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K. Baker and G. Paternò: NUMERICAL CALCULATION OF THE ZERO BIAS CONDUCTIVITY FOR A SUPERCONDUCTING TUNNEL JUNCTION IN THE PRESENCE OF DEPAIRING.
II. - VOLTAGE APPLIED TO THE JUNCTION.

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We have previously[(1)] presented a numerical calculation of the differential conductance σ for a metal-insulator-superconductor tunnel junction in the presence of depairing. The calculation was made for different values of termperature and of Z (pair breaking parameter), when there was no voltage applied to the junction.

In this work we extend our calculation of σ to the case in which there is a voltage applied to the junction.

In this case the integral by which we compute σ is no longer symmetric so it must be divided into two parts. Each part can then be calculated separately. If V is the potential applied to the junction, we have for the conductivity:

$$\sigma = \int_{0}^{+\infty} G(\varepsilon, Z) \frac{1}{KT} \frac{\exp\left(\frac{\varepsilon - eV}{KT}\right)}{\left[\exp\left(\frac{\varepsilon - eV}{KT}\right) + 1\right]^2} d\varepsilon \; +$$

$$+ \int_{-\infty}^{0} G(\varepsilon, Z) \frac{1}{KT} \frac{\exp\left(\frac{\varepsilon - eV}{KT}\right)}{\left[\exp\left(\frac{\varepsilon - eV}{KT}\right) + 1\right]^2} d\varepsilon$$

where[2]:

$$G(\varepsilon, Z) \; \begin{cases} = \dfrac{\varepsilon}{\Delta} \dfrac{1}{Z^{1/2}} I_m \dfrac{1}{(\bar{x} - z)} & \text{for } |\varepsilon| > \varepsilon_m \\ = 0 & \text{for } |\varepsilon| \leq \varepsilon_n \end{cases}$$

For the meaning of the symbols, we refer to our previous paper[1].

Measuring as usual the energy ε with respect to Δ (energy gap in the presence of depairing) and using the symmetry properties of $G(\varepsilon, Z)$ with respect to $\varepsilon = 0$, we get:

$$\sigma(\beta, Z, V') = \frac{\Delta}{KT} \left\{ \int_{\varepsilon_m/\Delta}^{\infty} G(E, Z) \frac{\exp\left[\frac{\Delta}{KT}(E - V')\right]}{\left(\exp\left[\frac{\Delta}{KT}(E - V')\right] + 1\right)^2} dE \; + \right. \tag{1}$$

$$\left. + \int_{\varepsilon_m/\Delta}^{\infty} G(E, Z) \frac{\exp\left[\frac{\Delta}{KT}(E + V')\right]}{\left(\exp\left[\frac{\Delta}{KT}(E + V')\right] + 1\right)^2} dE \right\}$$

with

$$E = \frac{\varepsilon}{\Delta}, \qquad V' = \frac{eV}{\Delta}$$

where Δ is the energy gap in the presence of depairing.

The Fortran program used to compute (1) is presented at the end of this paper. A table of values of σ computed by this program is given as well. Also included are graphs of σ as a function of V', for three values of Δ_o/KT and three values of Z. (Δ_o is the energy gap in the absence of depairing). For the list of subroutines referenced in our main program, we refer to our previous paper(1).

REFERENCES. -

(1) - K. Baker and G. Paternò, Frascati Report LNF-72/10 (1972).

(2) - P. Fulde, Phys. Rev. 137 A, 783 (1965).

