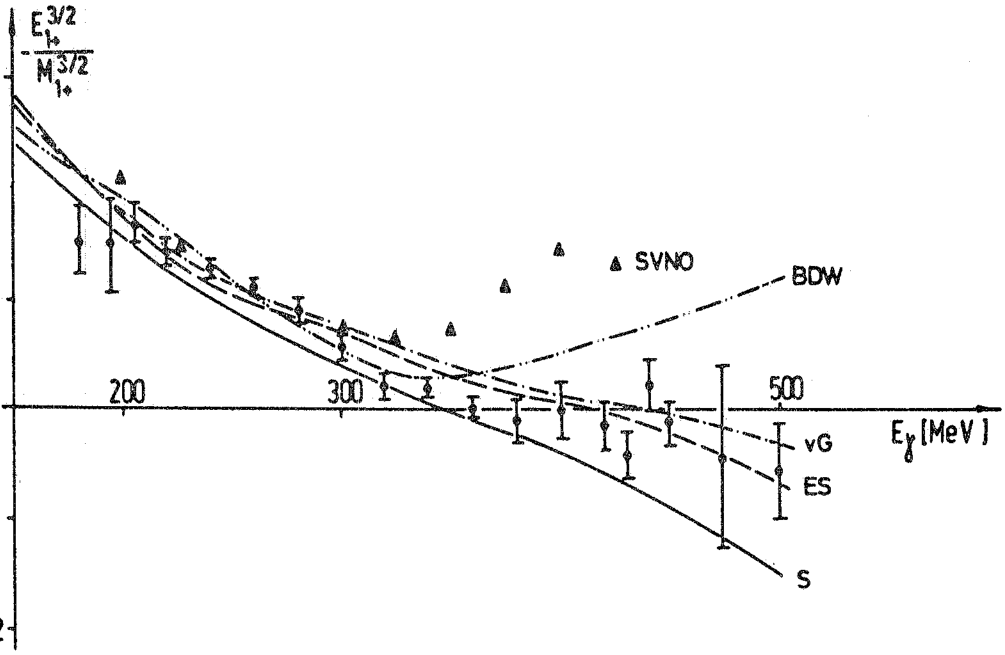

FIG. 18 - The ratio $E_{1+}^{(3)}/M_{1+}^{(3)}$ vs $E_\gamma = k_{Lab}$ as resulting from phenomenological fits, compared to calculated values. Triangles (SVNO) : fit of Spillantini et al., Nuclear Phys. B13, 320 (1969); circles : fit of Noelle et al. [37]; solid curve (S) : calculations of the Bonn group[7, 8]; dashed curve (ES) : calculations of Engels and Schmidt[32]; dash-dot curve (vG) : calculation of v. Gehlen, Nuclear Phys. B20, 102 (1970); dash-double dot curve (BDW) : calculation of Berends et al.[11]. (Ref. (37))

One should stress however, that the reliability of multipoles calculated in Ref. (11) should be discussed in a context much broader than allowed by these lectures.

8. - THE FIXED ANGLE APPROACH. -

Our discussion of various attempts to calculate the multipoles $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ will be completed by a summary of (preliminary) results obtained within the framework of the fixed-angle dispersion approach which we already mentioned in Sec. 2. The details of this technique have been exposed in Ref. (12, 23) and some recent calculations were presented in Ref. (13).

We recall that fixed angle dispersion relations for photoproduction amplitudes were introduced in order to avoid troubles with multipole expansions of invariant amplitudes in the un physical cosine region inevitably appearing in fixed-t dispersion relations. We do not want to report here on all the calculations which can be found in original papers[12, 23]. The point important for us right now is that at some stage of analytic calculations we get a system of equations of the type (4. 1). This system is a starting point for numerical work of Ref. (13).

As independent variable v of formula (4. 1) has been taken the squared c. m. three momentum of the pion (denoted ν) and $K_{kq}^{(\alpha\beta)}(\nu',\nu)$ should now be interpreted as coupling kernels calculated in the fixed angle formalism.

The last point is quite important since these kernels are different from e.g. those of Ref. (9, 11, 25). The calculated photoproduction amplitudes may therefore show differences when compared to the amplitudes obtained in the fixed-t formalism.

The system we shall ultimately deal with may be written as follows :

$$\mathcal{M}_i^{(3)}(\nu) = B_i^{(3)}(\nu) + \frac{1}{\pi}\int_0^{\nu_c} \frac{d\nu'}{\nu'-\nu}\,\mathrm{Im}\,\mathcal{M}_i^{(3)}(\nu') + \sum_{j=2,3}\int_0^{\nu_c} d\nu'\, K_{ij}^{(3,3)}(\nu',\nu)\,\mathrm{Im}\,\mathcal{M}_j^{(3)}(\nu') \,. \tag{8.1}$$

In order to get (8. 1), the system (4. 1) should be handled in the following way:

Firstly we separate a subsystem of coupled equations for $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ by putting $K_{2j}^{(3\beta)}(\nu',\nu) = K_{3j}^{(3\beta)}(\nu',\nu) = 0$ if indices are different from j = 2, 3 ; β = 3.

Secondly, we introduce a discontinuity in the $\delta_2^{(3)} = \delta_3^{(3)}$ multipole phase by assuming that the phase discontinously drops to zero at a point $\nu = \nu_c$ whose position will be adjusted in such a way as to obtain reasonable values of $\mathrm{Im}M_{1+}^{(3)}$ around resonance. The conjecture is analogous to those exploited in Refs. (5, 32, 51). The polynomial term of (4. 1) is neglected in view of the discussion of Sec. 4.

As to the first point above, one should observe that the separation of the discussed subsystem can not be at the moment supported by arguments similar to those contained in the table of the Bonn group[8, 9] showing the relative importance of various coupling kernels. We hope that the possibility of isolating the equation for $M_{1+}^{(3)}$ from the rest of the system persists also in the fixed angle approach. This point requires however a more detailed study. As to $E_{1+}^{(3)}$ the calculations indicate the importance of the coupling with $M_{1+}^{(3)}$. There are hints[26] that coupling with $E_{0+}^{(1)}$ and $E_{0+}^{(3)}$ may also be important.

