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A. Bramon and M. Greco : NONET STRUCTURE OF THE  
 $J^{PC} = 3^{--}$  MESONS.

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**Nonet Structure of the  $J^{PC} = 3^{--}$  Mesons (\*)**

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**Summary.** — The meson resonances  $g(1660)$ ,  $\varphi_N(1650)$ ,  $\varphi_N(1830)$  and  $K_N(1760)$  are tentatively grouped in an «ideal»  $SU_3$  nonet having  $J^{PC} = 3^{--}$ . A discussion of their masses and their decay modes is presented showing general agreement with the available experimental data. Our results are also discussed in the light of finite-energy sum rules for the reactions  $\pi K \rightarrow \omega K$  and  $\pi K \rightarrow \varphi K$ .

**1. — Introduction.**

It is generally accepted that two of the main features of our understanding of hadron interactions, at least from a phenomenological point of view, are the approximate validity of unitarity symmetry schemes and the Regge behaviour of the scattering amplitudes. Only in a reduced number of cases however the immediate application of these models leads to a successful description of the experimental facts. When mesons are considered, one observes that only the natural, spin-parity  $J^{PC} = 1^{--}$  and  $2^{++}$  nonets are well understood in the framework of a fundamental quark model or  $SU_3$  <sup>(1)</sup>. Similarly the only well-established Regge trajectories are those containing these mesons. For the large majority of the remaining cases the introduction of supplementary and sometimes arbitrary assumptions is often unavoidable. In this sense we think that an attempt to study the  $3^{--}$  mesons, belonging to the

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(\*) To speed up publication, the authors of this paper have agreed to not receive the proofs for correction.

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<sup>(1)</sup> See for example the rapporteurs' talks by B. FRENCH, H. HARARI and G. MORPURGO: at the *XIV International Conference on High-Energy Physics* (Vienna, 1968).

natural series and lying on the leading Regge trajectories, would be particularly attractive.

On the other hand a new  $K^*$ -resonance decaying mainly into  $K\pi$  and  $K\pi\pi$  has been very recently observed <sup>(2)</sup>, suggesting a natural spin-parity assignment to this state.

In the present paper a phenomenological analysis of the  $3^-$  resonances is done, showing that the observed properties of these states can be fairly well understood in terms of an «ideal»  $SU_3$  nonet structure. In Sect. 2 we discuss the mass properties of these  $3^-$  mesons suggesting the identification of some of the actually known resonances with the members of this nonet. Further evidence in favour of these assignments is obtained in Sect. 3 where the decay widths and branching ratios are analysed. Finally we study the implications of our results in the framework of finite-energy sum rules for the reactions  $\pi K \rightarrow \omega K$  and  $\pi K \rightarrow \phi K$ .

## 2. - Masses and linear trajectories.

The  $g$  or  $\rho_N(1660)$  has been for some time the only meson showing some evidence for a  $J^P = 3^-$  assignment <sup>(3)</sup>. This choice was also supported by the value of its mass,  $(1660 \pm 20)$  MeV, which allowed to identify the  $g$ , in a world of linear Regge trajectories with a common slope, as the first recurrence of the  $\rho$ -meson. Because of the experimental confusion existing in the  $R$ -region, to which the  $g$ -meson belongs, it is however very hard to establish clearly its main properties and decay modes.

In a recent paper CARMONY *et al.* <sup>(2)</sup> have reported on the observation of a  $K_N(1760)$  strange meson with a probable  $3^-$  spin-parity assignment and whose mass and width are  $(1759 \pm 10)$  MeV and  $(60 \pm 20)$  MeV, respectively. The interpretation of this state as the Regge recurrence of the  $K^*(890)$ -meson follows quite naturally.

The whole situation is shown in Fig. 1, where a common slope  $\alpha' \simeq (0.9 \text{ GeV})^{-2}$  has been assumed for all trajectories. The very approximate  $\rho$ - $\omega$  mass degeneracy implies a similar equality between the masses of the  $g$  and its isoscalar partner. This suggests the identification of the  $\phi_N(1650)$  of mass  $(1664 \pm 14)$  MeV and natural spin-parity as such an  $I = 0$  state. Analogously the recurrence of the  $\phi$ -meson is predicted to have a mass roughly corresponding to the  $\phi_N(1830)$  enhancement observed in different experi-

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<sup>(2)</sup> D. D. CARMONY, D. CORDS, H. W. CLOPP, A. F. GARFINKEL, R. F. HOLLAND, F. J. LOEFFLER, H. B. MATHIS, L. K. RAUGAN, J. ERWIN, R. L. LANDER, D. E. PELLETT, P. M. YAGER, F. T. MEIERE and W. L. YEN: *Phys. Rev. Lett.*, **27**, 1160 (1971).