MAINPGM

```
      REAL*8 NEWINT
      DOUBLE PRECISION EO,ABTEST,FINT,COM,FOUR,H,CONDU,V,DENS,Z,E,PIGR,
     1FUNC,FUN1,EINC,EMIN,EMAX,BETA,A,TOTINT,ATEST,FUNMIN,FUNMAX,OMEGA,
     2EXPO,PESO,TWO
      DOUBLE PRECISION TEST, TOTAL, TTEST, VOLT
      DIMENSION V(28)
      DIMENSION TEST(2,30),ATEST(2,30),FUNMIN(2,30),FUNMAX(2,30),ZA(30)
      DIMENSION BETO(30),SIGZER(30),BETA(30),NEWINT(2,30),TOTINT(2,30)
      DIMENSION FINT(2,30),FOUR(2,30),TWO(2,30),IFLAG(2,30),KFLAG(2,30)
      DIMENSION LFLAG(2),NFLAG(2),ABTEST(2),VOLT(2)
      DIMENSION TTEST(30),TOTAL(4,4,28)
      E=0.
      KMAX=26
      JBMAX=3
      IBMAX=3
      READ 700,(V(K),K=1,KMAX)
  700 FORMAT(F7.5)
      READ 91,(ZA(I),I=1,JBMAX)
   91 FORMAT(E13.6)
      READ 90,(BETO(J),SIGZER(J),J=1,IBMAX)
   90 FORMAT(2F7.5)
C     **********************************
      EPSI=.5E-06
      EPSEC=1.0E-03
C     **********************************
      PIGR=3.1415926535897932
      DO 701 KB=1,KMAX
      VV=V(KB)
      VOLT(1)=VV
      VOLT(2)=-VV
      DO 250 JB=1,JBMAX
      Z=ZA(JB)
      CALL ZFUNC(Z,A,EO)
C                     CALCULATE THE FUNCTION FOR E=1
      E=1.
      CALL DENSIT(Z,E,DENS,KER)
      FUNC=DENS
      FUN1=FUNC
      EINC=1.
      EMIN=EO
      EMAX=1.
      ABTEST(1)=0.
      ABTEST(2)=0.
      DO 5 IB=1,IBMAX
      DO 5 NB=1,2
      BETA(IB)=BETO(IB)*A
      TOTINT(NB,IB)=0.
      KFLAG(NB,IB)=0
      ATEST(NB,IB)=0.
      TEST(NB,IB)=0.
      FUNMIN(NB,IB)=0.
      OMEGA=BETA(IB)*(E+VOLT(NB))
      EXPO=DEXP(OMEGA)
      COM=(EXPO+1.)
```

