COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-72/30
31 Marzo 1972

E. Etim and Y. Srivastava: CONSERVATION CONSTRAINTS FOR e^+e^- ANNIHILATION AND ELECTROPRODUCTION. -

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E. Etim[(x)] and Y. Srivastava[(o)(+)] : CONSERVATION CONSTRAINTS FOR e^+e^- ANNIHILATION AND ELECTROPRODUCTION.

It is the purpose of this work to investigate some consequences of energy-momentum conservation[(1)] for hadron production in e^+e^- annihilation and electroproduction. It is shown that under certain assumptions the distribution function for e^+e^- annihilation,

$$f^h_\gamma = \frac{1}{\sigma^{tot}_{e^+e^-}(s)} \frac{d\sigma}{d^3p/E}$$

scales as $\underset{s \to \infty}{\sim} \frac{1}{s} F^h(x)$, where $x = \frac{2p \cdot q}{s}$, and that the n-particle invariant distribution function defined analogously (when averaged over all angles) scales as $s^{-n} F(x_1 \ldots x_n)$ where $x_i = 2E_i/\sqrt{s}$. These results agree exactly with naive dimensional counting reflecting the absence of any intrinsic dimensional parameters. When applied to the

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(x) - Work supported by the INFN.
(o) - Work supported by the National Science Foundation.
(+) - On sabbatical leave from North Eastern University, Boston, Mass. (USA).

total cross section, this argument implies that $\sigma^{tot}_{e^+e^-}(s) \longrightarrow \longrightarrow \frac{\text{constant}}{s}$. The statement that the structure functions for annihilation $\bar{F}_{1,2}(x)$ scale (in the Bjorken limit) is contained in the above results[2].

The variable x when crossed over into the "scattering" region, $\gamma(q) + p \to$ anything, becomes the Bjorken variable x_B. Therefore, if as is generally believed, $F_{1,2}(x)$ $(x \geq 1)$ are indeed the analytic continuations of $\bar{F}_{1,2}(x)$ $(x \leq 1)$, then Bjorken scaling follows[2]. Notice that for annihilation it is $(s \cdot f^h_\gamma)$ which scales unlike the case of particle production in scattering where f^h_{ab} scales. The difference arises because in the latter case there exists a dimensional parameter - the wellknown transverse momentum cutoff.

We also find that the average energy grows as $\sqrt{s}$ (for large s) and the multiplicities become independent of energy (although unlikely, a logarithmic growth cannot be excluded on general grounds). Similar considerations for electroproduction imply that not only does $f^h_{\gamma p}(s, x_B, x_B, p_\perp)$ scale (i.e. become independent of s) but may also become independent of x_B.

We first consider single hadron production in e^+e^- annihilation

$$\gamma(q) \longrightarrow \text{hadron } (p) + x.$$

Energy conservation reads

$$(1) \qquad \sqrt{s} = \sum_h \int pE\, dE\, d\Omega\, f^h_\gamma \ .$$

The angular integration is trivial if one considers the spin averaged cross-section (call this function $\bar{f}^h_\gamma(s, E)$). In terms of the scaling variable $x = 2E/\sqrt{s}$, eqn. (1) becomes

$$(2) \qquad 1 = \left(\frac{\pi}{2}\right) s \cdot \sum_h \int_{\frac{2m_h}{\sqrt{s}}}^{1} \sqrt{x^2 - \frac{4m_n^2}{s}}\; x\, dx\, \bar{f}^h_\gamma(s, x) \ .$$

The sum extends over stable hadrons only (hence is finite). If we assume that

(3) $$\bar{f}^{h}_{\gamma}(s,x) \xrightarrow[s\ \text{large}]{} s^{-\alpha}(\ln s)^{\beta} F^{h}(x) ,$$

uniformly in $x^{(\dagger)}$, then equation (2) gives

$$\alpha = 1 , \qquad \beta = 0 \qquad \text{and}$$

(4) $$1 \simeq \left(\frac{\pi}{2}\right) \sum_{h} \int_{0}^{1} x^{2}\, dx\, F^{h}(x) .$$

The average multiplicity is given by

(5) $$\langle n(s) \rangle \simeq \pi \int_{\frac{2m}{\sqrt{s}}}^{1} dx\, x\, F(x) .$$

Since F(x) is proportional to a measurable cross-section, we expect that this integral exists, in which case $\langle n(s)\rangle$ tends to a constant for large s. However, if one is willing to ahve F(x) diverge as x^{-2} as $x \to 0$, then $\langle n(s)\rangle$ can grow logarithmically. On physical grounds we find the latter behaviour unattractive (firstly because it will give unrealistically large cross sections for $x \to 0$) and secondly because in the case of scattering (with transverse momentum cutoff) one gets the lns growth of multiplicities not by having the scaled function diverge but by requiring that it has a finite limit in the pionization region ($x \to 0$).

For the average energy/particle, we have

(6) $$\langle E(s) \rangle \simeq \frac{\pi \sqrt{s}}{2 \langle n(s) \rangle} \int_{0}^{1} dx\, x^{2}\, F(x) ,$$

and is thus seen to grow like $\sqrt{s}$ (up to possible lns terms). This behaviour is similar to that found in strong interactions, while Bjorken and Brodsky[3] find $\langle E(s)\rangle \to$ constant and $\langle n(s)\rangle \sim \sqrt{s}$.