<sup>(3)</sup> PARTICLE DATA GROUP: *Rev. Mod. Phys.*, **43**, No. 2, Part. 2 (1971).

ments (4,5). The nonet of tensor mesons is also shown in Fig. 1 indicating that within the experimental uncertainties the expected degeneracy of trajectories  $\alpha_\rho = \alpha_{A_2} = \alpha_\omega = \alpha_f$ ,  $\alpha_{K^*} = \alpha_{K^{**}}$  and  $\alpha_\phi = \alpha_{f'}$ , is well satisfied (6).

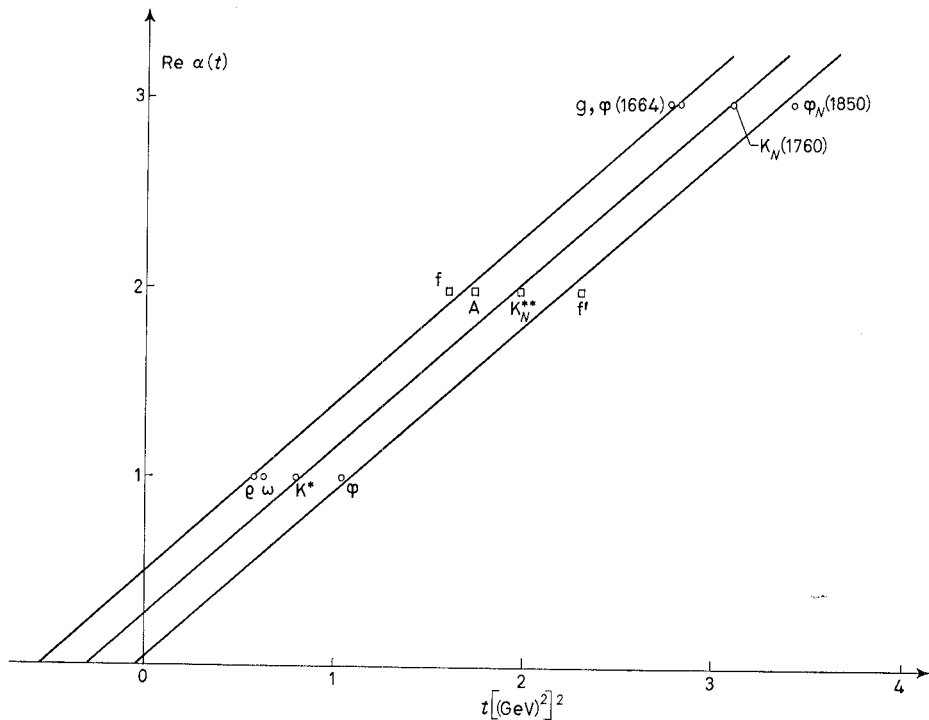


Fig. 1.

The quark model, on the other hand, predicts the following mass formula (7):

$$(1) \quad 2\hat{K}^* = \hat{\phi} + \rho, \quad \text{if } \hat{\omega} \simeq \hat{\rho},$$

where the symbol  $\hat{V}$  means the squared mass of the  $V$ -like mesons, which is well satisfied by the vector and tensor meson nonets. Since eq. (1) is linear

(4) J. A. DAUYSZ, B. R. FRENCH and V. SIMAK: *Nuovo Cimento*, **51** A, 801 (1967).

(5) B. R. FRENCH, J. B. KINSON, R. RIGOPOULOS, V. SIMAK, F. McDONALD, G. PETMEZAS and L. RIDDIFORD: *Nuovo Cimento*, **52** A, 438 (1967).

(6) C. B. CHIU and J. FINKELSTEIN: *Phys. Lett.*, **27** B, 510 (1968); see also M. JACOB: *Lectures on elementary particles and quantum field theory*, in *Brandeis Summer Institute*, edited by S. DESER, M. GRISARU and H. PENDLETON (1970).

(7) See for example the papers by R. H. DALITZ and by G. ZWEIG: in *Meson Spectroscopy*, edited by C. BALTAY and A. H. ROSENFELD (New York, 1968).

in the squared masses of the particles, it is also expected to be satisfied by the  $3^-$  nonet. It predicts in fact the mass of the  $\varphi_N(1830)$  to be 1.85 GeV. Note however that a very similar value is obtained using eq. (1) for the masses instead of the squared masses of the particles.