MAINPGM

```
      COM=COM*COM
      PESO=EXPO/COM
      FUNMAX(NB,IB)=PESO*FUN1
    5 CONTINUE
    9 DO 10 IB=1,IBMAX
      DO 10 NB=1,2
      IFLAG(NB,IB)=0.
      TWO(NB,IB)=0.
      FOUR(NB,IB)=0.
   10 CONTINUE
      H=(EMAX-EMIN)/2.
      N=1
      E=EMIN+H
      CALL DENSIT(Z,E,DENS,KER)
      FUNC=DENS
      DO 11 IB=1,IBMAX
      DO 11 NB=1,2
      IF(KFLAG(NB,IB).EQ.1) GO TO 11
      OMEGA=BETA(IB)*(E+VOLT(NB))
      EXPO=DEXP(OMEGA)
      COM=(EXPO+1.)
      COM=COM*COM
      PESO=EXPO/COM
      FUN=PESO*FUNC
      FOUR(NB,IB)=FOUR(NB,IB)+FUN
      FINT(NB,IB)=(H*(FUNMIN(NB,IB)+FUNMAX(NB,IB)+4.*FOUR(NB,IB)))/3.
      FINT(NB,IB)=BETA(IB)*FINT(NB,IB)
   11 CONTINUE
   25 H=H/2.
      N=2*N
      E=H+EMIN
      DO 12 IB=1,IBMAX
      DO 12 NB=1,2
      IF(IFLAG(NB,IB).EQ.1.OR.KFLAG(NB,IB).EQ.1) GO TO 12
      TWO(NB,IB)=TWO(NB,IB)+FOUR(NB,IB)
      FOUR(NB,IB)=0.
   12 CONTINUE
      DO 26 I=1,N
      CALL DENSIT(Z,E,DENS,KER)
      FUNC=DENS
      DO 13 IB=1,IBMAX
      DO 13 NB=1,2
      IF(IFLAG(NB,IB).EQ.1.OR.KFLAG(NB,IB).EQ.1) GO TO 13
      OMEGA=BETA(IB)*(E+VOLT(NB))
      EXPO=DEXP(OMEGA)
      COM=(EXPO+1.)
      COM=COM*COM
      PESO=EXPO/COM
      FUN=PESO*FUNC
      FOUR(NB,IB)=FOUR(NB,IB)+FUN
   13 CONTINUE
   26 E=E+H+H
      LFLAG(1)=0
      LFLAG(2)=0
```

```
      DO 14 IB=1,IBMAX
      DO 14 NB=1,2
      IF(KFLAG(NB,IB).EQ.1) IFLAG(NB,IB)=1
      IF(IFLAG(NB,IB).EQ.1) GO TO 17
      NEWINT(NB,IB)=(H*(FUNMIN(NB,IB)+FUNMAX(NB,IB)+2.*TWO(NB,IB)+4.*FOU
     1R(NB,IB)))/3.
      NEWINT(NB,IB)=BETA(IB)*NEWINT(NB,IB)
      TEST(NB,IB)=DABS(FINT(NB,IB)-NEWINT(NB,IB))
      ABTEST(NB)=TEST(NB,IB)/DABS(NEWINT(NB,IB))
