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S. Ferrara: CONFORMAL SYMMETRY, OPERATOR PRODUCTS AND CANONICAL SCALING. -

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I. - INTRODUCTION. -

In the present report I will summarize some recent results on exact and broken conformal invariance in field theory.

The relevance of conformal symmetry has been recently investigated by many authors in several directions(1). Let us remember the main directions of investigations:

the bootstrap approach to the construction of a conformal invariant quantum field theory(2÷11).

This approach offers an alternative to the usual canonical perturbative theory and overcomes the well known difficulties due to infrared and ultraviolet divergences. Moreover it allows, at least in principle, to evaluate the approximate anomaly of the dimensions by means of boostrap equations for vertices and propagators. The main advantage of using conformal symmetry is that the functional form of the vertex and propagator is unique so, when inserted into conformal covariant equations they are self reproducing and allow to obtain simple algebraic equations. The main difficulty of this approach however lies in the fact that nobody knows if the new improved perturbation series is itself rapidly convergent in order to get a "first order" solution for the anomalous dimension stable with respects to higher order corrections. Only future computations will give an answer to this question.

The other direction of investigation, which will be the subject of this paper, is the implications of conformal symmetry on operator product expansions(12÷14).

The usefulness of this approach is that it gives a natural generalization of the Wilson(15) short-distance expansion to light-like distances(16) and also in all space-time in the limiting situation of an exact conformal invariant massless theory.

We will discuss in some details the kinematical and dynamical constraints that the assumption of the existence of operator expansions put on the theory. Obviously this is a strong assumption on the underlying theory. The price that one has to pay is the fact that one is dealing with strong conformal invariance instead of weak conformal invariance as only required in the boostrop approach for example. However we point out that by strong conformal invariance we mean invariance under the algebra instead of the group. This is indeed a much weaker assumption, in fact the latter form of strong conformal invariance would imply a non causal theory (due to the reverberation phenomenon) while the former is completely consistent with causality. The reason lies in the fact that the group is infinitely many connected while the algebra, which is related to the universal covering group, allows the right causal prescription. Finally we emphasize that the main motivation of these researchs are related to the renormalization group. In fact if one identify the scale limit (in the sense of Callan-Symanzik(17)) with the Gell' Mann Low(18) limit of an ordinary renormalizable quantum field theory then one can show that this limit is also conformal invariant(19). So, in the framework of renormalizable interactions conformal symmetry is relevant to the real world when one explore processes in which properties of the limiting (interacting) massless theory are relevant.

II. - CONFORMAL SYMMETRY AND OPERATOR PRODUCT EXPANSIONS. -

Wilson(15) has given a very elegant interpretation of the singular behaviour of the operator product $A(x)\,B(0)$ of any two local operators at short-distances. Assume in fact that in this limit the operators of the theory are classified according to dilatation symmetry, then each operator is associated to an irreducible representation of this group i.e. carries a well definite dimension. In the decomposition of the product into irreducible representations of the dilatation group we get

$$A(x)\,B(0) \underset{x\to 0}{\sim} \sum_n C_n(x^2)\, x^{\alpha_1}\ldots x^{\alpha_n} O_{\alpha_1\ldots\alpha_n}(0) = \tag{II.1}$$

$$= \sum_n C_n(x^2)^{-1/2(l_A+l_B-l_n)} O_n(0)$$

Note that in the short-distance limit the spin is not an important concept, in fact tensor operators near $x \to 0$ behave like scalars.

Let us now consider the light-cone limit. In this case, as Brandt, Preparata and Frishman pointed out[16], to a specified light-cone singularity $(1/x^2)$ all operators contribute with spin such that $l_m = l_n + m - n$. So if $O_{\alpha_1 \dots \alpha_n}(0)$ has spin n, the local operators of this type give a contribution in the form

(II. 2) $$\left(\frac{1}{x^2}\right)^{\frac{l_A + l_B - l_n + n}{2}} \sum_n x^{\alpha_1} \dots x^{\alpha_m} O_{\alpha_1 \dots \alpha_m}(0) .$$

Thus we see that at light-like distances dimension and spin are both important as space-time quantum numbers.