The comment relative to the second point is the following. The multipole phase $\delta_2^{(3)} = \delta_3^{(3)}$ has been taken from fits to the (3, 3) phase shifts of Auvil et al. and of Roper[53] continued to the higher ($\sim$ 1 GeV) energy region. A sharp fall of the phase at ν_c (or equivalently a cutoff at ν_c) is introduced to (8. 1) (and consequently to formulae (4. 2) - (4. 4)) wherever the relative phase appears. In particular ν_c is present in the nonsingular coupling integral. The role of the discontinuity is therefore extended. It produces not only a CDD zero in the amplitude through a pole in the $D_2^{(3)}$ function (4. 4) but also affects the integrals (4. 2) and (4. 3) by cutting their high energy tail. For large values of ν_c the effect of this cutting is numerically negligible due to a rapid fall of $K_{kq}^{(\alpha\beta)}(\nu',\nu)$ with ν' tending to infinity. However, while lowering ν_c we actually have an interplay of two non negligible effects : a shift of the CDD zero and a shrinkage of the integration range.

Since in the calculations both multipole phases $\delta_2^{(3)}$ and $\delta_3^{(3)}$ are assumed equal (which is a consequence of extrapolating Watson's theorem to the whole integration region) a common value of ν_c is also adopted. This is of course an additional assumption since both phases may in principle differ far beyond the elastic region. It might be possible that a more extensive program similar to that presented in Ref. (8, 9) and Ref. (11) would require two distinct constants for $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ instead of a common one.

The procedure used to solve (8. 1) for $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ is (within the above discussed approximation scheme) a genuine solution of coupled integral equations. In particular, the "inhomogeneous term" of Refs. (5, 32) is (apart from the Born term) not viewed as a quantity taken from outside. Since this "inhomogeneity" or "left hand cut force term" is expressed through self coupling and coupling to the other multipole it builds up in the procedure of solving the system itself. Numerical results reveal an important role of the term (4. 3) in solving the equation for $M_{1+}^{(3)}$. This is easily seen when we look at values of ν_c which enable us to get correct values of $\mathrm{Im}M_{1+}^{(3)}$ around resonance with and without (4.3).

If we neglect the last term and retain (4.1) only, ν_c falls down to unreasonably low values. With (4.3) included, a value of $\nu_c \approx 12\ \mu^2$ (corresponding to $W \approx 11.4\ \mu$ close to the values suggested in Ref. (8) and Ref. (32)) seems to work pretty well.

In view of a preliminary character of the calculation no attempt was made to fit $\mathrm{Im}M_{1+}^{(3)}$ at resonance with greater precision. No doubts that such a fit leading to a more precise determination of ν_c could easily be done but it would be of little value unless an estimate (and a possible inclusion) of the neglected couplings were made.

Two solutions of (8.1) with $\nu_c = 12\ \mu^2$ deserve attention. The first one is the solution of the decoupled system (i.e. with $K_{23}^{(33)}(\nu',\nu) = K_{32}^{(33)}(\nu',\nu) = 0$) labeled SOLUTION 1. The second one is the solution of the system as a whole (i.e. with mutual coupling of both multipoles) labeled SOLUTION 2.

The two solutions may be characteristised as follows. There is almost no difference between the values of $M_{1+}^{(3)}$ in the two cases. The obvious conclusion is that the coupling of $E_{1+}^{(3)}$ with $M_{1+}^{(3)}$ is negligible. Our further discussion will be based on values of $M_{1+}^{(3)}$ as given by SOLUTION 2. The results are shown in Fig. 19. One has not to worry about a too high

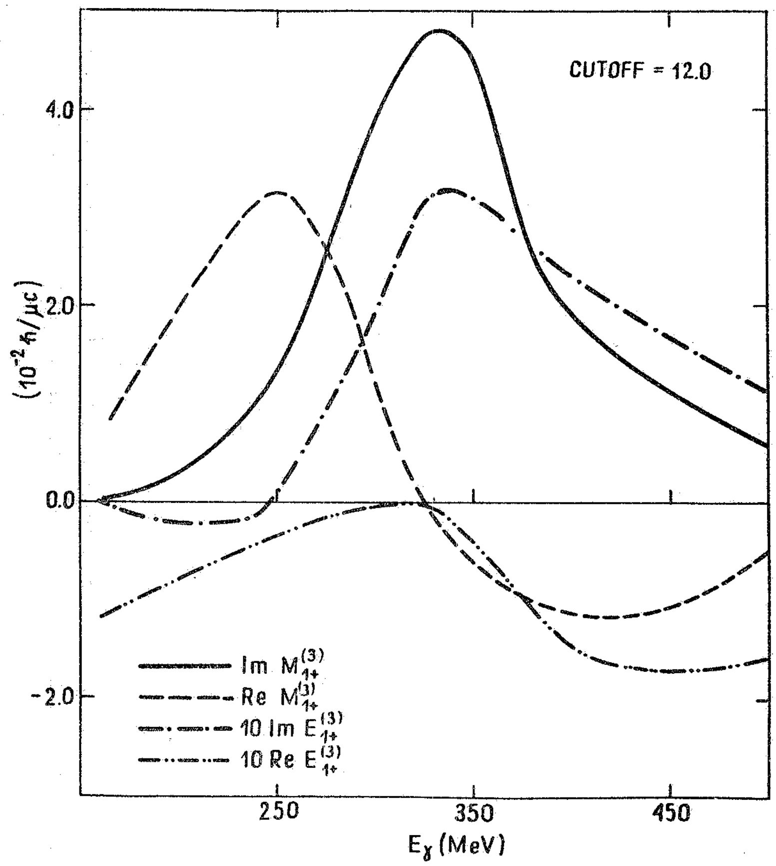

FIG. 19 - Values of $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ vs $E_\gamma = k_{Lab}$ as resulting from the fixed angle approach. Conceptually, the cutoff corresponds to the point of discontinuity of the phase[5]. (Ref. (13)).

value of $\mathrm{Im}M_{1+}^{(3)}$ around resonance. As mentioned before this value can be easily adjusted by small shifts in ν_c (and it is worth mentioning that around $12\ \mu^2$ there is a strong dependence of the solution on ν_c). The desired shape of $\mathrm{Im}M_{1+}^{(3)}$ is well reproduced and no low energy zero is present.

Contrary to $M_{1+}^{(3)}$, the electric quadrupole $E_{1+}^{(3)}$ results quite different in both solutions.

This is shown in Fig. 20. The coupling with $M_{1+}^{(3)}$ affects $E_{1+}^{(3)}$ so strongly that $ImE_{1+}^{(3)}$ in the two cases has different signs at resonance. According to generally accepted views (Refs. 32, 37, 47, 53) $ImE_{1+}^{(3)} < 0$ at resonance which property favours SOLUTION 1 over SOLUTION 2. Moreover, SOLUTION 2 yields an unpleasant low lying zero in $E_{1+}^{(3)}$ (around 250 MeV lab. photon energy) which in the case of SOLUTION 1 is shifted to a more reliable value of 400 MeV lab. photon energy. These features are evident from Fig. 21.