From the Gell-Mann–Okubo (linear or quadratic) mass formula the value of the mass of the isoscalar member of the octet is predicted to be 1.79 GeV. The mixing angle between the  $\varphi_N(1664)$  and  $\varphi_N(1850)$  states is then  $\theta \simeq 34^\circ$  in both cases and roughly coincides with that obtained for the well-established  $1^-$  and  $2^+$  nonets.

Consequently, the nonet of  $3^-$  mesons seems to present, as expected, an « ideal » quark model structure. In particular, the  $\varphi_N(1664)$  state is analogous to the  $\omega$  or  $f$  mesons in the sense that it can be considered as constructed by nonstrange quarks only. The  $\varphi_N(1850)$  on the contrary is a pure  $\lambda\bar{\lambda}$  state similar to the  $\varphi$  and  $f'$  mesons.

### 3. – Decay rates and coupling constants.

According to ref. (2) the only final states observed in the decay of the  $K_N(1760)$ -meson are  $K\pi$ ,  $K^*(890)\pi$  and  $K\rho$ . This suggests that the dominant decay modes of the other members of the  $3^-$  nonet, let us symbolize them by  $G$ , are also of the type  $G \rightarrow PP'$  and  $G \rightarrow VP$ , where  $P$  and  $V$  stand for pseudoscalar and vector mesons. They are explicitly listed in the first column of Table I.

The Lorentz-invariant matrix elements for the decays  $G \rightarrow PP'$  and  $G \rightarrow VP$  are given by (8)

$$(2a) \quad T_{GPP'} = g_{GPP'} G^{\alpha\beta\gamma} q_\alpha q_\beta q_\gamma$$

and

$$(2b) \quad T_{GVP} = g_{GVP} \varepsilon_{\alpha\beta\gamma\delta} V^\alpha p^\beta q^\gamma G^{\delta\mu\lambda} q_\mu q_\lambda,$$

where  $G^{\alpha\beta\gamma}$  and  $V^\alpha$  are the polarization tensors of the  $3^-$  and  $1^-$  mesons respectively,  $q$  is the four-momentum of one of the pseudoscalars and  $p$  that of the vector meson. Usual phase-space integrations lead to the expressions for the partial widths:

$$(3a) \quad \Gamma_{G \rightarrow PP'} = \frac{g_{GPP'}^2}{4\pi} \frac{1}{35} \frac{|\mathbf{q}|^7}{m_G^2},$$

$$(3b) \quad \Gamma_{G \rightarrow VP} = \frac{g_{GVP}^2}{4\pi} \frac{4}{105} |\mathbf{p}|^7,$$

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(8) G. CARBONE: *Nuovo Cimento*, **58** A, 23 (1968).

TABLE I.

Decay modes $G \rightarrow PP', VP$	Phase space $\cdot 10^4$ (GeV) <sup>(5,7)</sup>	Coupling constant	Branching ratios (%)	Partial width (MeV)
$K_N(1760) \rightarrow K\pi$	20	$1/2 g_F^2$	23.6	$14.2 \pm 5.9$
$K\eta$	8.3	$1/2 g_F^2$	9.4	$5.6 \pm 2.3$
$K^*\pi$	17	$9/20 g_D^2$	35.7	$21.4 \pm 8.9$
$\rho K$	12	$9/20 g_D^2$	22.5	$13.5 \pm 5.6$
$\omega K$	10	$3/20 g_D^2$	6.7	$4.0 \pm 1.7$
$\phi K$	1.0	$6/20 g_D^2$	1.4	$0.84 \pm 0.35$
$K^*\eta$	2.7	$1/20 g_D^2$	0.7	$0.42 \pm 0.18$
Total				$60 \pm 20$
$g(1660) \rightarrow \pi\pi$	26	$2/3 g_F^2$	42.1	$24.4 \pm 10.2$
$K\bar{K}$	6.0	$1/2 g_F^2$	4.9	$2.8 \pm 1.2$
$\omega\pi$	16	$3/5 g_D^2$	46.3	$26.8 \pm 11.2$
$K^*\bar{K}$	1.3	$3/5 g_D^2$	3.7	$2.1 \pm 0.9$
$\rho\eta$	3.2	$1/5 g_D^2$	3.0	$1.7 \pm 0.7$
Total				$58 \pm 24$
$\varphi_N(1664) \rightarrow K\bar{K}$	6.3	$1/3 g_F^2$	2.9	$2.9 \pm 1.2$
$\rho\pi$	18.7	$9/5 g_D^2$	93.0	$93 \pm 39$
$K^*\bar{K}$	1.5	$3/5 g_D^2$	2.5	$2.5 \pm 1.0$
$\omega\eta$	3.1	$1/5 g_D^2$	1.6	$1.6 \pm 0.7$
Total				$100 \pm 43$
$\varphi_N(1850) \rightarrow K\bar{K}$	14.6	$2/3 g_F^2$	28.5	$13.7 \pm 5.7$
$K^*\bar{K}$	9.5	$6/5 g_D^2$	64.2	$30.8 \pm 12.8$
$\phi\eta$	1.6	$4/5 g_D^2$	7.3	$3.5 \pm 1.5$
Total				$48 \pm 20$