In this connection we observe that in a zero (or continuous) mass theory there is just an additional operators which relates spin and dimension and this is just the generator K_λ of special conformal transformations which acts an space-time as

(II. 3) $$x_\mu \to \frac{x_\mu + c_\mu x^2}{1 + 2c\,x + c^2 x^2}$$

which together the Poincarè generators and the dilatation D close a 15 parameter Lie Algebra which is isomaphic to $O(4,2) \sim SU(2,2)$.

This fact is not surprising if we observe that this group of transformations on space-time is just the largest group which leads invariant the notion of two-events at vanishing relativistic distance $(x-y)^2 = 0 \Leftrightarrow (x_\varrho - y_\varrho)^2 = 0 \quad \rho \in O(4,2)$. Note that the unusual space-time transformation previously defined is more transparent if we define the coordinate inversion[20]

(II. 4) $$x_\mu \to \left(\frac{1}{x}\right)_\mu = \frac{x}{x^2}$$

In fact we have

$$x_\mu^c = R(Rx_\mu + c_\mu)$$

Moreover in Lagrangian theory, in all renormalizable scheme of interaction, the following relation holds[21]

(II. 5) $$\partial^{\mu} \mathcal{K}_{\mu\nu}(x) = 2x_{\nu} \partial^{\lambda} D_{\lambda}(x)$$

where $D_{\lambda}(x)$ and $K_{\mu\nu}(x)$ are the dilatation and conformal currents and therefore the scale-invariant limit is also conformal invariant.

III. - CONFORMAL INVARIANCE ON THE LIGHT-CONE: DEEP INELASTIC SCATTERING AND ASYMPTOTIC FORM FACTORS. -

We now review the constraints of conformal invariance on the light-cone[13].

Let us consider two local conformal scalars A(x), B(y) and write down a Wilson expansion on the light-cone

(III. 1) $$A(x)B(0) \underset{x^2 \to 0}{\sim} \sum_n \left(\frac{1}{x^2}\right)^{\Delta_n} C_n x^{\alpha_1} \dots x^{\alpha_n} O_{\alpha_1 \dots \alpha_n}(0)$$

where
$$\Delta_n = \frac{l_A + l_B - l_n + n}{2}$$

It is easily to prove that a sum of this type can be always arranged in such a way that one has an infinite sequence of irreducible tensor fields $O^n_{\alpha_1 \dots \alpha_n}(x)$, irreducible under the conformal algebra, i.e. of definite scale dimension l_n and moreover with $\left[O^n_{\alpha_1 \dots \alpha_n}(0), K_{\lambda}\right] = 0$ plus their gradients $\partial_{\alpha_{n+1}} \dots \partial_{\alpha_n} O^n_{\alpha_1 \dots \alpha_n}(x) = O^{nm}_{1 \dots \ m}(x)$.

Using the Jacobi identities with the generator K_{λ} and the properties that the algebra is closed on the light-cone we get

(II. 2) $$A(x)B(0) \underset{x^2 \to 0}{\sim} \sum_n \left(\frac{1}{x^2}\right)^{\frac{l_A + l_B - \tau_n}{2}} C_n x^{\alpha_1} \dots x^{\alpha_n} \int_0^1 du\, f_n^{AB}(u) O_{\alpha_1 \dots \alpha_n}(ux)$$

when $\tau_n = l_n - n$ is the twist of the local generator $O^n_{\alpha_1 \dots \alpha_n}$ and

$$f_n^{AB}(u) = u^{\frac{l_A - l_B + l_n + n}{2}} (1-u)^{\frac{l_B - l_A + l_n + n}{2} - 1}$$

and the sum is understood over inequivalent irreducible representations of conformal algebra. Note that each representation gives a "causal" contribution to the operator product and moreover manifestly satisfies translation invariance on hermitean basis as implied by the property

(III. 3) $$f_n^{AB}(u) = f_n^{BA}(1-u) \qquad 0 \leq u \leq 1$$

Let us consider the implications of this expansion on some physically relevant matrix-elements of the massive theory.

For forward-matrix elements (deep inelastic scattering) we have

(III.4) $$\langle p|A(x)\,B(0)|p\rangle \underset{x^2\to 0}{\sim} \sum_n \left(\frac{1}{x^2}\right)^{\frac{l_A+l_B-\tau_n}{}} C_n^{AB}\, x^{\alpha_1}\dots x^{\alpha_n}\langle p|O_{\alpha_1\dots\alpha_n}(0)|p\rangle$$

as

$$q_\mu = (p-p)_\mu = 0$$

Then we see that canonical Bjorken scaling implies $l_n = l+n$ with $l=2$ on inequivalent irreducible representations of conformal algebra.