On the other hand a strong influence of $M_{1+}^{(3)}$ on $E_{1+}^{(3)}$ (which is a new feature when compared to the conclusions of Schwela[8, 9] based on fixed-t equations) can not be arbitrarily switched off. A conclusion follows that most probably we should include other couplings while solving the equation for $E_{1+}^{(3)}$. As already mentioned, possible candidates are here couplings with the electric dipole in both isospin states : $E_{0+}^{(1)}$, $E_{0+}^{(3)}$ (which were also taken into account by Berends et al. [11]). Prior to checking the effect of this coupling any further conclusion would be premature.

Another possibility - and introduction of two distinct cutoff parameters for $E_{1+}^{(3)}$ and $M_{1+}^{(3)}$ has not been exploited but seems less consistent with the general line of the approach.

At any case, the fixed-angle dispersion model deserves more ambitious calculations in future.

9. - BRIEF CONCLUSIONS. -

In a summary of the problems discussed in the preceeding sections we can not ignore a possible impression that a great deal of work put in photoproduction calculations since 1957 has not borne the fruits which all of us would expect. There is no question that many important and interesting results have been obtained, but when taken as a whole, the theoretical description of pion photoproduction in the $\Delta(1236)$ resonance region is still far from being satisfactory.

Dramatic theoretical developments do not seem at present imminent hence more effort should be put in a search for better models or for refinements in the existing ones. As to the latter possibility it seems that models discussed in the preceeding sections have one common illness - they are underdefined in what concerns the bearing of high energy effects on interactions in the resonance region. The models will probably remain at the present stage of development until some progress will be noted in a general problem of connection between the high (asymptotic ?) and medium (resonance) energy ranges. If come true the hopes that such a connection may indeed be approximately expressed through a few parameters there will be opened new possibilities of constructing within the existing framework, a more sound model of photoproduction in a few hundreds MeV region. Otherwise prospects of constructing such a model look rather slim.

The need for new ideas in model calculations is as strong as the need for more complete and more precise data mainly on photoproduction with polarized particles. Discrimination between the existing calculations is in many cases impossible merely as a result of scanty data. We have seen that e.g. the situation with the electric quadrupole $E_{1+}^{(3)}$ could look much clearer if we had at our disposal data on forward and backward photoproduction. Many more examples of this kind could be quoted and there is no doubt that any important progress in this field would also stimulate theoretical studies.

10. - ISOTENSOR ELECTROMAGNETIC CURRENT. FORMAL PROBLEMS. -

The opinion that the electromagnetic current has only isoscalar ($I = 0$) and isovector ($I = 1$) components has for long been an unquestionable principle in the study of electromagnetic interactions of hadrons. In photoproduction processes the mentioned isospin properties of the current give rise to the selection rule $\Delta I \leq 1$.

A possibility that the electromagnetic current may have a more general isospin structure (which means in the first place a possible existence of an isotensor $I = 2$ component) has been

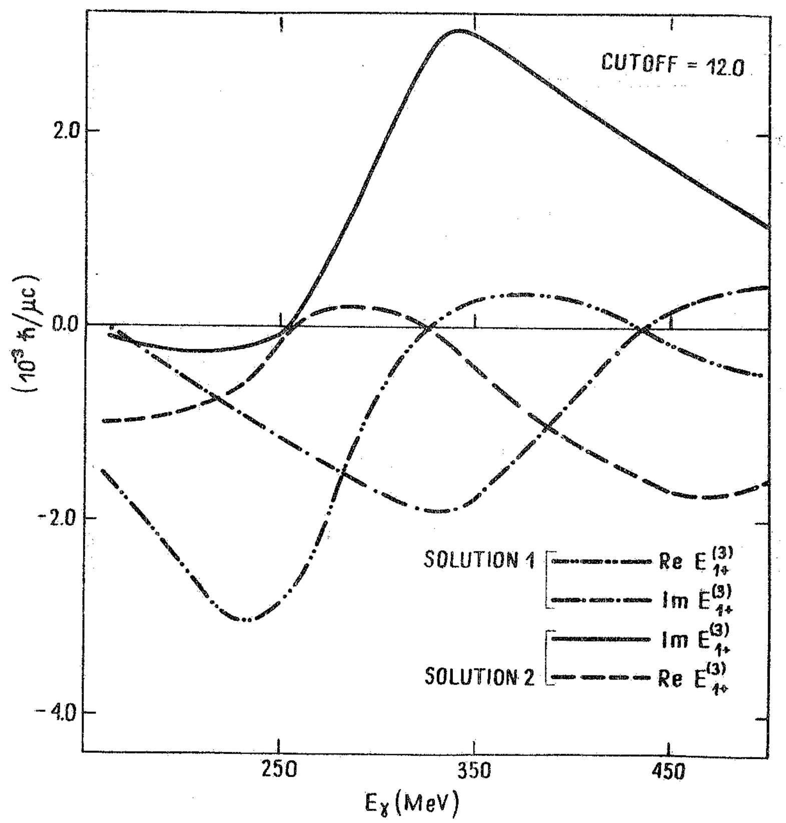

FIG. 20 - Values of $E^{(3)}_{1+}$ vs $E_\gamma = k_{Lab}$ as resulting from the fixed angle approach. SOLUTION 1 results from the coupled system of equations for $M^{(3)}_{1+}$ and $E^{(3)}_{1+}$; SOLUTION 2 results from the decoupled equation for $E^{(3)}_{1+}$. (Ref. (13)).

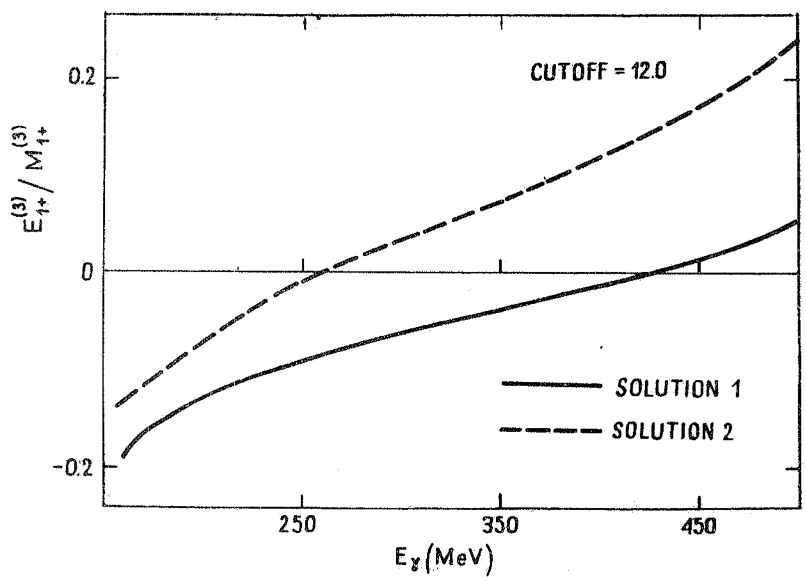

FIG. 21 - The ratio $E^{(3)}_{1+}/M^{(3)}_{1+}$ vs $E_\gamma = k_{Lab}$ according to the fixed angle approach. For details of notation see Fig. 20. (Ref. (13)).

