G. Parisi: ON SELF CONSISTENCY CONDITION IN CONFORMAL COVARIANT FIELD THEORY. -

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A very existing problem is to compute the anamalous dimensions[1] of the fundamental fields in a Lagrangian quantum field theory with scaling invariant interaction.

An important step in this direction was done indipendently by A. A. Migdal[2] and by L. Peliti[3] and the author: they have shown that in a confor mal invariant field theory with trilinear interaction it was possible to write a self consistency equation for the vertex and the conformal invariant solution of this equation is determined by the knowldge of the renormalized coupling costant g and of the anomalous dimension η. In this way the integral Dyson is equation was reduced to a simple numerical relation between and g.

(1) $$F(\eta, g) = 1$$

In the same paper[3] was also shown that unitarity implies a self consistency condition for the propagator and this condition is equivalent to a new indipendent costraint on η and g.

(2) $$H_V(\eta, g) = 1$$

The solution of the system

(3) $$F(\eta, g) = 1$$
$$H_V(\eta, g) = 1$$

yelds both the anomalous dimension of the field and the renormalized coupling costant for the zero mass theory[4, 5].


2.

K. Symanzik[6] noticed that it was possible to write a second self-consistency equation for the propagator using the so called Ward's differential equation for the propagator[7]. In this way he arrives to the system:

(4)
$$F(\eta, g) = 1$$
$$H_w(\eta, g) = 1$$

It was recently shown by G. Mack and K. Symanzik[8] the complitely equivalence of the two different approaches: the propagator satisfies the unitarity condition if the other two conditions are implemented and the systems 3 and 4 have the same solutions.

The aim of this letter is to find a very simple relation between the functions F and H in order to avoid unnecessary computations.

For sake of simplicity let us consider a 6-simensional $\lambda\phi^3$ theory, the extention of the whole argument to a 4-dimensional $g\bar{\psi}\gamma_5\psi\pi$ is straightforward. The self consistency equation for the existence of an operator 0 of dimension a is shown in Fig. 1: V is the vertex between 0 and the field ϕ and K is the kernel of the Bethe Salpeter amplitude. In Fig. 2 is shown the diagrammatical expansion of the vertex in product of the vertex Γ and of the propagator.

FIG. 1

FIG. 2
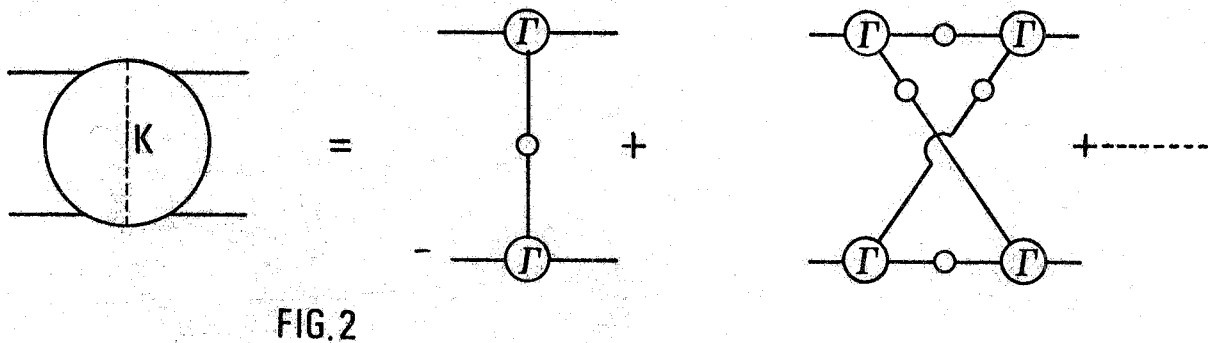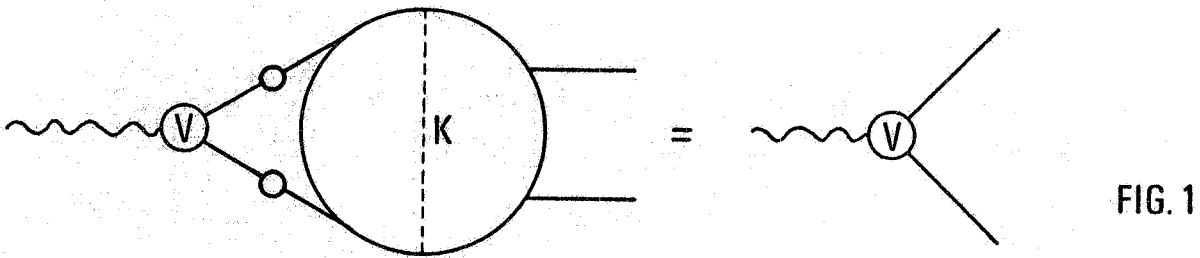
Using standard arguments[2 ÷ 5] the self consistency equation is equivalent to the following equation:

