A. Bramón and M. Greco: RADIATIVE DECAYS OF MESONS AND AN EXTENDED VECTOR MESON DOMINANCE MODEL. -

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ABSTRACT. -

A model describing the radiative $P \to \gamma\gamma$, $V \to P\gamma$, $P \to PP\gamma$ and the strong $V \to PPP$ decays is proposed. The amplitudes for proces ses with a three-particle final state are approximated by Finite Disper sion Relation and the coupling constants are related by the Quark Model. Predictions derived from usual VMD are only in qualitative agreement with experiments. Better agreement is achieved by extending VMD with the inclusion of further vector-mesons.

1. - INTRODUCTION. -

The introduction of Vector Meson Dominance (VMD) in the analy sis of the radiative decays of mesons dates from 1962 and is due to Gell- -Mann, Sharp and Wagner[1]. When combined with unitary symmetry arguments, VMD has shown to be a particulary predictive model indu cing several authors[2-6] to perform more extensive studies on the sub ject. However, in order to get a quantitative agreement with actual expe rimental results, some improvements must be added to the original sche me. The main tendency has consisted in considering SU(3) breaking effects. This amounts to a simple inclusion of ω-φ and η- η' mixing[3,4]

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to the use of chiral symmetries[5] and the most general form of octet-
-broken SU(3)[6]. As a consequence the number of arbitrary parameters
of the model is considerably increased and its predictive power is
therefore reduced.

In the present paper a model following a rather opposite approach
is proposed. The main hypothesis consists in an extension of VMD un-
der the assumption that further vector mesons exist (EVMD). Further-
more the amplitudes for decays into a three-particles final state are
evaluated using finite dispersion relations (FDR) implemented by dua-
lity and finite-energy sum rules (FESR), as suggested by Aviv and Nus_
sinov[7], instead of a simple pole dominance approximation. The re-
strictive predictions of the quark model are then invoked to relate the
different coupling constants. In this way all the processes under con-
sideration can be described in terms of only two unknown parameters.
Good agreement between theory and experiment is achieved.

2. - DECAY AMPLITUDES AND FDR. -

We restrict ourselves to consider the radiative decays of pseu-
doscalar (P) and vector (V) mesons

(1a) $$V \to P\gamma$$

(1b) $$P \to \gamma\gamma$$

(1c) $$P \to PP\gamma$$

and the strong interaction decays

(1d) $$V \to PPP$$

The decays (1a, b), with only two stable particles in the final
state, are described by the matrix element

(2) $$G\, \varepsilon_{\alpha\beta\gamma\delta}\, k_1^\alpha\, \varepsilon_1^\beta\, k_2^\gamma\, \varepsilon_2^\delta$$

where G is the coupling constant, $k_{1,2}$ are the four-momenta of the
spin-one particles and $\varepsilon_{1,2}$ their polarizations. The decay widths are
then given by the well known expressions

(3a) $$\Gamma(V \to P\gamma) = G_{VP\gamma}^2\, \frac{(m_V^2 - m_P^2)^3}{96\pi\, m_V^3}$$

(3b)
$$\Gamma(P \to \gamma\gamma) = G^2_{P\gamma\gamma} \frac{m^3_P}{64\pi}$$

quoted here for completeness.

In the remaining cases (1c, d), with a three-particle final state, the matrix element may be written as

(4)
$$T(\nu, t) = A(\nu, t)\ \varepsilon_{\alpha\beta\gamma\delta}\ \varepsilon^\alpha q^\beta_1 q^\gamma_2 p^\delta$$

where $4\nu \equiv s - u$, $s \equiv (p-q_1)^2$, $t \equiv (q_1+q_2)^2$, $q_{1,2}$ are the four-momenta of the pseudoscalar mesons in the final state and p that of the decaying particle. The pole model of Gell-Mann et al.[1] approximates $A(\nu, t)$ by the contribution of the poles of the intermediate vector mesons. Such a procedure however is rather crude and, as shown by several authors[7,8], some of the failures of the model can be attributed to this fact. Alternatively, we shall use the FDR approach which has been prooved to be a useful tool to get a more reliable approximation for $A(\nu, t)$.