whose numerical values, apart from the coupling constants  $g^2/4\pi$ , are given in Table I.

Let us now turn to the problem of relating different coupling constants under the assumption of exact  $SU_3$  symmetry. The pseudoscalar mesons are assumed to transform according to an octet representation considering that the small  $\eta$ - $\eta'$  mixing angle has negligible effects in this case (\*). The  $3^-$  and  $1^-$

(\*) In so doing no predictions can be made for processes involving the mainly singlet meson  $\eta'$ . But due to its large mass these processes are expected to be strongly depressed.

mesons, on the contrary, belong to two «ideal» nonets of  $SU_3$ , as discussed at the end of the preceding Section. Because of the violation of charge conjugation parity by the coupling  $G_1 P_8 P_8$ , all different  $G \rightarrow PP'$  decays can be expressed in terms of only one unknown  $g_F$ , as shown in third column of Table I. The situation concerning the vector-pseudoscalar decays is more involved since three different couplings,  $G_8 V_8 P_8$ ,  $G_1 V_8 P_8$  and  $G_8 V_1 P_8$ , are now allowed. We can use however one of the main features of the quark model (?) to relate these three couplings, which by simple  $SU_3$  should be regarded as independent. We shall make use of the fact that both  $\varphi$  and  $\varphi_N(1830)$  mesons are constructed only by strange quarks and, therefore, their couplings to any couple of mesons containing only nonstrange quarks are expected to be strongly suppressed. This suppression is known to occur in all cases where is expected, namely in the transitions  $\varphi \rightarrow \rho\pi$ ,  $\varphi \rightarrow \pi\gamma$  and  $f' \rightarrow \pi\pi$  (\*). Our final results are then listed, in terms of the  $D$ -type coupling constants  $g_D$  only, in the third column of Table I.

It is now clear that the different branching ratios of the decays  $G \rightarrow PP'$  and  $G \rightarrow VP$  are fixed once the value of the relative coupling strength  $g_F/g_D$  is given. The most direct way to estimate this ratio is to use the experimental results

$$(4) \quad \frac{K_N \rightarrow K\pi}{K_N \rightarrow K^*\pi + \rho K} = 0.4 \pm 0.1$$

quoted in ref. (3). The result

$$(5) \quad \frac{g_F^2}{g_D^2} = (0.52 \pm 0.13) (\text{GeV})^2$$

is easily obtained. It is worth observing that a similar result is obtained by evaluating the analogous ratio from the experimental data (3) on the  $V \rightarrow PP'$  and  $V \rightarrow VP$  decays:

$$(6) \quad \left( \frac{g_F^2}{g_D^2} \right)_V \simeq 0.46 (\text{GeV})^2.$$

Such an agreement can be explained by using  $SU_6$  arguments and is also pre-

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(\*) The coupling constants  $g_{\varphi\rho\pi}$  and  $g_{\varphi\gamma\pi}$  are known to be roughly a factor of 500 smaller than the corresponding  $g_{\omega\rho\pi}$  and  $g_{\omega\gamma\pi}$ . A small upper limit can be similarly found for the ratio  $g_{f'\pi\pi}^2/g_{f'K\bar{K}}^2$ .

dicted by a large variety of quark models (\*). The different branching ratios evaluated through eq. (5) are given in the fourth column of Table I.

Some comparisons with experimental data are now possible. The ratio

$$\frac{\mathbf{K}_N(1760) \rightarrow \mathbf{K}^*\pi}{\mathbf{K}_N(1760) \rightarrow \rho\mathbf{K}} = \frac{40 + 15}{60 \pm 25}$$

measured in ref. (2) is compatible, within the large errors, with our predictions. As far as the g-meson decay modes are concerned, the only noncontroversial experimental result is (9,10)  $(g \rightarrow \mathbf{K}\bar{\mathbf{K}})/(g \rightarrow \pi\pi) = 0.08 \pm 0.026$  in good agreement with our results. The prediction of the dominance of the  $\rho\pi$  mode in the  $\varphi_N(1650)$  decays is also quite satisfactory (3).