So the Bjorken scaling is a dynamical information on the spectrum of such representations as one can see by explicit computation of the second order Casimir operator

(III. 5) $$C_I = M^{\mu v}M_{\mu v} + 2PK - 2D^2 + 8\,D = 2n(n+2) + 2l_n(l_n - 4)$$

We remark that the previous analysis clarifies the kinematical role of conformal algebra. We see that scaling in deep-inelastic is a dynamical information not contained in conformal symmetry arguments.

Non-forward matrix elements.

The simplest case if the off-shell form factor[22]

(III. 6) $$W_\mu(x,p) = \langle 0|J_\mu(x)\,\phi(0)|p\rangle$$

where ϕ is a local field and J_μ is a conserved current. Let us consider the light-cone limit of this vertex and assume, according to Brandt and Preparata[23] that this limit is related to the asymptotic behaviour of the corresponding on-shell form factor, then one can use conformal symmetry on the light-cone and get

$$W_\mu(x,p) \underset{x^2\to 0}{\sim} \langle 0|T(J_\mu(x)\,\phi(0))|p\rangle \underset{x^2\to 0}{\sim}$$

(III. 7)

$$\sim (p\,\partial\,\partial_\mu - p_\mu \Box)\sum_n \left(\frac{1}{-x^2+i\varepsilon}\right)^{1/2(1-\tau_n)} (x\,p)^{n-1} \int_0^1 du\, f_n(u)\, e^{iupx}$$

and by Fourier transforming

$$W_\mu(p,q) = (p\;\;q^2 - p\,q\,q\;\;)\sum_n C_n(q^2)^{1/2(1-\tau_n)-2}\,(\omega)^{-1/2(1+\tau_n)} \;\; x$$

$$-q^2 \to \infty,\; \omega = 2m_\pi v/q^2 \text{ fixed}$$

(III. 8)

$$x\,(\omega-1)^{1-2}\,\theta(\omega-1)\,{}_2F_1\left(\frac{1-d_n}{2}, \frac{d_n+1-4}{2}+1; 1-1; 1-\frac{1}{\omega}\right)$$

where we have considered the π-electromagnetic form factor. The important point of eq. (III. 8) is that the threshold limit $\omega \to 1$ relevant to the mass shell is just

(III. 9)
$$W_2(q^2,s) \underset{\omega\to 1}{\sim} s^{1-2} \sum_n C_n(q^2)^{1-\frac{1}{2}(1+\tau_n)}$$

$(s = q^2(1-\omega))$ so

(III. 10)
$$F(q^2) = \operatorname{Im} G^{-1}(s)\, W_2(q^2,s) \underset{q^2\to\infty}{\sim} \sum_n (q^2)^{1-\frac{1}{2}(1+\tau_n)}$$

and the s-dependence in eq. (III. 9) exactly cancels the inverse propagator giving an unambiguous result in this limit.

From eq. (III. 10) we see that each term corresponds to a different light-cone singularity so if we assume light-cone dominance the term with the smallest decreasing behaviour dominates the form factor. As $\tau_n \geq 2$ we see that if $1 \leq 2$ we have the result

(III. 11)
$$F_{\pi}(q^2) \underset{-q^2\to\infty}{\sim} \left(\frac{1}{q^2}\right)^{l-1}$$

However if $l > 2$ the smallest twist operator such that $\tau_n < 1$ dominate the form factor.

This is what happens, under these assumptions, in the case of the canonical quark model where $l_{\pi} = 3$ and $\tau = 2$ for the axial current and we find

(III. 12)
$$F_{\pi}(q^2) \underset{-q^2\to\infty}{\sim} \left(\frac{1}{q^2}\right)^{3/2}$$

while the full conformal invariant result would give, as pointed out by Migdal $(1/q^2)^2$, as this behaviour corresponds just to the π-field itself in the operator product.

IV. - FULL CONFORMAL SYMMETRY. SELECTION RULES AND OPERATOR PRODUCT EXPANSIONS. -

In this Section we consider the constraints that full conformal symmetry put on the limiting theory, and moreover the consequences of the existence, in such a theory, of operator product expansions.

Polyakov[3, 6] firstly pointed out the constraints that conformal symmetry put on the general n-point correlation functions.

