Laboratori Nazionali di Frascati

LNF-72/23

G. Pancheri-Srivastava and Y. N. Srivastava: A MODEL DIFFRACTION TERM.

Estratto da: Lett. Nuovo Cimento 3, 411 (1972)

LETTERE AL NUOVO CIMENTO VOL. 3, N. 10 4 Marzo 1972

A Model Diffraction Term (*).

G. PANCHERI-SRIVASTAVA (**)

Laboratori Nazionali del CNEN - Frascati
Radcliffe Institute - Cambridge, Mass.

Y. N. SRIVASTAVA (***)

Laboratori Nazionali del CNEN - Frascati
Istituto di Fisica dell'Università - Roma

(ricevuto il 28 Dicembre 1971)

1. – In this letter we wish to present a modification of the Del Giudice-Veneziano ansatz [1] for the pomeron (diffraction term). This new model, besides enjoying all the good properties of the old one (*i.e.* crossing symmetry, proper large-angle behaviour etc.), now gives only small corrections to the dual-resonant model meson-meson scattering lengths and hence does not ruin previous agreement with experiments. We have also constructed off-shell pomeron amplitudes and imposed continuous PCAC in a manner analogous to ARNOWITT *et al.* [2] for the dual Veneziano terms. If we employ t-channel helicity conservation for diffraction dissociation processes (*e.g.*, $\pi\mathcal{N}\to A_1\mathcal{N}$) in agreement with recent experimental results [3] and data on A_1 production cross-section, we end up with an asymptotic prediction which seems to be borne out qualitatively by experiments.

For elastic $\pi\pi$ scattering DEL GIUDICE and VENEZIANO suggested the following expression for the pomeron term:

$$\left\{\begin{aligned} &P(s,t)=g\alpha'(t-t_0)H(s,t)\,,\\ &H(s,t)=\sigma^{\alpha(s)}\Gamma(1-\alpha(s))\Psi(1-\alpha(s),\tfrac{1}{2};\,-g\alpha'(t-4\mu^2))\,, \end{aligned}\right. \tag{1}$$

where α' is the universal slope of ordinary trajectories, $\alpha(s)$ the nonlinear pomeron trajectory normalized to $\alpha(0)=1$, g and σ positive dimensionless constants and Ψ is the confluent hypergeometric function [4]. The constant t_0 was fixed by the authors to be

(*) Work supported in part by the National Science Foundation Grant GP-23719.
(**) Fellow of the Radcliffe Institute, 1970-1971.
(***) On Sabbatical leave from Northeastern University, Boston, Mass.
([1]) E. DEL GIUDICE and G. VENEZIANO: *Lett. Nuovo Cimento*, **3**, 363 (1970).
([2]) R. ARNOWITT, M. H. FRIEDMAN, P. NATH and Y. N. SRIVASTAVA: *Phys. Rev. Lett.*, **22**, 1158 (1969).
([3]) AACHEN-BERLIN-BONN-CERN-CRACOW-HEIDELBERG-LONDON-VIENNA COLLABORATION: *Phys. Lett.*, **34** B, 160 (1971).
([4]) BATEMAN MANUSCRIPT PROJECT: *Higher Trascendental Functions*, vol. **1** (New York, 1953), chapt. 6.

$2\mu^2$ (μ being the pion mass) in order to eliminate a massless scalar particle. With this choice, however, the scattering lengths, as noted by the authors, turn out to be too large, a fact reflecting the absence of an Adler zero in expression (1). In the spirit of the Freund Harari conjecture about the dual role of the pomeron term, any candidate for such a role should possess the proper Adler zeros if it has to be added as a background builder to the usual Veneziano amplitude which does possess such zeros. We thus propose that expression (1) be modified as follows:

$$\tilde{P}(s,t)=\frac{\alpha' g}{2}\left(2t+s-\sum\right)\sigma^{\alpha(s)}\Gamma(1-\alpha(s))\,\Psi\left(1-\alpha(s),\frac{1}{2};-g\alpha'(t-4\mu^2)\right), \tag{2}$$