      IF(TEST(NB,IB).LT.EPSI.AND.ABTEST(NB).LE.EPSEC) IFLAG(NB,IB)=1
   17 LFLAG(NB)=LFLAG(NB)+IFLAG(NB,IB)
      FINT(NB,IB)=NEWINT(NB,IB)
   14 CONTINUE
      IF(N.GT.2000.OR.H.LE.EPSI) GO TO 27
      DO 1006 NB=1,2
      IF((LFLAG(NB)-IBMAX).LT.0) GO TO 25
 1006 CONTINUE
   27 CONTINUE
      NFLAG(1)=0
      NFLAG(2)=0
      DO 16 IB=1,IBMAX
      DO 16 NB=1,2
      IF(KFLAG(NB,IB).EQ.1) GO TO 15
      TOTINT(NB,IB)=NEWINT(NB,IB)+TOTINT(NB,IB)
      ATEST(NB,IB)=TEST(NB,IB)+ATEST(NB,IB)
      IF(FINT(NB,IB).LE.EPSI.AND.EMIN.GT.1.) KFLAG(NB,IB)=1
   15 NFLAG(NB)=NFLAG(NB)+KFLAG(NB,IB)
   16 CONTINUE
      DO 1007 NB=1,2
      IF((NFLAG(NB)-IBMAX).GE.0) GO TO 31
 1007 CONTINUE
      EMIN=EMAX
      EMAX=EMIN+EINC
      E=EMAX
      CALL DENSIT(Z,E,DENS,KER)
      FUNC=DENS
      DO 30 IB=1,IBMAX
      DO 30 NB=1,2
      IF(KFLAG(NB,IB).EQ.1) GO TO 30
      FUNMIN(NB,IB)=FUNMAX(NB,IB)
      OMEGA=BETA(IB)*(E+VOLT(NB))
      EXPO=DEXP(OMEGA)
      COM=(EXPO+1.)
      COM=COM*COM
      PESO=EXPO/COM
      FUNMAX(NB,IB)=PESO*FUNC
   30 CONTINUE
      GO TO 9
   31 CONTINUE
      DO 100 IB=1,IBMAX
      TOTAL(IB,JB,KB)=TOTINT(1,IB)+TOTINT(2,IB)
      TTEST(IB)=ATEST(1,IB)+ATEST(2,IB)
      CONDU=TOTAL(IB,JB,KB)/SIGZER(IB)
  100 CONTINUE
```

MAINPGM

```
250 CONTINUE
701 CONTINUE
    DO 301 IP=1,3
    PRINT 302,BETO(IP)
302 FORMAT(1H1,'DELTA/KT=',F10.5,/,' ',82('-'))
    PRINT 303,(ZA(JP),JP=1,3)
303 FORMAT(15X,'I',/,12X,'Z',2X,'I',7X,2(F6.3,14X),F6.3,/,15X,'I',/,6X
   1,'V',8X,'I',/,' ',14('-'),'I',67('-'),/,15X,'I')
    DO 301 KP=1,26
    PRINT 304, V(KP),(TOTAL(IP,JP,KP),JP=1,3)
304 FORMAT(3X,F10.5,2X,'I',3X,3(D14.6,6X),/,15X,'I')
301 CONTINUE
    STOP
    END
```