advanced in connection with some problems in η decay[54]. Later on difficulties with conventional interpretation of data on π^- photoproduction off neutrons led several authors[55-58] to the conclusion that the $\Delta I \leq 1$ rule should be relaxed. A more conservative attitude led to estimates of bounds on the magnitude of a possible isotensor term[59, 60].

The present discussion will be confined to problems arising in connection with the presumed presence of an isotensor current in photoproduction of pion on nucleons. One should however bear in mind that the problem is by itself more general and pertains to the whole of electromagnetic interactions of hadrons.

The electromagnetic current density $j_\mu(x)$ can in principle be decomposed in a series

$$j_\mu(x) = j_\mu^{(0)}(x) + j_\mu^{(1)}(x) + j_\mu^{(2)}(x) + \cdots\cdots \tag{10.1}$$

where $j_\mu^{(i)}(x)$ transform as components of an irreducible isotensor of order i, and there is no a priori limitation on the number of possible terms in (10.1).

The usual truncation of the series is motivated by the Gell-Mann-Nishijima relation which states that the charge $Q = \int j_0(x)\,dx$ transforms under isospin transformations as the sum of an isoscalar and third component of an isovector. We get therefore

$$\int (j_0^{(0)}(x) + j_0^{(1)}(x) + j_0^{(2)}(x) + \cdots)\,dx = I_3 + \frac{Y}{2} \tag{10.2}$$

where the isoscalar Y denotes the hypercharge(*). It follows that $\int j_0^{(i)}(x)dx = 0$ for $i \geq 2$ from which formula it has been traditionally inferred that also $j_0^{(i)}(x) = 0$ for $i \geq 2$. The latter and the former condition are of course not equivalent. One notes however that if only the former condition is satisfied, the Gell-Mann-Nishijima relation is not violated by the presence of higher isospin components in (10.1), and a possible existence of such terms should be examined.

In the particular case of pion photoproduction a typical transition matrix element may be written in the form :

$$T_\mu = \int dx \left\langle N\pi, (I = \tfrac{1}{2}, \tfrac{3}{2}) \right| j_\mu(x) \left| N, (I = \tfrac{1}{2}) \right\rangle \tag{10.3}$$

which implies (in virtue of the Wigner-Eckart theorem[61]) that I = 0, 1, 2 components of $j_\mu(x)$ are allowed. With the use of some simple algebra[19, 62] we get the following isospin decomposition of the four amplitudes of physical processes :

$$\begin{aligned}
\langle \pi^+ n | T | \gamma p \rangle &= -\frac{\sqrt{2}}{3}\left(V_3 + \sqrt{\tfrac{3}{5}}\,\mathcal{T}\right) + \sqrt{2}\left(\tfrac{1}{3}V_1 + S\right) \\
\langle \pi^0 p | T | \gamma p \rangle &= \frac{2}{3}\left(V_3 + \sqrt{\tfrac{3}{5}}\,\mathcal{T}\right) + \left(\tfrac{1}{3}V_1 + S\right) \\
\langle \pi^- p | T | \gamma n \rangle &= \frac{\sqrt{2}}{3}\left(V_3 - \frac{\sqrt{3}}{5}\,\mathcal{T}\right) - \sqrt{2}\left(\tfrac{1}{3}V_1 - S\right) \\
\langle \pi^0 n | T | \gamma n \rangle &= \frac{2}{3}\left(V_3 - \sqrt{\tfrac{3}{5}}\,\mathcal{T}\right) + \left(\tfrac{1}{3}V_1 - S\right)
\end{aligned} \tag{10.4}$$

where S, V_1, V_3, $\mathcal{T}$ denote I = 1/2 (scalar), I = 1/2 (vector), I = 3/2 (vector) and I = 3/2 (tensor) amplitudes, respectively(o).

(*) - We skip here possible transformation properties of $j_\mu(x)$ under higher symmetry groups.
(o) - The currently used notation for the isotensor term is not uniform. The following correspondence may be established :

$$\mathcal{T} = \sqrt{15}\,A^{(T)} \text{ (Refs. (19, 63))} = -A^{(2)} \text{ (Ref. (55))} = -\sqrt{15}\,H^2 \text{ (Ref. (59)) etc.}$$

It became customary to introduce the amplitudes ${}_pA^I$ (${}_nA^I$) describing photoproduction of final states of definite isospin (I = 1/2, 3/2) off protons (neutrons)[55, 63]. The respective expressions are :

$$
\begin{aligned}
\langle \pi^+ n | T | \gamma p \rangle &= \sqrt{2}\,({}_pA^{1/2} - \tfrac{1}{3}\,{}_pA^{3/2}) \\
\langle \pi^o p | T | \gamma p \rangle &= {}_pA^{1/2} + \tfrac{2}{3}\,{}_pA^{3/2} \\
\langle \pi^- p | T | \gamma n \rangle &= \sqrt{2}\,({}_nA^{1/2} + \tfrac{1}{3}\,{}_nA^{3/2}) \\
\langle \pi^o n | T | \gamma n \rangle &= -\,{}_nA^{1/2} + \tfrac{2}{3}\,{}_nA^{3/2}
\end{aligned}
\tag{10.5}
$$

and exhibit the otherwise obvious situation that a complete information on the four independent amplitudes is available through data on four photoproduction processes. The majority of available data pertains to the ${}_pA^I$ amplitudes only. To get definite information on the amplitudes ${}_nA^I$ new measurements on negative and neutral photoproduction on neutrons are necessary. We recall that under the constraint $\Delta I \leq 1$ the information on photoproduction of π^- is necessary to determine the isoscalar parts of the I = 1/2 amplitude, whereas reliable data on neutral photoproduction off neutrons if available in a more or less distant future, would merely serve as a consistency check. Present difficulty with getting more reliable data on the last two reactions is the main reason of ambiguities and controversial opinions on the issue of possible presence of an isotensor component in the electromagnetic current density. This point will be discussed later.