In order to get an estimate of the absolute partial widths a new input is required, which essentially fixes the absolute value of  $g_p$  or  $g_D$ . The experimental result (2)

$$(7) \quad \Gamma(\mathbf{K}_N(1760) \rightarrow \text{all}) = (60 \pm 20) \text{ MeV}$$

seems to be the most adequate for our purposes. As a consequence we obtain the predictions quoted in the last column of Table I, where the indicated errors result from those combined from eqs. (4) and (7). Alternatively we could have chosen as input the world average result (3)  $\Gamma(\varphi_N(1650) \rightarrow 3\pi, 5\pi) = (141 \pm 17) \text{ MeV}$ . The presence however of the  $5\pi$  decay mode, not of the type  $\text{PP}'$  or  $\text{VP}$  and therefore not included in our analysis, makes this choice less adequate to our purposes. Nevertheless the widths predicted in both ways would be mutually compatible because of the experimental uncertainties.

Our predictions concerning the total width of the  $\varphi_N(1850)$  is consistent with the available experimental information (3). In particular, FRENCH *et al.* (5) measured  $\Gamma_{\varphi_N(1850)} = (50 \pm 20) \text{ MeV}$  observing a clear bump in the  $\mathbf{K}\bar{\mathbf{K}} + n\pi$  mass spectrum which, according to Table I, should be the main decay mode of this resonance.

The experimental information about the g-meson is, as mentioned before, far from being clear, the measurements of its total width giving values ranging from 20 to 200 MeV. This may suggest that more than one resonance is present

(\*) Notice however that this result is not necessarily expected to hold for the  $2^{++}$  nonet because of the opposite behaviour under  $G$ -conjugation.

(9) D. J. CRENNELL, U. KARSHOWN, K. W. LAI, J. M. SCARR and I. O. SKILLICORN: *Phys. Lett.*, **28** B, 136 (1968).

(10) J. BARTSCH, G. KRAUS, N. TSANOS, C. GROTE, H. KOTZAN, A. MEYER, E. RYSECK, A. ANGELOPOULOS, G. ASCOLI, K. BARNHAM, V. T. COCCONI, P. F. DALPIAZ, R. ELY, J. D. HANSEN, D. R. O. MORRISON, J. SCHREIBER and H. TOFTE: *Nucl. Phys.*, **22** B 109 (1970).



in the  $g$ -meson region, decaying into the same states (\*). The data by CRENNELL *et al.* (9) based on the simultaneous observation of the  $K\bar{K}$  and  $\pi\pi$  decay modes, where contamination effects and background are probably less important than in the  $4\pi$  decay modes, are particularly suggestive. They find  $\Gamma(g \rightarrow \text{all}) = 79_{-25}^{+70}$  MeV in reasonable agreement with our predictions.

Finally we note that different decay modes, for example  $G \rightarrow VS$  or  $G \rightarrow TP$ , where S and T mean  $0^{++}$  and  $2^{++}$  meson states respectively, can obviously be present. There are, for example, indications (3) for a  $\phi_x(1650) \rightarrow 5\pi$  decay mode; in addition several enhancements of the states  $\Lambda_2\pi$ ,  $\rho\pi\pi$  have sometimes been associated with the  $g$ -meson (3). This should imply that the total widths of the  $3^-$  mesons are larger than the results of Table I. The stringent upper limits for the analogous decays of the  $K_x(1760)$  (2) would indicate a limited importance of the effect.

#### 4. - Finite-energy sum rules and the $3^-$ mesons.

We would like now to discuss the implications of our analysis using finite-energy sum rules. Quite apart from history, we believe that FESR provide a useful framework wherein our results can be tested with some confidence.