(5)
$$D(a, \eta, g) = 1$$

The result of this paper is that the system 4 is equivalent to the system:

$$D\ (\eta, \eta,\ g)\ =\ 1$$

(6)

$$\pi^{18} \frac{\Gamma^3(1+\frac{\eta}{2})\ \Gamma^4(1-\eta)\ \Gamma(\frac{3}{2}\eta)\ \Gamma(-1+\eta)}{\Gamma^3(2-\frac{\eta}{2})\ \Gamma^4(2+\eta)\ \Gamma(3-\frac{\eta}{2})\ \Gamma(4-\eta)}\ g^2\ \frac{d}{d\alpha}\ D\ (\alpha,\ \eta,\ g)\ \bigg|_{\alpha\ =\ \eta}\ =\ 1$$

The proof is simple: the functions $F(\eta,\ g)$ and $D(\eta,\eta,\ g)$ are identically by definition. The only problems arise from the second equation.

The Ward is equation for the self energy is written in Fig. 3, where we denote with a dashed line the Green function multiplied for the distance between the two points[9].
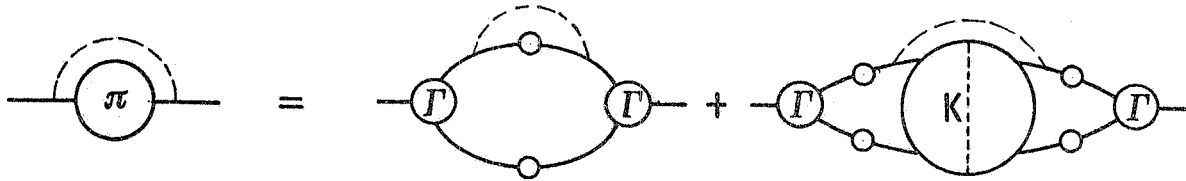

FIG.3

This equation can also be written as:

(7)
$$\pi_\mu\ =\ \Gamma G_\mu G \Gamma + \Gamma G G K_\mu G G \Gamma$$

where we designate the process of differentiation with respect to q_μ by the index μ, e. g. $\pi_\mu(q) = (d/dq_\mu)\ \pi(q^2)$.

Differentiating the Dyson is equation for the vertex

(8)
$$\Gamma\ =\ \Gamma G G K$$

we find:

(9)
$$\Gamma_\mu\ =\ \Gamma_\mu G G K + \Gamma G_\mu G K + \Gamma G G K_\mu$$

Using equations 7, 8, 9 together we arrive to the crazy result:

4.

(10) $$\pi_\mu = 0$$

Equation (10) is wrong: the initial integral was convergent but it was broken in the sum of not convergent integrals ($\Gamma G_\mu G\Gamma$ is convergent but $\Gamma_\mu GG\Gamma$ is always logarithmical divergent); the write conclusion is $\pi_\mu = \infty - \infty$, an indeterminate form.

The only possibility to obtain a non void formula is to make a slight modification of the initial integrals in order to have all the integrals convergent and return to the initial situation only in the final formula.

This can be realized in the following way: we compute the self energy relative to propagator of two operator, one of dimension η and the other of dimension $\eta + \varepsilon$, with ε small positive.

(11) $$\pi_\mu^\varepsilon = \Gamma^\varepsilon G_\mu G\Gamma + \Gamma^\varepsilon GGK_\mu GG\Gamma$$

All integral in (11) are convergent and

(12) $$\lim_{\varepsilon \to 0} \pi_\mu^\varepsilon = \pi_\mu$$

With this modification also the other integrals are no more divergent and we find for small :

(13) $$\pi_\mu^\varepsilon = - \varepsilon \left. \frac{d}{d\alpha} D(\alpha, \eta, g) \right|_{\alpha = \eta} \Gamma^\varepsilon GG\Gamma_\mu$$

GG can be easily computed in the limit of small from iden tity of ref. (10):

(14)
$$\Gamma^\varepsilon GG\Gamma_\mu = \frac{\pi^{12}}{-\varepsilon} \left[\frac{\Gamma(1 + \frac{\eta}{2})\,\Gamma(1-\eta)}{\Gamma(2-\frac{\eta}{2})\,\Gamma(2+\eta)} \right]^3 \frac{\Gamma(\frac{3}{2}\eta)}{\Gamma(3-\frac{3}{2}\eta)} g^2(x^2)^{-4+\eta} x_\mu =$$

$$= \frac{\pi^{18}}{\varepsilon} \frac{\Gamma^3(1+\frac{\eta}{2})\,\Gamma^4(1-\eta)\,\Gamma(\frac{3}{2}\eta)\Gamma(-1+\eta)}{\Gamma^3(2-\frac{\eta}{2})\,\Gamma^4(2-\eta)\,\Gamma(3-\frac{3}{2}\eta)\Gamma(4-\eta)} g^2 \pi_\mu$$

In the limit ε goes to zero we find the second equation of system 6.

The main interest in formula 6 is due to its simplifying effects on the effective computation of anomalous dimensions[11].

extended to him at the New York University where part of this work was done.

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