The dispersion formalism of photoproduction may be easily generalized in order to include the isotensor I = 3/2 amplitude. It can be easily shown that due to crossing properties of the isotensor part of the photoproduction matrix element[19] the equations for the $\mathcal{T}$ amplitudes decouple from the others just as it happens to the isoscalar part. Since any isotensor part is excluded from $B_i^{(\alpha)}$ due to the isospin structure of the conventional pole term (representing π and N exchange) the equation analogous to (2. 1) will have the form :

$$
\mathcal{M}_j^{(T)}(v) = \frac{1}{\pi} P \int_{v_o}^{\infty} \frac{dv'}{v'-v}\, \mathrm{Im}\,\mathcal{M}_j^{(T)}(v') + \sum_{k=1}^{\infty} \int_{v_o}^{\infty} dv'\, K_{jk}^{(TT)}(v',v)\, \mathrm{Im}\,\mathcal{M}_k^{(T)}(v')
\tag{10.6}
$$

Fortunately enough all the existing model calculations of photoproduction can be adapted to the ΔI = 0, 1, 2 situation. In order to obtain ${}_p\mathcal{M}_j^{(3)}(v)$ or ${}_n\mathcal{M}_j^{(3)}(v)$ of (10. 5) a solution of (10. 6) multiplied by an appropriate coefficient should simply be added to the previously calculated multipole $\mathcal{M}_j^{(3)}(v)$. Such calculations have not been performed as yet, but most probably would not help us very much. We know that the nonsingular integrals of (2. 1) are in general much smaller than the pole term. It follows that (apart from the terms involving arbitrary constants) equation (10. 6) will yield small $\mathcal{M}_j^{(T)}(v)$. Such small multipoles will be of little use to us in view of what we have learned in the preceeding sections about difficulties in quantitative determination of small multipoles.

A summary of these formal considerations could be that firstly, data available to us is at present too scanty for drawing model independent conclusions as to the presence of isotensor components in pion photoproduction and secondly, that dispersion model calculations of these components can hardly be expected to give reliable estimates of their magnitude. We shall see below that all efforts directed at proving (or disproving) the necessity of introducing isotensor amplitudes to pion photoproduction are actually little more than attempts to show how much the existing models (or fitting procedures) fairly explaining photoproduction off protons are able (or unable) to match the existing data on negative pion photoproduction off neutrons. Since the data itself leaves to speculate not only on the issue of violation of the $\Delta I \leq 1$ rule but also on problems with extracting π^- data from measurements on deuteron or even on possible violation of T-invariance, it is not surprising that any clear evidence against the $\Delta I \leq 1$ rule in pion photoproduction is still missing.

11. - ISOTENSOR ELECTROMAGNETIC CURRENT. INTERPRETATION OF PHOTOPRODUCTION EXPERIMENTS. -

Modifications due to the presence of isotensor terms do affect the $I = 3/2$ multipoles and most easly detectable changes will probably be those in the $M_{1+}^{(3)}$ multipole. It is therefore obvious that since the first suggestions of Shaw(54), Gittelman and Schmidt(59), and others, up to the present days(56, 57, 60, 63) great effort has been concentrated on studying thoroughly photoproduction at resonance (or equivalently - on looking after a possible isotensor excitation of $\Delta(1236)$). It became customary in this connection to introduce through the formula ${}_nM_{1+}^{(3)} = (1+x)_p M_{1+}^{(3)}$ the parameter x which gives a measure of the isotensor contribution to the total magnetic dipole excitation in the $I = 3/2$ state.

The best way of extracting possible isotensor terms would be to compare neutral photoproduction off neutrons with that off protons at resonance. The latter reaction is known to be well described by a strongly dominating $\Delta(1236)$ excitation with relatively small background. Were the description of the former process analogous we could estimate an isotensor admixture by plotting the difference of cross sections for the two processes in function of energy around resonance. Unfortunately such a test is not available as yet and we have rather to exploit data on charged photoproduction. It follows that some speculation on the role of non-resonant background terms cannot in general be avoided.

The difference of π^+ and π^- cross sections $\Delta\sigma$ may be written in the following short-hand notation exhibiting the isospin dependence:

$$\Delta\sigma \propto \mathrm{Re}\left\{ (V_1 - V_3)\left(S - \frac{1}{15}\mathcal{T}\right)^* \right\} \tag{11.1}$$

Sanda and Shaw(55) used this expression as a starting point of their considerations. They discussed the difference Δ' of the total cross sections around resonance written in such a way as to separate the slowly varying background from the rapidly varying contributions to the magnetic dipole excitation:

$$\Delta' \propto \mathrm{Re}\left\{ M_{1+}^{(0)} M_{1+}^{(3)*} + (M_{1+}^{(1)} - M_{1+}^{(3)}) \frac{M_{1+}^{(T)*}}{\sqrt{15}} \right\} + \text{slowly varying terms.} \tag{11.2}$$

In virtue of the Watson theorem the $M_{1+}^{(0)}$ multipole is almost real in the $\Delta(1236)$ region hence the first term will (roughly speaking) change sign at resonance producing a monotonic change in Δ' over that energy region if only $M_{1+}^{(T)}$ vanishes. If, on the contrary, the isotensor part does not vanish then the last term in (11.2) (actually equal to $|M_{1+}^{(3)}| \cdot |M_{1+}^{(T)}|$) will give rise to a contribution of constant sign and strong dependence on energy. It follows that a dip or a peak may appear in Δ' plotted against energy. The values of Δ' in function of energy around resonance are shown in Fig. 22. The experimental points have been calculated from π^+ data of Refs. (37, 50) and from π^- data of Refs. (65, 66, 67, 68). Taken at face value the clearly marked dip near the (3, 3) resonance could indeed be an evidence for isotensor excitation of $\Delta(1236)$. According to the calculations of Sanda and Shaw(55) and Donnachie and Shaw(56) a reasonable fit to the experimental points requires a negative 20 - 30 % admixture of isotensor part to the ${}_nM_{1+}^{(3)}$ multipole (or $x = -0.2 \div -0.3$)(*) which is a significant contribution.

Conclusions similar to those of Refs. (55, 56) have also been drawn in Ref. (58) in connection with measurements on radiative π^- capture process : $\pi^- p \to \gamma n$. Striking enough the respective data seem to differ significantly (by 30 %) from deuteron data just at the resonance $\Delta(1236)$, whereas both measurements seem to agree at a higher energy. This is shown in Fig. 23. Such a situation adds unexpected complications to the whole understanding of the photoproduction data at resonance. Suggestions as to a possible violation of T-invariance have been advanced in this connection(53, 63, 69) but one cannot exclude that the hitherto used methods of handling deuteron data are inadequate in the (3, 3) resonance region. We can not deal here at

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(*) - We leave aside the important issue of a possible isotensor current violating T (or C) invariance which has also been discussed in connection with interpreting the π^- data(56, 63, 69).