Essentially, FDR allows to decompose the amplitude $A(\nu, t)$ in two main contributions. The first one comes, as in the case of the pole model, from the nearby s-and u-channel singularities, for values of $\nu \leq N$, where N is the cut-off parameter. The second contribution accounts for the remaining resonant states and is dominated by the leading Regge trajectory exchanged in the t-channel provided N has been choosen suffi ciently large. In the spirit of duality, these two contributions are not independent and may be related by means of FESR. Moreover, in cases of actual interest both sides of the FESR are fairly well known and from the verification of the sum rule we estimate the most adequate value for N and at the same time the degree of reliability of our calculations.

We consider first the decay $\omega \to \pi^+\pi^-\pi^\circ$. The $\varrho^\pm$ poles dominate the s and u-channels whereas the ϱ-trajectory, $a \equiv a_\varrho(t) = 0.50 + a't$, with $a' = 0.9\,GeV^{-2}$, is exchanged in the t-channel.

Parametrizing the amplitude for high ν and fixed t as

(5)
$$A(\nu, t) \to g_{\varrho\omega\pi} g_{\varrho\pi\pi} \frac{\pi a'}{\Gamma(a)} \frac{1-e^{-i\pi a}}{\sin\pi a} (2a'\nu)^{a-1}$$

and using FDR techniques we obtain the following expression of the amplitude in the Dalitz region:

(6)
$$A(\nu, t) = g_{\varrho\omega\pi} g_{\varrho\pi\pi} \left\{ \frac{1}{s-m^2_\varrho} + \frac{1}{u-m^2_\varrho} + \frac{1}{t-m^2_\varrho} \frac{(2a'N)^{a-1}}{\Gamma(a)} \right\}$$

4.

The only difference between eq. (6) and the simple pole model is the factor $(2\alpha'N)^{\alpha-1}/\Gamma(\alpha)$ affecting the last term.

We also derive by standard techniques the first-moment FESR

$$(7) \qquad 2m_\varrho^2 - m_\omega^2 - 3\,m_\pi^2 + t = \frac{4\,\alpha'}{\Gamma(\alpha)}\ \frac{N^2(2\,\alpha'N)^{\alpha-1}}{\alpha+1}$$

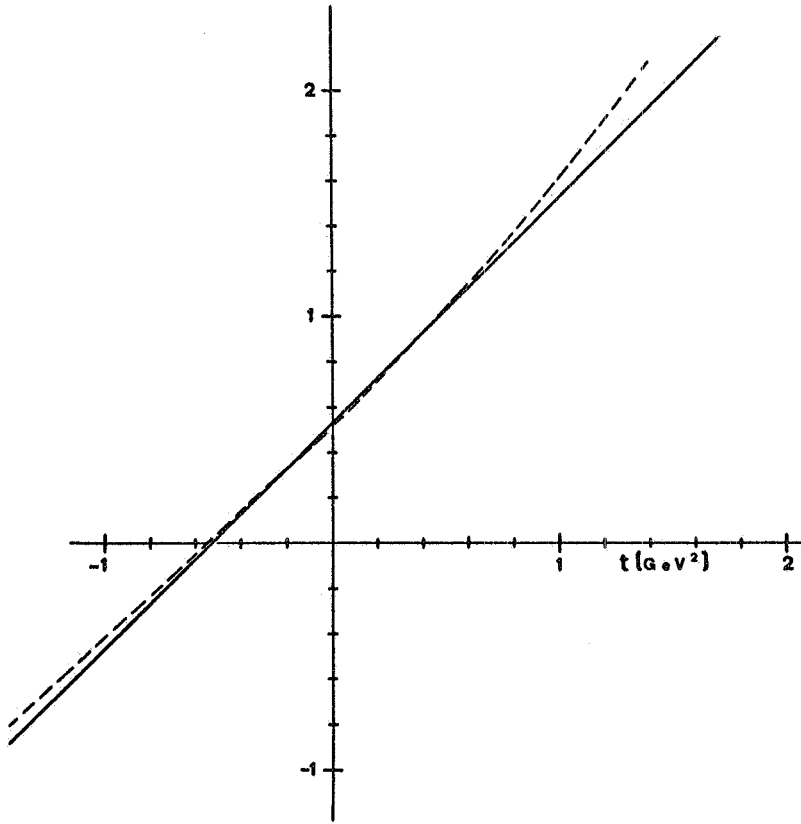The two sides of eq. (7) are plotted in Fig. 1 for $N = 0.64\,\text{GeV}^2 + 1/4\ t$.

FIG. 1 - Saturation of the sum rule, eq. (7) for the $\omega \rightarrow 3\pi$ amplitude. Full line represents the resonance side and dashed line the Regge side. Ordinates are in arbitrary units.