The  $g$ -meson has been studied by ADEMOLLO *et al.* (14) in the reaction  $\pi\pi \rightarrow \pi\omega$ , as the first recurrence of the  $\rho$  trajectory. It has been shown that the inclusion of the  $3^-$  state in the lowest-moment sum rule leads to an improvement of the equality between the resonance and the Regge sides of the equations, the agreement getting better in a larger region of the momentum transfer  $t$ . In particular, from the following parametrization of the leading Regge trajectory for high  $\nu$  and fixed  $t$ :

$$(8) \quad A(\nu, t) \xrightarrow{\nu \rightarrow \infty} \frac{\tilde{\beta}(t)}{\Gamma[\alpha(t)]} \frac{1 - \exp[-i\pi\alpha(t)]}{\sin \pi\alpha(t)} (2\alpha'\nu)^{\alpha(t)-1}$$

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(\*) In support of this hypothesis we note that evidence for the existence of a  $\rho'$  decaying into  $4\pi$  at this energy has been presented by DAVIER *et al.* (11) and BARBARINO *et al.* (12), as discussed in ref. (13).

(11) M. DAVIER, I. DERADO, D. C. FRIES, F. F. LIU, R. F. MOZLEY, A. C. ODIAN, W. P. SWANSON, F. VILLA and D. YOUNT: SLAC-PUB-972 (1971).

(12) G. BARBARINO, M. GRILLI, E. IAROCCHI, P. SPILLANTINI, V. VALENTE, R. VISENTIN, F. CERADINI, M. CONVERSI, L. PAOLUZI, R. SANTONICO, M. NIGRO, L. TRASATTI and G. T. ZORN: *Lett. Nuovo Cimento*, **3**, 689 (1972).

(13) A. BRAMÒN and M. GRECO: Frascati report LNF-71/97 (1971); *Lett. Nuovo Cimento*, to be published.

(14) M. ADEMOLLO, H. R. RUBINSTEIN, G. VENEZIANO and M. A. VIRASO: *Phys. Rev.*, **176**, 1904 (1968).

and with our definitions of the couplings, eqs. (2), they found

$$(9) \quad \frac{g_{\pi\pi\pi}g_{\pi\pi\omega}}{g_{\rho\pi\pi}g_{\rho\pi\omega}} = \frac{\tilde{\beta}(m_\pi^2)}{\tilde{\beta}(m_\rho^2)} 2\alpha'^2 = 1.4 \text{ (GeV)}^{-4},$$

where the last equality follows from the assumption  $\tilde{\beta}(t) = \text{const.}$

Furthermore, Shapiro's <sup>(15)</sup> extension of the Veneziano model <sup>(16)</sup> to  $\pi\pi$  scattering leads to the prediction  $\Gamma(g \rightarrow \pi\pi) = 38 \text{ MeV}$ . This latter result derived in the narrow-width approximation should be taken as only indicative.

Since, as seen before, the experimental evidence seems to weigh clearly in favour of the  $3^-$  state belonging to the  $K^*$  trajectory, we discuss in detail the implications of our results in the reactions  $\pi K \rightarrow \omega K$  and  $\pi K \rightarrow \varphi K$ . A remarkable property of these two processes is that all the natural-parity resonances lying on the degenerate  $K^*$  and  $K^{**}$  trajectories can be exchanged in the symmetric  $s$  and  $u$  channels, while the asymptotic behaviour is given by the  $\rho$  Regge trajectory exchanged in the  $t$ -channel, which couples strongly to the  $\pi\omega$  vertex and weakly to  $\varphi\omega$ .

The  $T$ -matrix for both processes is described in terms of a single invariant amplitude  $A(\nu, t)$  defined through

$$(10) \quad T = \varepsilon^{\mu\nu\sigma} V_\mu p_{1\nu} p_{2\sigma} p_{3\sigma} A(\nu, t),$$

where  $V_\mu$  is the polarization vector of the  $\omega$  ( $\varphi$ ) mesons, and the momenta are defined as follows:  $\pi(p_1) + K(p_2) \rightarrow \omega$ ,  $\varphi(q) + K(p_3)$ . Finally,  $s = (p_1 + p_2)^2$ ,  $t = (p_2 - p_3)^2$  and  $\nu = (s - u)/4$ .

The contribution of the leading Regge pole to the amplitude is parametrized for high  $\nu$  and fixed  $t$  as in eq. (8). The lowest-moment sum rule therefore reads

$$(11) \quad \int_{\text{thresh}}^{\bar{\nu}} \nu \text{Im} A(\nu, t) = \frac{\tilde{\beta}(t)}{\Gamma[\alpha(t)]} \frac{1}{\alpha(t) + 1} (2\alpha' \bar{\nu})^{\alpha(t)-1} \bar{\nu}^2.$$