DELTA/KT= 0.80000

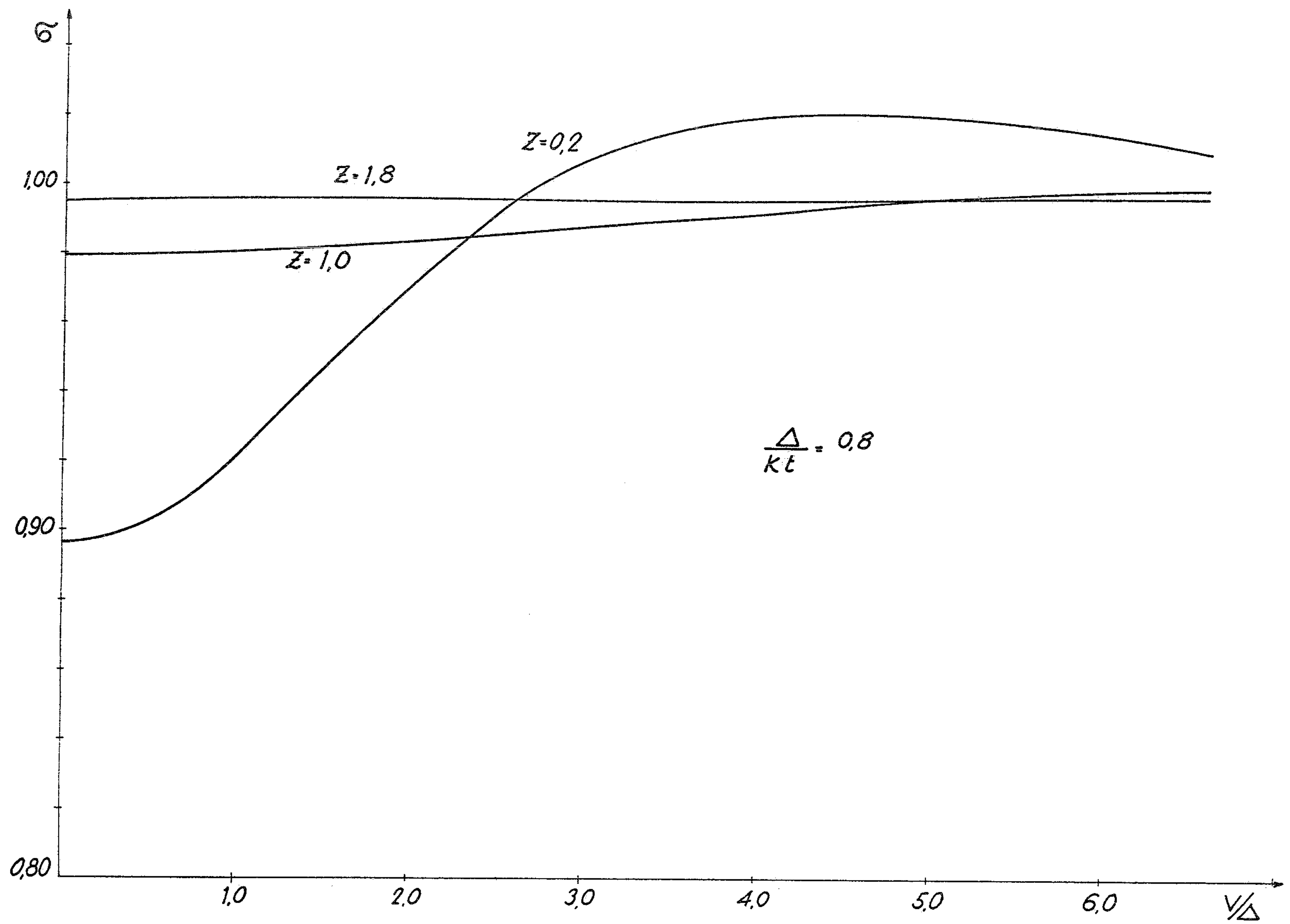
Z / V	0.200	1.000	1.800
0.0	0.896276D 00	0.979598D 00	0.995958D 00
0.20000	0.897413D 00	0.979647D 00	0.995960D 00
0.50000	0.903216D 00	0.979903D 00	0.995970D 00
0.80000	0.913285D 00	0.980370D 00	0.995988D 00
1.00000	0.921823D 00	0.980794D 00	0.996006D 00
1.50000	0.946644D 00	0.982205D 00	0.996064D 00
2.00000	0.971640D 00	0.984032D 00	0.996146D 00
2.50000	0.992485D 00	0.986153D 00	0.996248D 00
3.00000	0.100734D 01	0.988435D 00	0.996371D 00
3.20000	0.101160D 01	0.989366D 00	0.996425D 00
3.50000	0.101640D 01	0.990759D 00	0.996510D 00
3.70000	0.101860D 01	0.991670D 00	0.996571D 00
4.00000	0.102079D 01	0.993010D 00	0.996667D 00
4.20000	0.102161D 01	0.993875D 00	0.996733D 00
4.40000	0.102200D 01	0.994711D 00	0.996800D 00
4.60000	0.102183D 01	0.995504D 00	0.996870D 00
4.80000	0.102158D 01	0.996272D 00	0.996942D 00
5.00000	0.102110D 01	0.997003D 00	0.997016D 00
5.20000	0.102046D 01	0.997695D 00	0.997091D 00
5.40000	0.101968D 01	0.998346D 00	0.997165D 00
5.60000	0.101781D 01	0.998936D 00	0.997243D 00
5.80000	0.101670D 01	0.999504D 00	0.997321D 00
6.00000	0.101552D 01	0.100003D 01	0.997401D 00
6.20000	0.101427D 01	0.100052D 01	0.997481D 00
6.40000	0.101296D 01	0.100096D 01	0.997558D 00
6.60000	0.100718D 01	0.100133D 01	0.997639D 00