With this choice the agreement is particularly good for physical values of t and remains reasonable for a larger range of t values outside the Dalitz region. Notice that the cut-off N has been chosen between, but not exactly midway, the last included resonance and the first left out, the g-meson in this case. Equation (6) leads by numerical integration to

$$(8a) \qquad \Gamma(\omega \rightarrow \pi^+ \pi^- \pi^0) = g_{\varrho\omega\pi}^2\ \frac{g_{\varrho\pi\pi}^2}{4\pi}\ (10.5 \times 10^3\ \text{MeV}^3).$$

The present result agrees rather well, as could be expected, with that obtained by Goldberg and Srivastava[9] within the framework of the Veneziano model and is roughly 20% smaller than the simple pole model result[1,4]. A similar calculation for the $\varphi \to \pi^+ \pi^- \pi^0$ decay gives

$$(8b) \qquad \Gamma(\varphi \to \pi^+ \pi^- \pi^0) = g_{\rho\varphi\pi}^2 \frac{g_{\rho\pi\pi}^2}{4\pi} (38 \times 10^4 \text{ MeV}^3).$$

Eq. (8b) essentially coincides with the pole model expression[4] due to the fact that the two final pions are roughly on the ϱ mass.

Let us now turn to the $\eta \to \pi^+ \pi^- \gamma$ transition. The ϱ-trajectory α is again assumed to dominate the t-channel exchange while the A_2 poles describe the low energy s- and u-channels contribution. We obtain:

$$(9) \qquad A(\nu, t) = \frac{2g_{A_2\eta\pi} \, g_{A_2\pi\gamma}}{s - m_{A_2}^2} \left\{ 2t + s - \Sigma - \frac{m_\pi^2}{s} (m_\eta^2 - m_\pi^2) \right\} +$$

$$+ (s \leftrightarrow u) + \frac{2g_{\varrho\pi\pi} \, g_{\varrho\eta\gamma}}{t - m_\varrho^2} \frac{(2\alpha'N)^{\alpha-1}}{\Gamma(\alpha)}$$

where

$$\Sigma = s + t + u = m_\eta^2 + 2m_\pi^2$$

The first-moment FESR is written as

$$(10) \qquad (t + 2m_{A_2}^2 - \Sigma)\left\{ 2t + m_{A_2}^2 - \Sigma - \frac{m_\pi^2}{m_{A_2}^2} (m_\eta^2 - m_\pi^2) \right\} = K \frac{4\alpha'}{\Gamma(\alpha)} \frac{N^2 (2\alpha'N)^{\alpha-1}}{\alpha+1}$$

where $K \cong g_{\varrho\pi\pi} g_{\varrho\eta\gamma} / g_{A_2\eta\pi} g_{A_2\pi\gamma}$ and the coupling constants have been defined as in reference (10). From the experimental data[11] $\Gamma(A_2 \to \eta\pi) \cong$ $\cong 15.3$ MeV, $\Gamma(A_2 \to \varrho^0\pi) \cong 32.5$ MeV and $\Gamma(\varrho \to \pi\pi) \cong 140$ MeV we obtain $g_{A_2\eta\pi} \cong 2.8$ GeV^{-1}, $g_{A_2\pi\varrho} \cong 7.7$ GeV^{-2} and $g_{\varrho\pi\pi} \cong 5.9$. Introducing in eq. (8a) the experimental result[11] $\Gamma(\omega \to \pi^+ \pi^- \pi^0) = 10.2$ MeV and using SU(3) arguments, as described in the next Section, we also obtain $g_{\varrho\eta\varrho} =$ $= 17.$GeV^{-1}. These results and usual VMD lead to the relations

$$(11) \qquad K \cong g_{\varrho\pi\pi} g_{\varrho\eta\varrho} / g_{A_2\eta\pi} g_{A_2\pi\varrho} \cong (4.6 \pm 0.9) \text{ GeV}^2$$

6.

where the quoted error represent a reasonable estimate of the uncertainties on the above experimental data[(x)]. Notice that the numerical value of eq. (11) agrees with that reported in reference (10). The two sides of the FESR[(10)] are then plotted in Fig. 2. with a cut-off $N = 0.90$ GeV$^2 + 1/4$ t. Again this choice implies the exact verification of eq. (10) for the physical values of the two pion invariant mass, $t \simeq 0.16$ GeV^2, and a reasonable agreement for a larger t region. From equation (9) we finally deduce