The low-energy absorptive part is assumed to be dominated by the exchange in the  $s$  and  $u$  channels of the  $K^*$ ,  $K^{**}$  and  $K_N(1760)$  mesons. We obtain

$$(12a) \quad A_\omega^{K^*}(\nu, t) = \frac{g_{K^*K\pi}g_{K^*\omega K}}{s - m_{K^*}^2} + (s \leftrightarrow u),$$

$$(12b) \quad A_\omega^{K^{**}}(\nu, t) = \frac{g_{K^{**}K\pi}g_{K^{**}\omega K}}{s - m_{K^{**}}^2} p p' \cos \theta_s + (s \leftrightarrow u),$$

$$(12c) \quad A_\omega^{K_N}(\nu, t) = \frac{g_{K_N K\pi}g_{K_N \omega K}}{s - m_{K_N}^2} p^2 p'^2 \left( \cos^2 \theta_s - \frac{1}{5} \right) + (s \leftrightarrow u),$$

<sup>(15)</sup> J. A. SHAPIRO: *Phys. Rev.*, **179**, 1345 (1969).

<sup>(16)</sup> G. VENEZIANO: *Nuovo Cimento*, **57 A**, 190 (1968).

and similarly for  $A_{\phi}^{K^*}(\nu, t)$ ,  $A_{\phi}^{K^{**}}(\nu, t)$  and  $A_{\phi}^{K_N}(\nu, t)$ . We have defined

$$(13a) \quad p = \frac{1}{2\sqrt{s}} \{(s - m_{\pi}^2 - m_K^2)^2 - 4m_{\pi}^2 m_K^2\}^{\frac{1}{2}},$$

$$(13b) \quad p' = \frac{1}{2\sqrt{s}} \{(s - m_{\omega}^2 - m_K^2) - 4m_{\omega}^2 m_K^2\}^{\frac{1}{2}},$$

$$(13c) \quad \cos \theta_s = \frac{1}{4pp'} \left\{ 2t + s - \Sigma + \frac{1}{s} (m_{\pi}^2 - m_K^2)(m_{\omega}^2 - m_K^2) \right\},$$

with  $\Sigma = 2m_K^2 + m_{\pi}^2 + m_{\omega}^2$ . The coupling constants involved in the eqs. (12) are taken from the experimentally observed decays and, if necessary, by using  $SU_3$  symmetry. Numerically they are

$$(14a) \quad \frac{g_{K^*K\pi}^2}{4\pi} = 10, \quad \frac{1}{4\pi} g_{K^*\omega K} = \frac{1}{4\pi} \left( \frac{1}{\sqrt{2}} g_{K^*\phi K} \right)^2 = 6.5 \text{ (GeV)}^{-2}.$$

$$(14b) \quad \frac{g_{K^{**}K\pi}^2}{4\pi} = 21.6 \text{ (GeV)}^{-2}, \quad \frac{1}{4\pi} g_{K^{**}\omega K}^2 = \frac{1}{4\pi} \left( -\frac{1}{\sqrt{2}} g_{K^{**}\phi K} \right)^2 = 8.85 \text{ (GeV)}^{-4},$$

$$(14c) \quad \frac{g_{K_N K\pi}}{4\pi} = 7 \text{ (GeV)}^{-4}, \quad \frac{1}{4\pi} g_{K_N \omega K}^2 = \frac{1}{4\pi} \left( \frac{1}{\sqrt{2}} g_{K_N \phi K} \right)^2 = 4.08 \text{ (GeV)}^{-6}.$$

As a first check of the values we have used, we notice that from the exchange degeneracy of the  $K^*$  and  $K^{**}$  trajectories one would get the relation

$$g_{K^{**}K\pi} g_{K^{**}\omega K} = 2\alpha' g_{K^*K\pi} g_{K^*\omega K},$$

which is very well verified by eqs. (14a) and (14b).

Using eqs. (12) the sum rule (11) is explicitly written as

$$(15a) \quad g_{K^*K\pi} g_{K^*\omega K} (t + 0.47) + \frac{1}{2} g_{K^{**}K\pi} g_{K^{**}\omega K} (t + 2.83)(t + 0.41) + \\ + \frac{1}{4} g_{K_N K\pi} g_{K_N \omega K} (t + 5.06)[(t + 0.97)^2 - 0.18] = \frac{4}{\pi} \frac{\bar{\beta}_{\omega}(t)\alpha(t)}{I[2 + \alpha(t)]} (2\alpha' \bar{p})^{\alpha(t)-1} \bar{p}^2$$

for the  $\omega$ -meson and

$$(15b) \quad g_{K^*K\pi} g_{K^*\phi K} (t + 0.04) + \frac{1}{2} g_{K^{**}K\pi} g_{K^{**}\phi K} (t + 2.41)(t + 0.17) + \\ + \frac{1}{4} g_{K_N K\pi} g_{K_N \phi K} (t + 4.64)[(t + 0.74)^2 - 0.10] = \frac{4}{\pi} \frac{\bar{\beta}_{\phi}(t)\alpha(t)}{I[2 + \alpha(t)]} (2\alpha' \bar{p})^{\alpha(t)-1} \bar{p}^2$$

for the  $\phi$ .