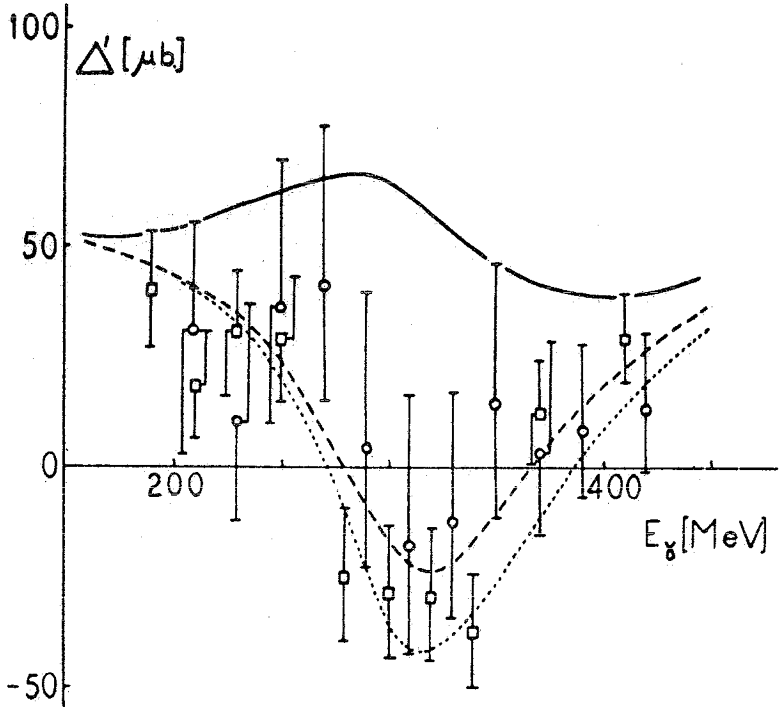

FIG. 22 - The difference $\Delta' \propto (\sigma_T^{(\pi^-)} - \sigma_T^{(\pi^+)})$ vs $E_\gamma = k_{Lab}$ from the fit (dashed curve) and for the conventional (no isotensor terms) model (solid line). The dotted curve was obtained when cross sections on radiative π^- capture were used instead of $\sigma_T^{(\pi^-)}$. Data points are from Ref. (65) (squares) and Ref. (66) (circles). (Ref. (56)).

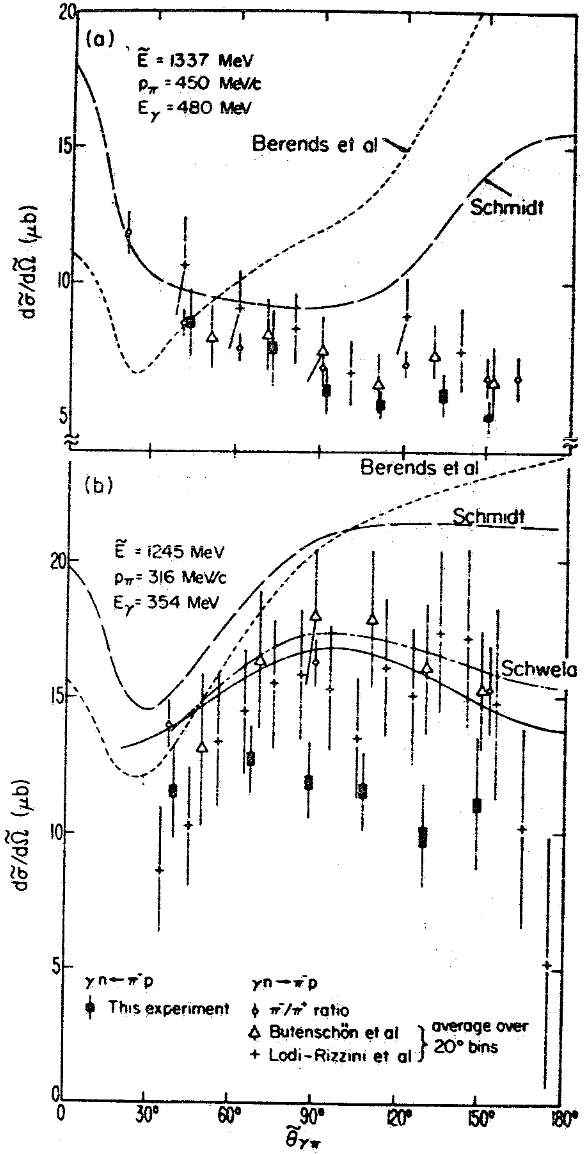

FIG. 23 - Differential cross sections for $\gamma n \rightarrow \pi^- p$ and the inverse process $\pi^- p \rightarrow \gamma n$. Tilded symbols refer to the c. m. system, in particular $\tilde{E}$ denotes the total energy, $\tilde{\theta}_{\gamma\pi}$ the production angle. Experimental points are from Berardo et al.(58) ("This experiment"), Refs. (38, 45, 50) (π^-/π^+), Butenschön et al.(66) and Lodi-Rizzini et al.(65). Theoretical curves are of Berends et al.(11), Schmidt(72), and Schwela(8). One notes a clear discrepancy between data on $\gamma n \rightarrow \pi^- p$ and on the inverse process at $\tilde{E}$ = 1245 MeV close to resonance, in contrast to fair agreement between both sets of data at $\tilde{E}$ = 1337 MeV. (Ref. (58)).

length with the latter problem but we should bear in mind that its existence weakens many conclusions reported in the following.

Turning back to Ref. (58) we note that the authors did not get acceptable fits when various conventionally calculated photoproduction amplitudes were used[10, 11, 37, 39, 70] (*). By relaxing the $\Delta I \leq 1$ rule and using the model of Sanda and Shaw[55] a reasonable fit to radiative capture data could be obtained for $-0.4 < x < -0.2$ which agree with suggestions of Ref. (55). The $\pi^- p \to \gamma n$ and deuteron data can be fitted simultaneously if in addition to the isotensor term some T violation is allowed (in this connection see also Refs. (56, 63)). The results are shown in Fig. 24.