(12a)
$$\Gamma(\eta \to \pi^+ \pi^- \gamma) = g_{\eta \varrho \gamma}^2 \frac{g_{\varrho \pi \pi}^2}{4\pi} (53 \pm 9) \, \text{MeV}^3 .$$
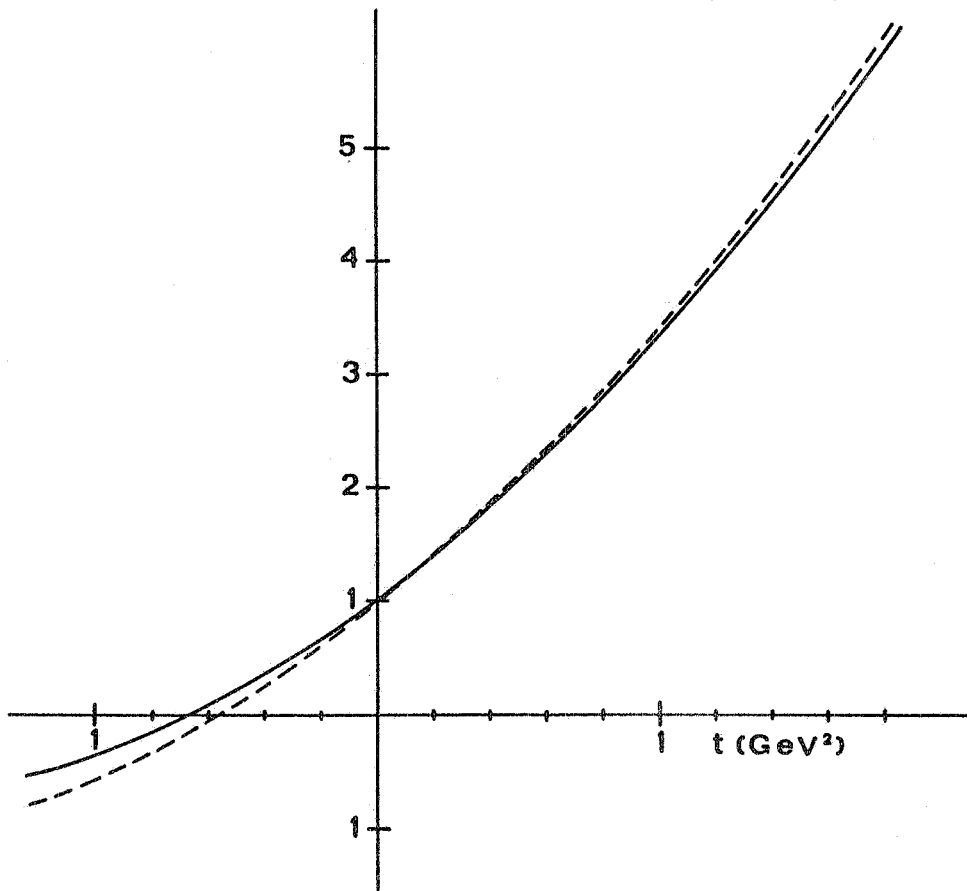

FIG. 2 - Saturation of the sum rule, eq. (10), for the $\eta \to \pi^+ \pi^- \gamma$ amplitude, as in Fig. 1.

(x) - In deriving the first equality of eq. (11) we have assumed that modifications of VMD, as described in next Section, have negligible effects on the ratio K.

This result is again in good agreement with the Veneziano model evalua tion[9] and represents a considerable correction, of about a factor of 3, to the pole model result[1,4]. In Fig. 3, we show the photon-energy spec trum computed by FDR technique and by the pole model, as compared with the experimental data[12]. The agreement is good in both cases.

