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G. Parisi : ON THE IMPOSSIBILITY OF ANALITIC CONTINUATION OF STRUCTURE FUNCTIONS FROM THE DEEP INELASTIC SCATTERING TO THE ANNIHILATION REGION. -

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ABSTRACT. -

Using a very simple physical argument we show that if the interaction is strong enough to form bound states, the structure functions of inclusive e^+e^- annihilation into hadrons, cannot be the analitic continuation of those of deep inelastic scattering.

It is well known that the differential cross section for the inclusive experiment $e^-+p \to e^- +$ anything, depends from two independent functions $W_1(q^2, \nu)$, $W_2(q^2, \nu)$[1].

It was shown from the SLAC experiment[2] that these two functions scale, i.e. the two limits:

$$
(1) \qquad \lim_{\substack{q^2 \to -\infty \\ \frac{-2\nu}{q^2} = \omega}} W_1(q^2, \nu) = F_1^N(\omega)
$$

$$
\lim_{\substack{q^2 \to -\infty \\ \frac{-2\nu}{q^2} = \omega}} W_2(q^2, \nu) = F_2^N(\omega)
$$

exist and they define two non zero function of ω.

If scale invariance is valid also in the process $e^+e^- \to p +$anything it is possible to define in the same way two other functions[1] $A^N_{1,2}(\omega)$[1,3].

Many authors have suggested that scaling in the deep inelastic e p scattering implies scaling in the deep annihilation region and that $A^N_{1,2}(\omega)$ are the analytic continuation of $F^N_{1,2}(\omega)$[1,3].

In this letter we present a physical argument which implies that the two structure functions in the two different regions cannot be represented by the same analytic function. This result is derived under the hypothesis that there is scaling in the deep annihilation region.

The argument is simple: let us suppose that there is in the theory a continuous parameter g and the number of stable particles is not a continuous function of g. The physical meaning of the coupling constant g may be for example the SU(3) breaking parameter: at exact SU(3) the Δ (1236) and the Ω^-(1672) have the same mass and therefore they must be both stable or unstable.

If we denote by g_c the value of the simmetry breaking parameter at which the Δ becomes stable (or the Ω^- becomes unstable), we find that the deep annihilation structure functions are discontinuous functions of g near g_c, while the deep inelastic structure functions are at least continuous functions of g.

Physical intuition tell us that the total and the exclusive cross sections are continuous functions of g: the only difference between g a slightely bigger or smaller than g_c is that in the first case the produced Δ decay in $N\pi$ with a very long mean life, in the second case they are produced with roughly the same rate, but now they are stable and do not decay in $N\pi$.

We define the following functions:

$$(2) \quad F^N_{1,2^+}(\omega), \quad F^N_{1,2^-}(\omega), \quad A^N_{1,2^+}(\omega), \quad A^N_{1,2^-}(\omega), \quad A^\Delta_{1,2^+}(\omega), \quad A^\Delta_{1,2^-}(\omega)$$

where we denote by + or - the limit $g \to g_c$ from above or from below. The function $A^\Delta_{1,2\pm}(\omega)$ are the ones related to the process $e^+ + e^- \to \Delta +$ + anything. For g greater than g_c the Δ in the final state is identified by looking to its decay product; this procedure is not ambigous the limit $g \to g_c$: in this situation the Δ has zero whith.

According to the scaling hypothesis all this functions are different from zero. We define an other function in the annihilation region $\bar{A}^N_{1,2}(\omega)$: we consider only the protons which are not decays products of a Δ. $\bar{A}^N_{1,2}(\omega)$ is equal to $A^N_{1,2}(\omega)$ for g smaller than g_c but is different for g greater than g_c e.g.:

$$(3) \qquad A^N_{1,2^+}(\omega) = \bar{A}^N_{1,2^+}(\omega) + A^+_\Delta\left(\frac{m_N}{m_\Delta}\omega\right)$$

From the above discussion it follows that

(4) $F^{N}_{1,2^+}(\omega) = F^{N}_{1,2^-}(\omega)$ $\bar{A}^{N}_{1,2^+}(\omega) = \bar{A}^{N}_{1,2^-}(\omega)$, $A^{\Delta}_{1,2^+}(\omega) = A^{\Delta}_{1,2^-}(\omega)$

but

(5) $$A^{N}_{1,2^+}(\omega) \neq A^{N}_{1,2^-}(\omega)$$

provided only that $A^{\Delta}\rightarrow \neq 0$.

The physical meaning is clear: when g becomes less than g_c the Δ does not more decay in $N\pi$ so there is a discontinuous change in the cross section for inclusive proton production in e^-e^+ annihilation: we losė all the proton coming from a Δ , its branching ratio in $N\pi$ changes discontinuously from 1 to 0.

We arrive to the conclusion that $F^{N}_{1,2^+}(\omega) = F^{N}_{1,2^-}(\omega)$ but $A^{N}_{1,2^+}(\omega) \neq A^{N}_{1,2^-}(\omega)$ and this would be impossible if both F^N and A^N are the same analytic function.

We conclude that it is impossible that both above and under g_c the cross section for inclusive proton production in e^+e^- annihilation are the analytic continuation of deep inelastic cross section. This result follows from the osservation that in a expansion in powers of $g-g_c$ we find contributions of first order in the deep inelastic region while we find contribution of zero order in the annihilation region. These discontinuitines in single particle inclusive experiment are related to the sudden change of the structure of the in and out Hilbert spaces at $g = g_c$.

Our conclusions based on a physical arguments are not in disagreement with some field theoretical investigations: it has been recently prooved that, if we introduce elementary unstable particles even in simple Feynmann diagrams, the possibility of connecting the two different region with the same analytic function is lost[(4, 5)].

In this letter we have shown that the same conclusions are reached in any theory whose interaction is enough strong to produce bound states. We exclude therefore that in the real world there is any simple exact analytic relation between the cross sections for inclusive proton production in e^+e^- collisions and for e p scattering in the deep inelastic region.

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