where $\sum$ is the sum of the squares of the external masses through which the off-shell continuation of the Adler point has to be made. This is, of course, not a smooth continuation but it should be remembered that the continuation in the Lovelace-Veneziano formula was not smooth either. Consider, for instance, a typical term in $\pi\pi$ scattering

$$V(s,t)=\alpha'(2\mu^2-s-t)\,B(s,t)=\alpha'\left(u-\sum+2\mu^2\right)B(s,t)\,,$$

where B is the Euler beta function. The Adler zero is therefore obtained by continuing $\sum$ off shell. Our modification (2) is entirely analogous.

Notice that our expressions (2) has no scalar massless pole even for one particle off mass shell since, for any elastic amplitude with one particle off shell, $2t+s-\sum=$ $=t-u=4p_sq_sz_s$, where z_s is the scattering angle and p_s, q_s denote the initial and final 3-momenta in the s-channel c.m. system.

With expression (2), the contribution to a scattering amplitude of a pomeron trajectory exchanged in a given channel, say t, can be written in the very compact form

$$\tilde{P}_{ts}+\tilde{P}_{tu}=g\alpha'\frac{s-u}{2}\left[H(t,s)-H(t,u)\right].$$

Of course, the detailed test of the model rests in obtaining phenomenological fits to the diffraction term in high-energy scattering processes. Pending such tests we present below two theoretical applications of the model. The first is in calculating the contributions to scattering lengths for $\pi\pi$ and πK, where the Veneziano resonance formula makes good predictions. The second is to impose the continuous PCAC constraints which give us rather interesting results.

2. – $\pi\pi$ and πK scattering lengths.

Using the same notation as in ref. ([1]), we call S_I^{pom} the pomeron pole contribution to the $\pi\pi$ amplitudes with pure isospin I ($I=0,1,2$) in the s-channel. It is now

$$S_0^{\text{pom}}=\frac{cg\alpha'}{2}\{(s-u)[H(t,s)-H(t,u)]+$$
$$+(s-t)[H(u,s)-H(u,t)]+3(t-u)[H(s,t)-H(s,u)]\}\,,$$

$$S_1^{\text{pom}}=\frac{cg\alpha'}{2}\{(s-u)[H(t,s)-H(t,u)]-(s-t)[H(u,s)-H(u,t)]\}\,,$$

$$S_2^{\text{pom}}=\frac{cg\alpha'}{2}\{(s-u)[H(t,s)-H(t,u)]+(s-t)[H(u,s)-H(u,t)]\}\,,$$

where the normalization is fixed so that

$$\sigma_{\text{tot}}^{\pi\pi}(s) \xrightarrow[s\to\infty]{} 8\pi^2 cg\alpha'\sigma\,.$$

At threshold we have

$$(3)\qquad S_{0,2}^{\text{pom}} \xrightarrow[\substack{s\to 4\mu^2\\ t,u\to 0}]{} 2c\sigma z_0\left\{\sqrt{\pi z_0}-\sum_{n=1}^{\infty} z_0^n\left(A_n - B_n\sqrt{\pi z_0}\right)\right\},$$

$$(4)\qquad S_1^{\text{pom}} \xrightarrow[\substack{s\to 4\mu^2\\ t,u\to 0}]{} c\sigma g\alpha'(t-u)\left\{\tfrac{3}{2}\sqrt{\pi z_0}-\sum_{n=1}^{\infty} z_0^n\left(a_n - b_n\sqrt{\pi z_0}\right)\right\}$$

with

$$z_0 = 4\mu^2 g\alpha'\,,\qquad A_n = \frac{2^{n-1}}{(2n-1)!!\,n}\,,\qquad B_n = \frac{1}{n!(2n+1)}$$

and

$$a_n = A_n\{n+1-4\mu^2\alpha_{\text{P}}'(0)[\Psi(n)-\Psi(\tfrac{1}{2})]\}\,,$$

$$b_n = B_n\{n+\tfrac{3}{2}-4\mu^2\alpha_{\text{P}}'(0)[\Psi(n+\tfrac{1}{2})-\Psi(\tfrac{1}{2})]\}\,.$$