We also have

(12b)
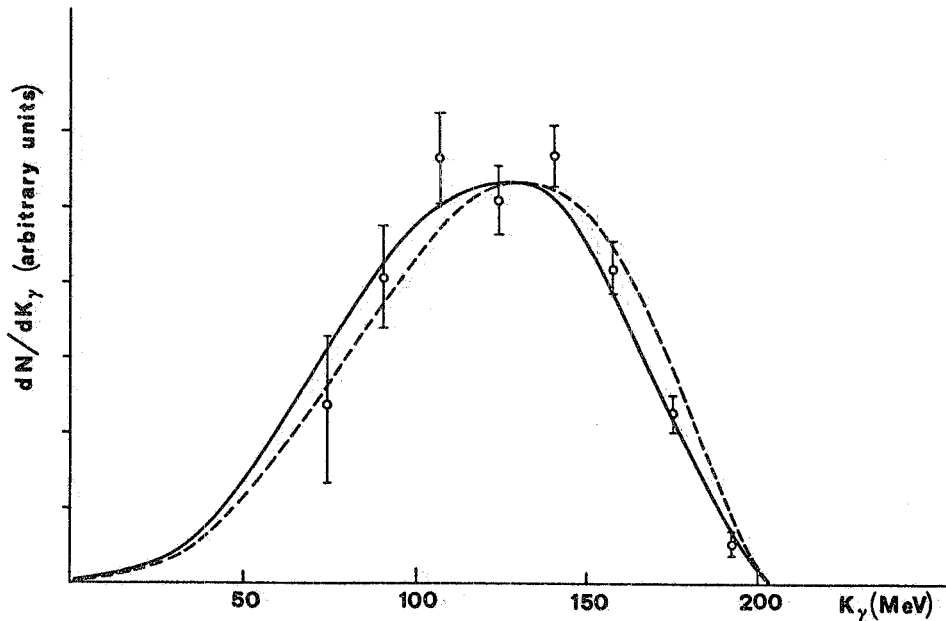$$\Gamma(\eta' \to \pi^+\pi^-\gamma) = g^2_{\eta'\varrho\gamma} \frac{g^2_{\varrho\pi\pi}}{4\pi} (1.6 \times 10^5 \,\text{MeV}^3)$$

FIG. 3 - Photon energy distribution in the $\eta \to \pi^+\pi^-\gamma$ decay as predicted by VMD (dashed line) and by FDR techniques (full line).

which has been derived in the same way or, more simply, calculating $\Gamma(\eta' \to \varrho^0\gamma)$ through the matrix element (2). This simplification is due to the fact that the invariant mass of the two pions is now essentially equal to m^2_ϱ and the corrections therefore to the simple pole model are negligible.

3. - EXTENDED VMD AND QUARK MODEL. -

The coupling constants appearing in the decay rates discussed in the preceding Section can be easily related by means of VMD as down in references (3-6).

If is however well known that simple VMD can fail when applied

to relate different processes$^{(x)}$. The most striking failure is the rela-
tion between total γp cross section and vector meson photo-production[13].
The value $g_\varrho^2/4\pi = 1.4 \pm 0.2$ obtained in this way is indeed in sharp disa_
greement with the colliding beams result[14] $f_\varrho^2/4\pi = 2.54 \pm 0.23$. An
obvious possible reason for discrepancies is the existence of additional
hadronic vector states, as suggested by many theoretical models, which
couple to the photon. The rather large cross sections for e^+e^- annihila-
tion into hadrons measured at Frascati[15] seem to confirm this hypothe_
sis. The channel $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, in particular, presents a bump
structure[16] at energies $2E = \sqrt{s} \simeq 1.6$ GeV which could be interpretated
as a ϱ'-meson[17].

Motivated by these reasons we analyze in what follows the pre-
dictions of the VMD model extended by the inclusion of these higher sta-
tes. We shall also assume, as suggested by the quark model, that the
new vector mesons V' are classified according to SU(3) nonets, similar
ly to the known vector states V. In what follows we'll make use of
only one additive nonet of vector mesons, which will account in the ave-
rage for all the set, presumably infinite, of higher states.

Defining the V, V'-photon couplings as

$$g_{V\gamma} \equiv em_V^2/f_V, \qquad g_{V'\gamma} \equiv em_{V'}^2/fV'$$

and the strong vertices VVP and VV'P through

$$g \equiv g_{\varrho\varrho\eta_8} \quad, \qquad h \equiv g_{\omega\varrho\eta_8} \quad, \qquad f \equiv g_{\varrho\varrho\eta} \quad,$$
$$g' \equiv g_{\varrho\varrho'\eta_8} \quad, \qquad h' \equiv g_{\omega\varrho'\eta_8} \quad, \qquad f' \equiv g_{\varrho\varrho'\eta} \quad,$$

we can express, as down in ref. (18), the different coupling constants
for the decays (1) in terms of these eight parameters.