DELTA/KT= 2.20000

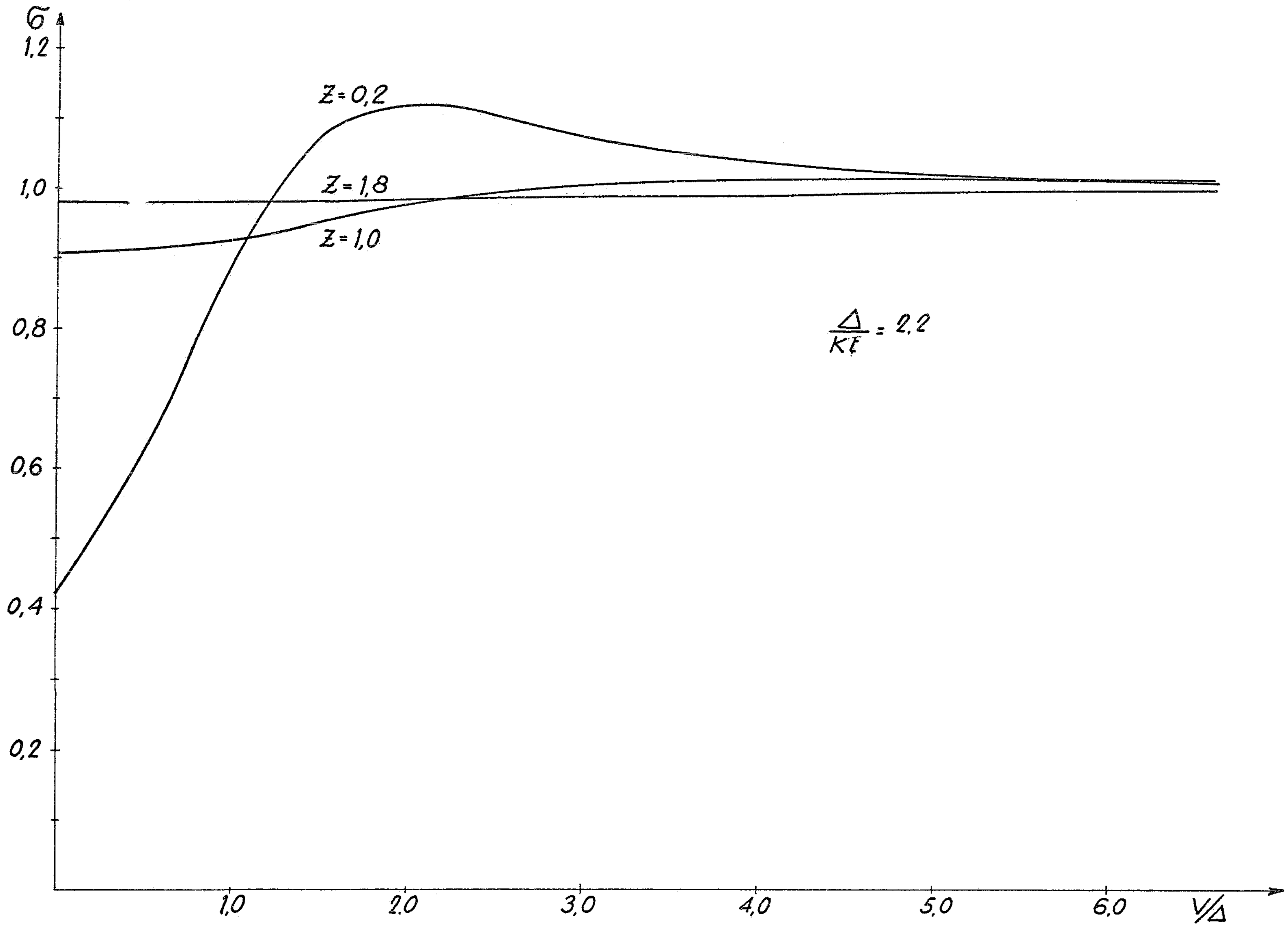
V \ Z	0.200	1.000	1.800
0.0	0.502693D 00	0.907642D 00	0.981758D 00
0.20000	0.524177D 00	0.908780D 00	0.981803D 00
0.50000	0.628268D 00	0.914549D 00	0.982039D 00
0.80000	0.784522D 00	0.924409D 00	0.982469D 00
1.00000	0.891217D 00	0.932622D 00	0.982859D 00
1.50000	0.107595D 01	0.955720D 00	0.984151D 00
2.00000	0.112328D 01	0.977799D 00	0.985819D 00
2.50000	0.110544D 01	0.995229D 00	0.987743D 00
3.00000	0.107722D 01	0.100701D 01	0.989803D 00
3.20000	0.106742D 01	0.101026D 01	0.990638D 00
3.50000	0.105506D 01	0.101380D 01	0.991884D 00
3.70000	0.104829D 01	0.101541D 01	0.992700D 00
4.00000	0.104002D 01	0.101692D 01	0.993890D 00
4.20000	0.103555D 01	0.101743D 01	0.994654D 00
4.40000	0.103176D 01	0.101764D 01	0.995391D 00
4.60000	0.102853D 01	0.101761D 01	0.996097D 00
4.80000	0.102577D 01	0.101738D 01	0.996771D 00
5.00000	0.102339D 01	0.101702D 01	0.997410D 00
5.20000	0.102133D 01	0.101654D 01	0.998013D 00
5.40000	0.101954D 01	0.101599D 01	0.998579D 00
5.60000	0.101798D 01	0.101540D 01	0.999108D 00
5.80000	0.101660D 01	0.101477D 01	0.999601D 00
6.00000	0.101538D 01	0.101413D 01	0.100006D 01
6.20000	0.101429D 01	0.101349D 01	0.100048D 01
6.40000	0.101332D 01	0.101286D 01	0.100086D 01
6.60000	0.101245D 01	0.101225D 01	0.100121D 01