In the above expressions $\alpha_{\text{P}}'(0)$ is the slope of the pomeron trajectory which we shall put equal to $\frac{1}{2}\alpha'$ and $\Psi(z)=[(\mathrm{d}/\mathrm{d}z)\Gamma(z)]/\Gamma(z)$.

As in ref. (1), $(\sigma g\alpha')^{-1}$ is the scale factor in a Regge-type expansion so that assuming it to be of order $1\,(\text{GeV})^2$ we see that $\sigma g \simeq 1$. Since σ has to be larger than 1 to ensure Regge behaviour in s at fixed t of expression (1) or (2) (provided $\alpha(s)\xrightarrow[s\to\infty]{}-\infty$) the constant g will be less than 1 and we can neglect in eqs. (3) and (4) terms of order, z_0^n with $n>1$. We then have

$$\mu a_0^{(0)} = \mu a_0^{(2)} \simeq \frac{\mu^2\sigma_{\text{tot}}^{\pi\pi}(\infty)}{2\pi^2}\left(\sqrt{\pi z_0}-z_0\right),$$

$$\mu a_1^{(1)} \simeq -\frac{\mu^2\sigma_{\text{tot}}^{\pi\pi}(\infty)}{8\pi^2}\left\{\sqrt{\pi z_0}-\frac{4}{3}z_0\left[1-4\mu^2\alpha_{\text{P}}'(0)\ln 2\right]\right\}.$$

To estimate the order of magnitude we have taken $\sigma_{\text{tot}}^{\pi\pi}(\infty)=16\text{ mb}$ and computed the scattering lengths for a few g values: we thus get Table I.

TABLE I.

g	1	$\frac{1}{2}$	$\frac{1}{4}$
$\mu a_0^{(0)}$	0.017	0.013	0.09
$-\mu a_1^{(1)}$	0.004	0.003	0.002

These values for, say, $g=\frac{1}{2}$ represent a 10%, 15% and 25% correction to the current algebra $I=0,1,2$ scattering lengths and are smaller if $\sigma_{\text{tot}}^{\pi\pi}(\infty)$ and/or g are smaller. They seem to be of the correct magnitude thus encouraging us to use expression (2) for the calculation of the pomeron contribution to πK scattering amplitude. The pomeron trajectory being exchanged only in the $I_t = 0$ channel, we can write

$$S_{\frac{1}{2}}^{\text{pom}} = S_{\frac{3}{2}}^{\text{pom}} = c' g\alpha' \frac{s-u}{2} [H'(t, s) - H'(t, u)] ,$$

where

$$H'(t, s) = \sigma^{\alpha(t)} \Gamma(1-\alpha(t)) \Psi\big(1-\alpha(t), \tfrac{1}{2}; -g\alpha'(s-(m+\mu)^2)\big) ,$$

m being the K-meson mass. Again the above amplitude, when extrapolated off the mass shell in the manner already specified, has zeros when any of the external momenta is sent to zero. Proceeding as before, we get

$$(m+\mu) a_0^{(\frac{1}{2})} = \frac{m\mu}{2\pi^2} \sigma_{\text{tot}}^{\pi\text{K}}(\infty) \left[\sqrt{\pi x} - \sum_{n=1}^{\infty} x^n (A_n - B_n \sqrt{\pi x})\right]$$

with $x = 4m\mu\alpha' g$. Taking $\sigma_{\text{tot}}^{\pi\text{K}}(\infty) = 12$ mb, we again estimate the scattering lengths for different g-values as shown in Table II.