The n-particle distribution functions defined as

(7) $$f_\gamma^{(h_1\ldots h_n)} = \frac{1}{\sigma^{tot}_{e^+e^-}(s)} \frac{d\sigma^{(h_1\ldots h_n)}}{\prod_{i=1}^{n}\left[d^3p_i/E_i\right]} ,$$

when averaged over all angles, satisfy the energy momentum constraint, if they scale as

(8) $$\bar{f}^{(h_1\ldots h_n)}(s, x_1\ldots x_n) \sim \frac{1}{s^n} F^{(h_1\ldots h_n)}(x_1\ldots x_n) ,$$

where the F's satisfy the recursion relation

(9) $$\left[1 - \frac{x_1+x_2+\cdots+x_n}{2}\right] F^{(h_1\ldots h_n)}(x_1\ldots x_n) = \\ = \left(\frac{\pi}{2}\right) \sum_{h'} \int_0^1 dx'\, x'^2\, F^{(h_1\ldots h_n, h')}(x_1\ldots x_n, x') .$$

This equation is obviously consistent with factorization if the x_i's are small.

Energy-momentum conservation for electroproduction also yields rather interesting results. The invariant distribution function for $\gamma(q)$ + proton (p) $\to$ hadron (p') + anything is again defined as

$$f^h_{\gamma p} = \frac{1}{\sigma^{tot}_{\gamma p}(Q^2, x_B)} \frac{d\sigma^h_{\gamma p}}{(d^3p'/E')} ,$$

and satisfies

(10) $$(q + p)^\mu = \sum_h \int \frac{d^3p'}{E}\, p'^\mu f^h_{\gamma p} .$$

The $\mu = 0$ component gives

$$(11) \qquad 1 = \frac{\pi}{2} \sum_h \int_{-1}^{+1} dx_F \, \bar{f}^h_{\gamma p} (s, x_B, x_F, \langle p_\perp \rangle),$$

where $x_B = \frac{2p \cdot q}{Q^2}$, $x_F = \frac{2p_{\parallel}'^*}{\sqrt{s}}$ and $\bar{f}(\ )$ is obtained by integrating f over $p_\perp$ (under the standard assumption of a transverse momentum cutoff). One obtains another sum rule by multiplying equ. (10) by q^μ:

$$(12) \quad (x_B - 2) = (\tfrac{\pi}{2}) \sum_h \int_{-1}^{+1} dx_F \left\{ (x_B - 2) - \frac{x_F \, x_B}{x_F} \, f^{-h}_{\gamma p} (s, x_B, x_F, \langle p_\perp \rangle) \right\}.$$

Repeating arguments similar to those discussed above, eqn. (11) shows that $\bar{f}$ scales (i. e., is independent of s)[4]. While this equation only says that $\bar{f}^h$, when integrated over x_F and summed over h, is independent of x_B, we conjucture that in fact the individual $\bar{f}^h$ are independent of x_B. From a comparison of eqs. (11) and (12) we further suggest that the way the latter equation is satisfied is that $\left[\sum_h \bar{f}^h\right]$ is symmetric under $x_F \leftrightarrow - x_F$.(++) Moreover, if we assume that $\left[\sum_h \bar{f}^h_{\gamma p}\right]$ has a structure in x_F similar to that found in strong interaction (since they satisfy identical sum rules this is not unreasonable), then there is some evidence for the symmetry in x_F in $\pi^- p$ reaction[6].

The other conservation constraints (due to charge, baryon number and strangeness) can be used in conjunction to derive other useful sum rules to which we hope to return elsewhere.

In conclusion, we have obtained some interesting consequences from momentum conservation constraints for e^+e^- annihilation as well as for electroproduction, which show different scaling properties. Under certain assumptions, we were able to show that for annihilation naive dimensional counting is indeed correct, whereas forelectroproduction a different function scales due to the presence of a dimensional parameter in the form of $p_\perp$ cutoff. In addition, we have given support for, through not a proof of, factorization and symmetry properties of the electroproduction distribution functions.

Some of the results can be derived in specific models (e. g; partons, light cone, multiperipheral models, etc.). However their generality can be appreciated from our analysis.

FOOTNOTES and REFERENCES. -

(+) - Such an asymptotic expansion can always be made; however, that the dominant term is uniformly the same is an extra assumption. That this technicality is indeed crucial can be seen in a model by Bjorken and Brodsky (ref. (3)), where results different from ours are obtained only by violating it. In this model the "wee" region ($x \lesssim 2E_0/\sqrt{s}$) is the important one and the function f behaves differently in the wee and the non-wee regions. This has been accomplished by an extra intrinsic dimensional parameter E_0.

(++) - No data are available to test the independence of $\bar{f}$ on x_B. There is some evidence for symmetry in x_F for the proton spectra from Desy (ref. (5)) which is at a rather low value of s ($\simeq 7$ Gev^2) and $Q^2 \simeq 1.15$ GeV^2.

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