DELTA/KT= 4.00000

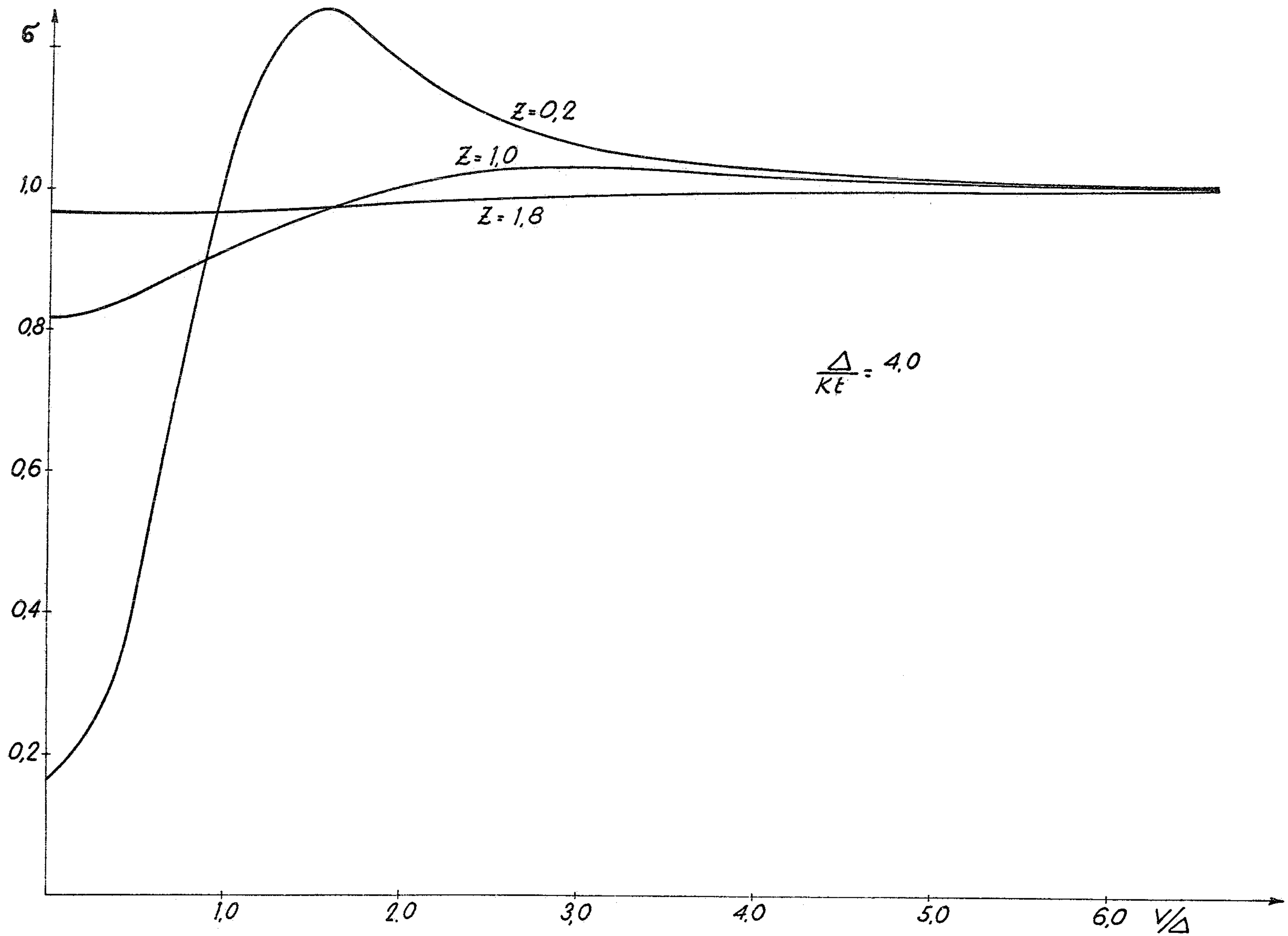
Z / V	0.200	1.000	1.800
0.0	0.169411D 00	0.820920D 00	0.964757D 00
0.20000	0.209347D 00	0.825945D 00	0.964950D 00
0.50000	0.430375D 00	0.850099D 00	0.965948D 00
0.80000	0.809086D 00	0.886711D 00	0.967732D 00
1.00000	0.104810D 01	0.913218D 00	0.969301D 00
1.50000	0.125917D 01	0.971534D 00	0.974230D 00
2.00000	0.117972D 01	0.100847D 01	0.979973D 00
2.50000	0.110677D 01	0.102588D 01	0.985799D 00
3.00000	0.106786D 01	0.103098D 01	0.991150D 00
3.20000	0.105802D 01	0.103104D 01	0.993074D 00
3.50000	0.104694D 01	0.102997D 01	0.995691D 00
3.70000	0.104129D 01	0.102877D 01	0.997246D 00
4.00000	0.103462D 01	0.102661D 01	0.999293D 00
4.20000	0.103106D 01	0.102508D 01	0.100047D 01
4.40000	0.102804D 01	0.102355D 01	0.100151D 01
4.60000	0.102546D 01	0.102206D 01	0.100241D 01
4.80000	0.102322D 01	0.102063D 01	0.100319D 01
5.00000	0.102128D 01	0.101929D 01	0.100385D 01
5.20000	0.101957D 01	0.101803D 01	0.100440D 01
5.40000	0.101807D 01	0.101686D 01	0.100486D 01
5.60000	0.101674D 01	0.101578D 01	0.100523D 01
5.80000	0.101555D 01	0.101478D 01	0.100552D 01
6.00000	0.101448D 01	0.101386D 01	0.100575D 01
6.20000	0.101353D 01	0.101302D 01	0.100591D 01
6.40000	0.101266D 01	0.101224D 01	0.100602D 01
6.60000	0.101188D 01	0.101153D 01	0.100609D 01



Conductivity v. s. voltage for Δ/KT = 0. 8.



Conductivity v. s. voltage for Δ/KT = 2. 2.



Conductivity v. s. voltage for Δ/KT = 4. 0.