TABLE II.

g	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{16}$
$\mu a_0^{(\frac{1}{2})}$	0.017	0.013	0.01	0.005

We see that this contribution to πK scattering length is, for $\frac{1}{16} \leqslant g \leqslant 1$, of the same order of magnitude as the one provided by the usual Veneziano term (2) (the current algebra value is similar). It is interesting to calculate the threshold cross-section for πK scattering using our pomeron term summed with the dual Veneziano term. For $\text{K}^+\pi^-$ we obtain

$$\frac{\sigma_{\text{K}^+\pi^-}^{\text{pom+Ven}}(\text{threshold})}{\sigma_{\text{K}^+\pi^-}^{\text{Ven}}(\text{threshold})} \approx (1 + 0.14\sqrt{g})^2 ,$$

which for $g \leqslant 1$ can reduce the discrepancy between the measured value by TRIPPE *et al.* and the Veneziano model value (see footnote 16 of ref. (2)).

3. – Continuous PCAC.

Consider for example $\text{K}\pi \to \text{K}\pi$ and $\text{K}\pi \to \text{KA}_1$ amplitudes with last π and A_1 off their mass shell. We try to relate the pomeron parts of these amplitudes via PCAC. Let the pomeron term for the process

$$\text{K}(p_1) + \pi(p_2) \to \text{K}(p_3) + \pi(q) ,$$

be

$$T_\pi = c_0(t)(s-u)[H(t, s) - H(t, u)] ,$$

where $c_0(t)$ has not off-shell momentum dependence. For the process

$$K(p_1) + \pi(p_2) \to K(p_3) + A_1(q)$$

let us write

$$T_A = \varepsilon_\mu T_A^\mu \,,$$

where

$$T_A^\mu = (p_1 - p_3)^\mu D_1 + (p_1 + p_3)^\mu D_2 + q^\mu D_3 \,.$$

PCAC implies that [2]

$$\frac{2F_\pi m_A^2}{g_A} T_\pi = (t - \mu^2 + q^2) D_1 + (s - u) D_2 + 2q^2 D_3 \,. \tag{5}$$

In eq. (5) m_A is the A_1-meson mass and the PCAC constants appearing at the left-hand side can be fixed from KSFR and Weinberg sum rules [2] to be

$$F_\pi m_A = g_A \,.$$

Asymptotic t-channel helicity conservation for the A_1 production process requires that the leading term in $T^t_{\lambda=1}$ vanish, $T^t_{\lambda=1}$ being the helicity $+1$ amplitude for the A_1-meson in the t-channel c.m. system. Since $T^t_{\lambda=1} \sim D_2 \sim \beta_2(t) s^{\alpha(t)-1}$, t-channel helicity conservation implies $\beta_2(t) = 0$.

In the spirit of ref. [2], we assume that, for not too large values of the off-shell momentum q, we need to keep only up to quadratic dependence in q in the invariant amplitudes. This leads us to write for the D_i's:

$$D_{1,3} = d_{1,3}(s - u)[H(t, s) - H(t, u)] \,,$$
$$D_2 = d_2(q^2 - m_A^2)[H(t, s) - H(t, u)] \,,$$

where the d_i's are constants. Notice that the form chosen for D_2 ensures on-shell (for the A_1-meson) t-channel helicity conservation. Using the above equations, we obtain

$$\begin{cases} d_3 = -\dfrac{1}{2}(d_1 + d_2) \,, \\[2ex] c_0(t) = \dfrac{1}{2m_A}[(t - \mu^2) d_1 - m_A^2 d_2] \,. \end{cases} \tag{6}$$

From (6) the reader will notice why we did not simply set $d_2 = 0$, since this choice would imply a zero in the pion-residue function at $t = \mu^2$, which in itself is unreasonable besides the embarassing consequence that it leads to the ratio of A_1 cross-section to π cross-section much larger than unity instead of *vice versa*.