It can be shown that this function (we assume conformal scalars for simplicity).

(VI. 1)
$$W(x_1 \ldots x_n) = \langle 0|\phi_1(x_1)\ldots\phi_n(x_n)|0\rangle$$

can be written in term of an arbitrary function of $N = n(n-3)/2$ conformal invariant variables, the so called harmonic ratios.

For example for $n = 4$ $N = 2$ and we get

(IV.2)
$$\langle 0|\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)|0\rangle = \left[\frac{1}{(x_1-x_2)^2}\right]^{1/2(l_1+l_2-l_3-l_4)} \times$$

$$\text{(IV.2)}\quad x\left[\frac{1}{(x_1-x_3)^2}\right]^{1/2(l_1+l_3-l_2-l_4)}\left[\frac{1}{(x_1-x_4)^2}\right]^{l_4}\left[\frac{1}{(x_2-x_3)^2}\right]^{1/2(l_2+l_3+l_4-l_1)} f(X,Y)$$

where

$$X=\frac{(x_1-x_2)^2(x_3-x_4)^2}{(x_1-x_3)^2(x_2-x_4)^2}\qquad Y=\frac{(x_1-x_4)^2(x_2-x_3)^2}{(x_1-x_2)^2(x_3-x_4)^2}$$

for $n=3$ $N=0$ and we get, according to Polyakof

$$\text{(IV.3)}\quad \langle 0|\phi_1(x_1)\phi_2(x_2)\,d_3(x_3)|0\rangle=\left[\frac{1}{(x_1-x_2)^2}\right]^{1/2(l_1+l_2-l_3)} x \left[\frac{1}{(x_1-x_3)^2}\right]^{1/2(l_1+l_3-l_2)}\left[\frac{1}{(x_2-x_3)^2}\right]^{1/2(l_2+l_3-l_1)}$$

for $n=2$ $N=-1$ the solution is overdetermined and we get the Migdal's[22] selection rule

$$\text{(IV.4)}\quad \langle 0|\phi_1(x_1)\phi_2(x_2)|0\rangle = C\left[\frac{1}{(x_1-x_2)^2}\right]^{l_1} \quad l_1=l_2$$
$$= 0 \quad l_1\neq l_2$$

Selection rule for the two-point correlation functions involving arbitrary conformal tensors have been derived by Gatto, Grillo and the author[1]

$$\text{(IV.5)}\quad \langle 0|O_n(x_1)O_m(x_2)|0\rangle=0 \quad \text{unless} \quad \begin{matrix} l_n=l_m \\ n=m \end{matrix}$$

In the case of arbitrary conserved conformal tensors these authors derived also a selection rule for the vertex. If $\partial^{\alpha_1} O_{\alpha_1\cdots\alpha_n}(x)=0$ and then $l_n=2+n$

(IV.5) $$\langle 0|O_{\alpha_1 \dots \alpha_n}(x)\,A(y)\,B(z)|0\rangle = 0 \quad \text{unless} \quad 1_A = 1_B$$

This selection rule, in the particular case of n = 1 was independently derived by Migdal[22].

Let us write explicitely two interesting three-point functions:
Electromagnetic vertex

(IV.6) $$\langle 0|J_\mu(x)\,O(y)\,O(0)|0\rangle = \left[\frac{1}{x^2(x-y)^2}\right]^2 \left(\frac{1}{y^2}\right)^{1-1} \left[x^2(x-y)_\mu - (x-y)^2 x_\mu\right]$$

Gravitational vertex

$$\langle 0|\theta_{\mu\nu}(x)\,O(y)\,O(0)|0\rangle = \left[\frac{1}{x^2(x-y)^2}\right]^3 \left(\frac{1}{y^2}\right)^{1-1} \times$$

(IV.7) $$\times\left[x^4(x-y)_\mu(x-y)_\nu + (x-y)^4 x_\mu x_\nu - x^2(x-y)^2\left((x-y)_\mu x_\nu + x_\mu(x-y)_\nu\right) - \frac{1}{4}x^2 y^2 (x-y)^2 g_{\mu\nu}\right]$$

Another important consequence of conformal symmetry is the "vertex graph identity" of Parisi, Peliti, D'Eramo[9] for time-ordered three-point functions (scalar fields)

(IV.8) $$\langle 0|T(\phi_1(x_1)\,\phi_2(x_2)\,\phi_3(x_3))|0\rangle = \int d^4\xi\, \langle 0|T(\phi_1(x_1)\,\phi_2(x_2)\,\phi_3^x(x_3))|0\rangle \, \langle 0|T(\phi_3(\xi)\,\phi_3(x_3))|0\rangle$$

where $\phi_3^x(x_3)$ is a conventional operator, called the shadow of $\phi_3(x_3)$ of dimension $1^x = 4 - 1$.

