Laboratori Nazionali di Frascati

LNF-72/22

G. Parisi : A DUALITY SUM RULE FOR DEEP INELASTIC SCATTERING.

Estratto da : Lett. Nuovo Cimento 3, 395 (1972)

LETTERE AL NUOVO CIMENTO VOL. 3, N. 10 4 Marzo 1972

A Duality Sum Rule for Deep Inelastic Scattering.

G. PARISI

Laboratori Nazionali del CNEN - Frascati

(ricevuto il 23 Dicembre 1971)

Recently, BLOOM and GILMAN (¹) have proposed to extend duality to deep inelastic scattering.

Their hypothesis is that $F_2(\omega) = \lim_{q^2\to\infty} \nu W_2(\nu, q^2)$ can be decomposed into two terms, resonances and background: the background is only $I = 0$ in the t-channel and the resonance contribution is not zero.

In this letter we want to show that from the assumptions of duality, SU_3 simmetry, absence of s-channel and t-channel exotic contributions and validity of Fubini–Dashen–Gell-Mann (²) sum rule, we are able to derive a sum rule (³) for the difference between proton and neutron structure functions

$$\int_0^1 [F_{2P}(\omega) - F_{2N}(\omega)] \frac{d\omega}{\omega} = \frac{1}{3}. \tag{1}$$

This sum rule is nearly satisfied: the experimental value for the left hand side is 0.19 with an error of 40% (³). Of course, we do not expect our sum rule to be exactly true in the real world: it may have a 20% error due to SU_3 breaking. This sum rule was also obtained by LANDSHOFF and POLKINGORNE (⁴) in the framework of the quark-parton model.

The derivation of eq. (1) is very simple. We denote by $F_{2B}^{ik}(\omega)$ the structure functions relative to $\langle B|J_i^\mu(x) J_k^\nu(0)|B\rangle$, where B is or the one-proton or the one-neutron state and $J_\mu^i(x)$ are the eight SU_3 currents.

Duality allows us to decompose $F_{2B}^{ik}(\omega)$ into resonances and background, and the background is only a t-channel SU_3 singlet.

(¹) E. BLOOM and F. GILMAN: *Phys. Lett.*, **25**, 277 (1970).
(²) S. FUBINI: *Nuovo Cimento*, **43** A, 475 (1966); R. F. DASHEN and M. GELL-MANN: *Lecture at the Third Coral Gables Conference on Symmetry Principles at High Energies* (1966).
(³) E. D. BLOOM, G. BUSCHHORN, R. COTTRELL, D. H. COWARD, H. DE STAEBLER, J. DREES, G. L. JORDAN, G. MILLER, L. MO, H. PIEL and R. E. TAYLOR: SLAC-PUB-796 (1971).
(⁴) V. P. LANDSHOFF and J. C. POLKINGORNE: *Nucl. Phys.*, **28** B, 240 (1971).

Under the hypothesis of absence of exotic t-channel exchange $(10, \overline{10}, 27)$, we can write the resonance contribution in the following way:

$$F_{2B}^{ik}(\omega) = \varepsilon(\omega)[\varrho(\omega)\,\delta_{ik} + D(\omega)\,d_{ik8} \pm d(\omega)\,d_{ik3}] + F(\omega)\,f_{ik8} \pm f(\omega)\,f_{ik3}\,, \tag{2}$$

where we must take the sign + for the proton and — for the neutron.

The absence of s-channel exotic resonances $(\overline{10}, 27)$ fixes

$$d(\omega) + f(\omega) = 0\,. \tag{3}$$

From the Fubini–Dashen–Gell-Mann [2] sum rules

$$\int_0^1 f(\omega)\,\frac{\mathrm{d}\omega}{\omega} = \frac{1}{4}\,, \tag{4}$$

follows that

$$\int_0^1 [F_{2P}(\omega) - F_{2N}(\omega)]\,\frac{\mathrm{d}\omega}{\omega} = -\frac{4}{3}\int_0^1 \frac{\mathrm{d}(\omega)}{\omega}\,\mathrm{d}\omega = \frac{1}{3}\,. \tag{5}$$

* * *

We thank Profs. R. A. BRANDT, N. CABIBBO, L. MAIANI and M. TESTA for discussions.