Going now to the positive- $t$  region one notes that at  $t = m^2$  the ratio

$$R \equiv \bar{\beta}_{\omega}(m_{\phi}^2) / \bar{\beta}_{\phi}(m_{\phi}^2) = g_{\rho\omega\pi} / g_{\rho\phi\pi}$$

is approximately equal to 32, in agreement with the very weak coupling of the  $\varphi$  to the  $\rho\pi$  system, as compared to the  $\omega$ -meson. It is worth noticing that this nice feature is only due to the inclusion of the  $K_N(1760)$  in the sum rule, and provides therefore a good check for our calculations. Equations (15) in fact, limited in the l.h.s. to the contributions of the  $K^*$  and  $K^{**}$ , would lead to  $R = 2.2$ , in strong disagreement with experiments. We observe also that eq. (9), predicted by ADEMOLLO *et al.* <sup>(14)</sup> assuming  $\bar{\beta}(t) = \text{const}$ , provides the products  $g_{K_N K \pi} g_{K_N \omega K}$  and  $g_{K_N K \pi} g_{K_N \varphi K}$  a factor two bigger than ours, and leads to  $R \simeq 3.7$  also in disagreement with the data. Similar considerations hold at  $t = 0$ , where we find  $\bar{\beta}_\omega(0)/\bar{\beta}_\varphi(0) = g_{\gamma\omega\pi}/g_{\gamma\varphi\pi} \simeq 38$  to be compared with the value 3.4 obtained saturating the sum rule with only the  $K^*$  and  $K^{**}$  mesons.

As far as the negative- $t$  region is concerned, the behaviour of the resonance side of sum rules (15) goes differently in the two cases. The first zero for  $\alpha(t) = 0$  is in fact approximately present in each term in the l.h.s. of (15a) and the insertion at each step of a new term slightly shifts its position. The inclusion of the  $3^-$  state furthermore improves the position of the zero at  $\alpha(t) = -2$ . In the  $\varphi$  case, on the contrary, the first zero, which occurs at  $t \simeq 0$  when the  $K^*$  alone is considered, is obtained through the contribution of next resonances which add together with alternate signs. Notice that it is this peculiarity of the signs responsible for the smallness of the resulting coupling, while in the  $\omega$  case all the resonances add together with the same sign to build up a strong coupling of the  $\rho$  trajectory at the  $\omega\pi$  vertex.

We do not go into great detail in the discussion of the saturation problem which goes beyond the scope of this paper because it requires both an accurate knowledge of the residue functions  $\bar{\beta}_{\omega,\varphi}(t)$  and the inclusion of other resonances on both the parent and daughter trajectories.

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#### ● RIASSUNTO

Le risonanze mesoniche  $g(1650)$ ,  $\varphi_N(1650)$ ,  $\varphi_N(1830)$  e  $K_N(1760)$  sono classificate in un nonetto « ideale » di  $SU_3$  avente  $J^{PC} = 3^{--}$ . Si presenta una discussione delle loro masse e dei relativi modi di decadimento che risultano essere in buon accordo con i dati sperimentali a disposizione. I nostri risultati sono anche discussi alla luce di regole di somma ad energia finita applicate alle reazioni  $\pi K \rightarrow \omega K$  e  $\pi K \rightarrow \varphi K$ .

Структура нонета мезонов  $J^{PC} = 3^{--}$ .

**Резюме (\*).** — Мезонные резонансы  $g(1660)$ ,  $\varphi_N(1650)$ ,  $\varphi_N(1830)$  и  $K_N(1760)$ , в порядке эксперимента, группируются в «идеальный»  $SU_3$  нонет, имеющий  $J^{PC} = 3^{--}$ . Проводится обсуждение их масс и мод распада, которые обнаруживают общее согласие с имеющимися экспериментальными данными. Наши результаты рассматриваются также в свете правил сумм при конечной энергии для реакций  $\pi K \rightarrow \omega K$  и  $\pi K \rightarrow \varphi K$ .

(\*) *Переведено редакцией.*

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