This vertex graph identity has been already generalized to any tensor operator in the form[24]

(IV.9) $$\langle 0|T(O_{\alpha_1 \dots \alpha_n}(x_1)\,P_{\beta_1 \dots \beta_m}(x_2)\,Q_{\gamma_1 \dots \gamma_j}(x_3))|0\rangle =$$

$$= \int d^4\xi \, \langle 0 | T(O_{\alpha_1 \ldots \alpha_n}(x_1) P_{\beta_1 \ldots \beta_m}(x_2) Q^{x\, \bar{\gamma}_1 \ldots \bar{\gamma}_j}(x_3)) | 0 \rangle$$

(IV.9)

$$\langle 0 | T(Q_{\bar{\gamma}_1 \ldots \bar{\gamma}_5}(\xi) Q_{\gamma_1 \ldots \gamma_j}(x_3)) | 0 \rangle$$

Conformal covariance of eq. (IV.9) can be easily checked using the R operation (recall eq. (II.4)) and the transformation property of any conformal tensor ($[O_{\alpha_1 \ldots \alpha_n}(0), K_\lambda] = 0$) under R[14,20,24]

$$\text{(IV.10)} \qquad R\, O_{\alpha_1 \ldots \alpha_n}(x) = \left(\frac{1}{x^2}\right)^{l_n} M_{\alpha_1}^{\beta_1}(x) \ldots M_{\alpha_n}^{\beta_n}(x)\, O_{\beta_1 \ldots \beta_n}\left(\frac{1}{x}\right)$$

Where $M_{\mu\nu}(x)$ is the conformal metric

$$\text{(IV.11)} \qquad M_{\mu\nu}(x) = g_{\mu\nu} - 2 \frac{x_\mu x_\nu}{x^2}$$

with the covariance property

$$\text{(IV.12)} \qquad M_{\mu\nu}(x-y) = M_\mu^\varrho(x)\, M_\nu^\sigma(y)\, M_{\varrho\sigma}\left(\frac{1}{y} - \frac{1}{x}\right)$$

Using coordinate-inversion symmetry one can easily verify that the most general conformal covariant ansatz for two and three-point functions is respectively[24]

$$\text{(IV.13)} \quad \langle 0 | O_{\alpha_1 \ldots \alpha_n}(x)\, O_{\beta_1 \ldots \beta_n}(0) | 0 \rangle = C_n \left(\frac{1}{x^2}\right)^{l_n} M_{\alpha_1\beta_1}(x) \ldots M_{\alpha_n\beta_n}(x) - \text{traces}$$

$$\langle 0 | O_{\alpha_1 \ldots \alpha_n}(x)\, B(y)\, G(z) | 0 \rangle = C \left[\frac{1}{(y-z)^2}\right]^{1/2(l_B + l_C - \tau_n)} \times$$

$$\text{(IV.14)} \quad \times \left[\frac{1}{(x-y)^2}\right]^{1/2(\tau_n + l_B - l_C)} \left[\frac{1}{(x-y)^2}\right]^{1/2(\tau_n + l_C - l_B)} \times$$

$$\times \left[\left(\frac{1}{z-x}\right)_{\alpha_1} + \left(\frac{1}{x-y}\right)_{\alpha_1}\right] \ldots \left[\left(\frac{1}{z-x}\right)_{\alpha_n} + \left(\frac{1}{x-y}\right)_{\alpha_n}\right] - \text{traces}$$