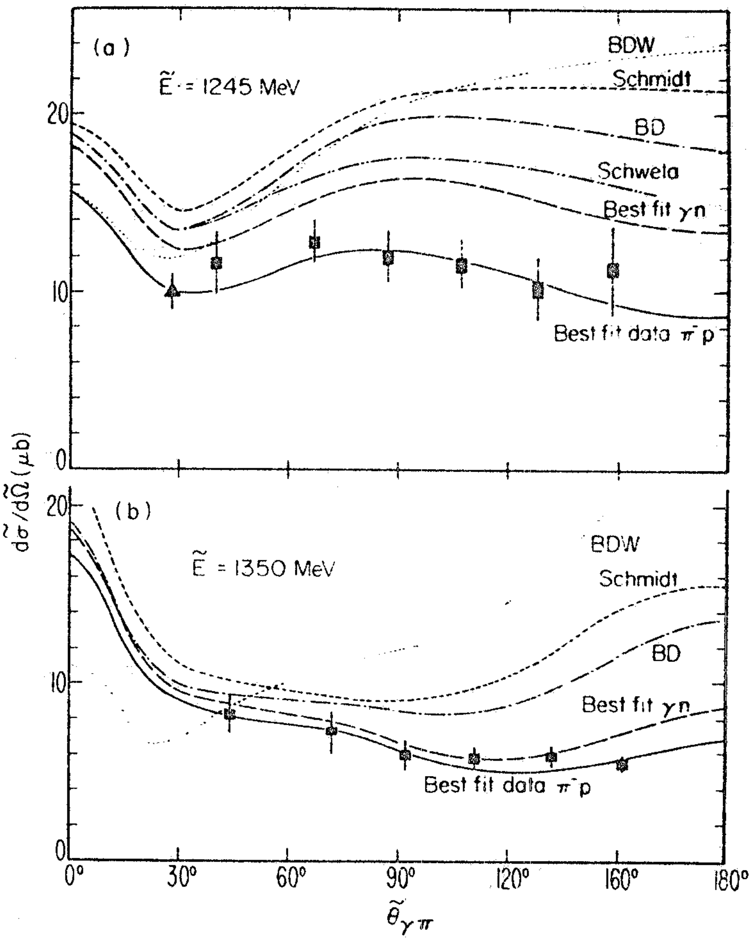

FIG. 24 - Comparison of experimental cross sections for $\gamma n \to \pi^- p$ obtained via detailed balance from $\pi^- p \to \gamma n$ with theoretical predictions. Data points and fits to data (solid curves for fits to $\gamma n \to \pi^- p$ and long-dash curves for $\pi^- p$ γn) are of Berardo et al.[58]. Theoretical predictions are by Berends et al.[11] (BDW), Berends and Donnachie[70] (BD), Schwela[8] and Schmidt[72]. (Ref. (58)).

From what was told above one may be inclined to think that there is indeed strong evidence in favour of an isotensor component of the $\Delta(1236)$ magnetic excitation and, that the magnitude of the $M_{1+}^{(T)}$ amplitudes is appreciable. We shall comment on this conclusion in a while.

The question to be answered now is, how was it possible to get fair agreement with data on photoproduction off protons at the $\Delta(1236)$ resonance within the framework of models ne-

(*) - Unfortunately in some fits of Ref. (58) multipoles resulting from different calculations seem to have been used simultaneously.

glecting such a large isotensor component as suggested by fits to π^- data. The static model of CGLN[1] may be here the first example.

An answer to this question comes almost directly from a look on equation (10. 6). If we specify this equation to the $M_{1+}^{(T)}$ case and (according to the usual approximation scheme) neglect all nonsingular coupling terms except self coupling, we get a homogeneous equation. Under the usual assumption that the (3, 3) phase tends to π at infinity, our homogeneous solution is identical (up to an arbitrary multiplicative constant, see Sec. 4) to the homogeneous part of $M_{1+}^{(3)}$. While calculating ${}_pM_{1+}^{(3)}$ the two constants merge into one which is the object of fits of e. g. Ref. (8). Only an analogous fit to neutron data can discriminate between the two constants thus giving information on an isotensor component. Corrections to other multipoles in the $\Delta(1236)$ region are probably masked by uncertainties in calculating those multipoles.

The above reasoning is of course far from being general but it relies on exactly the same approximation scheme as has been used in most dispersion model calculations discussed in the preceding sections.

Following a more conservative attitude (which is also shared by the author of this text) the situation is not that clear. The first point that needs comment is the phenomenology of data. In contrast to the abundant proton measurements, neutron data is still scanty and practically confined to cross section measurements. They are still subject to sensible alterations and in particular the data of Ref. (66) has been reanalyzed by Benz et al.[71] yielding larger cross sections than before. As to the above mentioned dip in Δ', Berends and Weaver[63] estimate that it is a one standard deviation effect based on two points. They claim that the dip disappears if one uses the data of Ref. (66) together with those of Ref. (67). Amusing enough these are exactly the measurements which according to Ref. (56) produced the dip in question although Ref. (56) and Ref. (63) differ as to the treatment of proton data.

The discrepancy between deuteron data and the inverse process (π^- radiative capture) has been mentioned before. It should be taken seriously since whatever are the reasons for this discrepancy it seems impossible to get satisfactory fit to both types of data simultaneously, unless T invariance is relaxed (although radiative capture could be fitted simultaneously with data from π^-/π^+ ratio[60]). Other possibilities of reconciling both types of data have not been exploited. Here the importance of having the π^-/π^+ ratio from deuteron data should be stressed as this quantity is less affected by unknown deuteron corrections.

One should therefore not be astonished if the apparent successes with relaxing the $\Delta I \leq$ ≤ 1 rule and T-invariance were merely successes in fitting data with an increased number of parameters. To answer this question much more measurements are necessary. The existing data does not allow for model independent tests and in the absence of such tests any progress in understanding the current problems in pion photoproduction at resonance can hardly be attained.

This leads to the second point which needs comments. Most hitherto quoted calculations aiming at a quantitative explanation of negative π photoproduction data are based on dispersion models and therefore each specific calculation is worth as much as the model itself. The models of Sanda and Shaw[55] and of Berends and Weaver[63] are based on dispersion calculations of Berends, Donnachie and Weaver[11]. Discrepancies between "conventional calculations" and measurements on $\pi^- p \rightarrow \gamma n$ as reported in Ref. (58) and shown in Figs. 23 and 24 also refer to earlier dispersion model calculations. It is therefore important to realize that some "small multipoles" of Ref. (11) compared with the results of phenomenological fits of Refs. (37, 52) show substantial deviations which amount to 100%, e. g. discrepancies in $E_{0+}^{\pi^0}$ are serious.