Now if we use the factorization property, we can obtain for the ratio of πA_1 coupling *vs.* $\pi\pi$ coupling to the pomeron:

$$\frac{\beta^{\lambda=0}_{\pi A_1}(t)}{\beta_{\pi\pi}(t)} = \frac{d_1\sqrt{[t - (m_A - \mu)^2][t - (m_A + \mu)^2]}}{m_A^2 d_2 + (\mu^2 - t) d_1} \,. \tag{7}$$

Assuming the same t dependence of the pomeron term in $\pi\mathcal{N} \to \pi\mathcal{N}$ and $\pi\mathcal{N} \to A_1\mathcal{N}$. eq. (7) allows us to compute the ratio of the integrated diffraction cross-sections, *i.e*,

$$\frac{\sigma(\pi\mathcal{N} \to \pi\mathcal{N})}{\sigma(\pi\mathcal{N} \to A_1\mathcal{N})} \approx \frac{\beta^2_{\pi\pi}(0)}{\beta^2_{\pi A_1}(0)} \approx \left(\frac{d_2}{d_1}\right)^2,$$

having assumed $|d_2| \gg |d_1|$. To compute the ratio d_1/d_2 we use the experimental value of $\sigma^{el}_{\pi\mathcal{N}}(\infty) \approx 4$ mb and the fact that A_1-production corss-section at 16 GeV/c is about (250 ± 50) μb ([5]). This tells us that $|d_1/d_2| \approx \frac{1}{4}$. We now have a prediction: if d_1 and d_2 have opposite signs, then from eqs. (6) or (7) we notice that the $\pi\pi$ coupling to the pomeron, $c_0(t)$, has a zero at the momentum transfer value t_0 such that

$$-t_0 = -\mu^2 + m^2_A\left(-\frac{d_2}{d_1}\right) \approx 4m^2_A = 4.4\ (\text{GeV/c})^2 .$$

Thus we expect a dip in the diffraction part of $d\sigma/dt$ for $\pi\mathcal{N}$ scattering at $t \approx \approx -4.4$ (GeV/c)^2. In fitting $\pi\mathcal{N}$ data at large t, BARGER and PHILLIPS ([6]) find that their pomeron term has a minimum around $t \approx -3$ (GeV/c)^2. Also, BERETVAS and BOOTH ([7]) have obtained from experiment the t dependence of the pomeron coupling in $\pi\mathcal{N}$ system. Figure 1 of ref. ([7]) shows that the pomeron coupling at $t = -3$ (GeV/c)^2 is $\approx 1.5 \cdot 10^{-3}$ times its values at $t = 0$ and that at higher (*i.e.* more negative) t values it rises again. In a recent letter AKERLOF *et al.* ([8]) have done a careful experimental analysis of the reactions $\pi^\pm p \to \pi^\pm p$ and $\pi^+ p \to K^+\Sigma^+$ in the region $2.2 < -t < 3.5$ (GeV/c)^2. They find a dip in the elastic reactions at $-t = (2.8$ GeV/c)^2 but not in the inelastic one. Due to this reason they reach the conclusion (in agreement with us) that the dip structure is diffractive in origin. Our calculation in fact shows a qualitative agreement with the above experimental results.

([5]) J. BALLAM, A. BRODY, G. CHADWICK, D. FRIES, Z. GUIRAGOSSIÀN, W. JOHNSON, R. LARSEN, D. LEITH, F. MARTIN, M. PERL, E. PICKUP and T. YAN: *Phys. Rev. Lett.*, **21**, 934 (1968).

([6]) V. BARGER and R. PHILLIPS: *Phys. Rev. Lett.*, **22**, 116 (1969).

([7]) A. BERETVAS and N. BOOTH: *Phys. Rev. Lett.*, **22**, 113 (1969).

([8]) C. AKERLOF, P. CALDWELL, P. KALBACI, D. MEYER, K. STANFIELD, P. KIRK, A. LESNIK, D. RUST, C. WARD and D. YOVANOVITCH: *Phys. Rev. Lett.*, **27**, 219 (1971).