Using exact conformal symmetry the program of working out a completely conformal invariant operator product expansions has already been carried out by Gatto, Grillo and the author(14). Formulas are complicated, at present, when spin operator are involved. However, for the scalar case we have

$$\text{(IV.15)}\quad A(x)\,B(0) = \left(\frac{1}{x^2}\right)^{\frac{l_A+l_B-l}{2}} \int_0^1 du\, f(u)\; {}_0F(l-1;-(\tfrac{x}{2})^2\Box u(1-u)\,O(ux)$$

when

$$f(u) = u^{\frac{l_A-l_B+l}{2}-1}\,(1-u)^{\frac{l_B-l_A+l}{2}-1}$$

Note that eq. (IV.15) is connected to the Wightman function $\langle 0|A(x)\,B(0)\,O(z)|0\rangle$ from the selection rule (IV.4). It can be also shown that expansion (IV.15) is definitely different from the operator generalization (IV.9)(25). This is due to the fact that, the convolution (vertex graph identity) of Wightman functions

$$\text{(IV.16)}\quad W(\phi(x_1)\,\phi(x_2)\,\phi(x_3)) = \int d^4\xi\; W(\phi(x_1)\,\phi(x_2)\,\phi^{x}(\xi))\,W(\phi(x_3)\,\phi(x_3))$$

is not conformal covariant.

However, as recently shown by Grillo, Parisi and the author, if one make the substitution

$$\text{(IV.17)}\qquad W(\phi(x_1)\,\phi(x_2)\,\phi^{x}(x_3)) \rightarrow f(x_1 x_2 x_3)$$

where

$$f(x_1x_2x_3)\Big|_{x_2=0} = (-x_1^2)^{1/2(l_3^x-l_1-l_2)}\Big\{\Big[-(x_1-x_3)^2+i\varepsilon\,\Big]^{1/2(l_1-l_1-l_3^x)} +$$

$$+\Big[-x_3^2+i\varepsilon\Big]^{1/2(l_1-l_2-l_3^x)} + \Big[-(x_1-x_3)^2+i\varepsilon\,(x_3^o-x_1^o)\Big]^{1/2(l_2-l_1-l_3^x)} +$$

$$+\Big[-x_3^2+i\varepsilon\, x_3^o\Big]^{1/2(l_1-l_2-l_3^x)}$$

then one can verify that eq. (IV.16) becomes conformed covariant. This is due to the fact that eq. (IV.16) with the substitution (IV.17) is nothing but the discontinuity of the conformal covariant vertex graph identity for T-products. In particular, in this way one gets an operator expansion which exactly coincides with eq. (IV.15).

V. - CANONICAL SCALING AND FREE LIMITING THEORY. -

In this last section we give the restrictions of a massive theory which in the scale limit become conformal invariant, where the scale limit has to be intended in the sense of Callan, Symanzik, Gell'Mann and Low, as explained in the introduction.

From the previous section we have learned that conformal symmetry gives all the two-point Wightman functions for arbitrary conformal tensors save for an overall constant factor.

Moreover we have the following result[13]:

- the divergence of an irreducible conformal tensor $O_{\alpha_1 \dots \alpha_n}$ (or order n) is itself conformal covariant provided the scale dimension of the tensor is fixed to be $l_n = 2+n$. From the selection rules of conformal algebra this implies the vanishing of the two-point correlation function of $\partial^{\alpha_1} O_{\alpha_1 \alpha_2 \dots \alpha_n}(x)$. But this implies the vanishing of this tensor and then the conservation law[26]

$$\partial^{\alpha_1} O_{\alpha_1 \dots \alpha_n}(x) = 0 \tag{V.1}$$

Note that eq. (V.1) is immediate if the Hilbert space has a positive definite metric.

However this result can be also derived under the assumption that all representations of conformal algebra are of the type alrealy described in sect. III.

From the previous discussion we then obtain that it is very hard to understand the canonical Bjorken scaling observed at SLAC. In fact it is well known that, from such scaling law it follows the existence, on the light-cone of a tower of operators with $l_n = 2+n$ and infinitely increasing spin which couple to the product of currents.

Assuming the usual connection with a limiting conformal invariant theory then this theory might exhibit an infinite set of symmetric traceless conformal tensors with dimension $l_n = 2+n$ and then necessarily conserved. As a consequence we have the following result: A renormalizable theory cannot have canonical scaling unless it does manifest an infinite number of conserved tensors. This last fact a forti ori implies a free-limiting theory (parton model) or a mechanism of (spontaneously) symmetry breaking which invalidates the previous theorem. However recently Schroer(11) pointed out that there could be a particular class of models, like the vector gluon model, where one has to fullfil gauge invariance, which are renormalizable and which moreover could possess canonical scaling. In fact, due to the gauge invariance constraints, these models become non-interacting in the conformal invariant scale limit.

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