As to the multipoles of Schwela[10] they have been calculated from a dispersion model[7-10] so as to fit proton data and π^- data on total cross sections[66]. The results of Ref. (10) clearly demonstrate that the best fit to data of Ref. (66) matches rather poorly the data on π^-/π^+ ratio. It was shown more recently by Schwela[60] that without introducing isotensor terms and by merely changing the open constants (pertaining to the multipoles $E_{0+}^{(0,1)}$ and $M_{1-}^{(0,1)}$) the fit of Ref. (10) can be modified in such a way as to fit π^-/π^+ data (at least partly) and data on the radiative π^- capture simulaneously. The multipoles mostly affected by the change are $E_{0+}^{(0,1)}$ and $M_{1-}^{(0,1)}$ which include the multipoles-principal troublemakers of charged photoproduction. It is worth mentioning that an attempt was made to improve the fit to π^- data by allowing also the open constant of $M_{1+}^{(3)}$ to change (and we have seen before that such an assumption is equivalent to allowing for some isotensor component). The modified fit to π^- cross section data yields a 3% upper limit on a possible isotensor admixture, which is a rather conservative estimate and agrees with an earlier result of Gittelman and Schmidt. The estimate of Ref. (60) is unfortunately of little value since according to Schwela the modified fit better agrees with the measured assymetry ratio but remains poor for the π^-/π^+ ratio. On the other hand a 20% isotensor admixture as suggested in Ref. (55) is claimed in Ref. (60) to give no reasonable fit either to the deuteron or to the π^- capture data.

Summarising all that, there seems to be only one clear point in the confusing situation and namely that there exists discrepancy between various measurements on the reaction $\gamma n \rightleftarrows \pi^- p$. Are these discrepancies genuine (due e.g. to violation of T invariance) or are they merely a result of erroneous extracting of π^- data from measurements on deuteron is a question which requires further study but stands essentially beyond the existing model calculations of photoproduction off nucleons.

For the present confusion we should blame not only the ambiguous data but also, and before all the important differences between the multipoles calculated according to different models. It seems that the situation does not allow for decisive statements and for that reason any concluding remark seems at this place inappropriate.

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APPENDIX ADDED IN PROOF

The discussion of the isotensor electromagnetic current in photoproduction given in Sec. 11 of this text represents the state of affairs as known to the author in the summer of 1971.

Since then, some developments have been reported, and they will be briefly summarized below.

As to theoretical studies, efforts were concentrated on explaining the dip of Fig. 22. Continuing the previous line of the Bonn group(60, 37) Noelle and Pfeil(A1) performed a multipole analysis of π^- data. Using these multipoles together with those previously obtained for π^+ photoproduction(37) Noelle and Pfeil concluded that no appreciable isotensor contribution is necessary to explain the dip. However, the dip of Ref. (A1) results much shallower than predicted e. g. in Ref. (66). Actually the curve of Noelle and Pfeil follows the upper limit of error bars in the dip region. This result has been achieved by allowing the $E_{0+}^{(0)}$ multipole to be strongly energy dependent around the (3, 3) resonance - a result which is incompatible with our previous ideas about this multipole, based on dispersion calculations.

In their successive paper Pfeil and Schwela(A2) included data of Ref. (58) on radiative capture of π^- to their fit and allowed for violation of T invariance. The analysis was performed at two energies only (350 MeV and 360 MeV γ energy in lab.). The picture resulting from this analysis is quite different from that suggested by Refs. (55, 56). In particular at 350 MeV best fits of Pfeil and Schwela require large isotensor components in the multipoles E_{0+} and M_{1-} (apart from M_{1+}) whereas E_{1+} is less affected. Since at 360 MeV it is just the opposite what results the whole analysis leaves an impression of being highly confusing.

The study of Donnachie and Shaw briefly summarized in Ref. (56) has become available in a detailed version(A3). The model is based on "Born+ Δ(1236) dominance" and allows for T violating isovector and isotensor M_{1+} multipoles. Such structure of the model implies that the remaining multipoles essentially have conventional values (with the exception of $M_{1-}^{(0,1)}$ which due to their inelasticity are explicitly involved in the fitting procedure). Donnachie and Shaw conclude that within this model one can fairly fit $\pi^{\pm}$ and π^0 data (including data on π^- capture(58)) only with an isotensor admixture $x \approx -0.3$, and a non negligible T violating phase. Comprehensive surveys of these problems can be found in the review papers by Donnachie(A4, A5) and also in the talk by Shaw(A6).

For completeness one should also mention a paper(A7) whose authors try to estimate the isotensor contribution to photoproduction amplitudes by studying the sum of differential cross sections for π^+ and π^- photoproduction as predicted by a dispersion approach at 350 MeV γ energy in lab. Unfortunately the assumptions made are so model dependent that the final result can be considered as conclusive.

Model independent tests for checking the $\Delta I \leq 1$ rule, consisting in inequalities between cross sections in various charge states have been analyzed by Field(A8) and Pais(A9).

The situation as described above may be subject to substantial changes due to the advent of new data. Namely counter data of Fujii et al.(A10) on the π^-/π^+ ratio and corrected data of the ABBHHM collaboration(71, A11) seem to indicate that the dip of Fig. 22 fills up, hence showing no necessity for isotensor terms. On the other hand these results when compared with those on π^- capture(58) indicate an enhancement of T violating terms. The absence of $I = 2$ excitation can also be inferred from data on electroexcitation of bound protons and neutrons in the Δ(1236) region measured by Bleckwenn et al.(A12).

On the other hand the results of the PFRN collaboration(A13) seem to confirm the existence of a dip. The discrepancy between the results of Ref. (A11) and Ref. (A13) seems at this moment quite serious.

The most up-to-date summary of the experimental situation can be found in the review by Susinno(A14).

Making use of the new data of Ref. (A11) (but apparently excluding the data on π^- capture) Pfeil and Schwela(A15) performed a multipole analysis of π^+, π^0 and π^- photoproduction in the energy region from threshold up to 450 MeV. They found obviously no need for introducing isotensor terms to the analysis but it should be remarked incidentally that non negligible discrepancy arose between the multipoles as found in Ref. (A15) and those predicted by dispersion calculations(9, 10, 11).

One may conclude that as far as experimentalists tell us, the question of the existence of an $I = 2$ component of the electromagnetic current remains open. Measurements of the π^-/π^+ ratio on deuterons and data on photoproduction of π^0 on neutrons would help us to clarify the situation. As to the problem of violation of T invariance, all present speculation rely on one experiment only, and new data on radiative π^- capture by protons would be strongly wanted.

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