We prefer however to reduce the number of these unknown quan-
titatives taking into account the well known quark model predictions

(13) $\qquad\qquad h = f = \sqrt{2}\, g \qquad$ and $\qquad h' = f' = \sqrt{2}\, g'$

In this way all the processes under consideration depend exclusively on
two independent parameters which, for convenience, are choosen to be

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(x) - For a detailed discussion on the subject see for instance ref. (13).

$g_{\rho\omega\pi}$ and $\lambda = g_{\rho'\omega\pi} f_\rho / g_{\rho\omega\pi} f_{\rho'}$. Introducing the ω-ρ and η-η' mixing angles, θ_V and θ_P, through the corrections

$$\varphi = -\cos\theta_V \,\omega_8 + \sin\theta_V \,\omega_r \qquad \eta = \cos\theta_P\,\eta_8 - \mathrm{sen}\,\theta_P\,\eta_1$$

(14)

$$\omega = \sin\theta_V\,\omega_8 + \cos\theta_V\,\omega_{1'} \qquad \eta' = \sin\theta_P\,\eta_8 + \cos\theta_P\,\eta_1$$

and the "ideal" mixing angle $\theta_o = \mathrm{arctg}\,1/\sqrt{2} = 35.3^\circ$ we obtain

$$g_{\rho\varphi\pi} = \mathrm{tg}(\theta_V - \theta_o)\,g_{\rho\omega\pi}$$

(15a)

$$g_{\varphi\pi^o\gamma} = \mathrm{tg}(\theta_V - \theta_o)\,g_{\rho\omega\pi}\,\frac{e}{f_\rho}(1+\lambda)$$

$$g_{\pi^o\gamma\gamma} = g_{\rho\omega\pi}\,\frac{2e^2}{f_\rho f_\omega}\left\{1 - \mathrm{tg}(\theta_V - \theta_o)\frac{f\omega}{f\varphi}\right\}(1+2\lambda)$$

and with the simplifying approximation $\theta_V = \theta_o$

$$g_{\omega\pi^o\gamma} = g_{\rho\omega\pi}\,\frac{e}{f_\rho}(1+\lambda)$$

$$g_{\rho\pi\gamma} = \frac{1}{3}g_{\rho\omega\pi}\,\frac{e}{f_\rho}(1+\lambda)$$

(15b)

$$g_{\rho\eta\gamma} = \frac{1}{\sqrt{3}}g_{\rho\omega\pi}(\cos\theta_P - \sqrt{2}\sin\theta_P)\,\frac{e}{f_\rho}(1+\lambda)$$

$$g_{\omega\eta\gamma} = \frac{1}{3\sqrt{3}}g_{\rho\omega\pi}(\cos\theta_P - \sqrt{2}\sin\theta_P)\,\frac{e}{f_\rho}(1+\lambda)$$

$$g_{\varphi\eta\gamma} = \frac{2}{3\sqrt{3}}g_{\rho\omega\pi}(\sqrt{2}\cos\theta_P + \sin\theta_P)\,\frac{e}{f_\rho}(1+\lambda)$$

$$g_{\eta\gamma\gamma} = \frac{2}{3\sqrt{3}}g_{\rho\omega\pi}(\cos\theta_P - 2\sqrt{2}\sin\theta_P)\,\frac{e^2}{f_\rho^2}(1+2\lambda).$$

The coupling constants for the processes involving the η'-meson can be deduced from those of the η by simply changing θ_P into $\theta_P + \pi/2$.

Let us finally note that in deriving the $g_{P\gamma\gamma}$ coupling constants we have neglected contributions coming from the transitions $P \to V'+V' \to$ $\to \gamma + \gamma$, retaining only first order terms in the correction to the VMD model.

4. - DISCUSSION AND RESULTS. -

Before proceeding to compare results of the present model with the available experimental information the numerical values of the quantities f_V, $g_{\rho\pi\pi}$, θ_V and θ_P are needed. According to the new data on e^+e^- colliding beams from Orsay[14] we use $f_\rho^2/4\pi = 2.54 \pm 0.23$, $f_\omega^2/4\pi = 19.2 \pm 2.0$ and $f_\varphi^2/4\pi = 11.0 \pm 0.9$ and, taking also into account the most recent results listed in ref. (11), $\Gamma(\varrho \rightarrow \pi\pi) = (140\pm10)\,\text{MeV}$, $m_\rho = 776\,\text{MeV}$ which imply $g_{\rho\pi\pi}^2/4\pi = 2.66 \pm 0.19$.

The question arises now of the choice of the mixing angles, according to a linear or quadratic Gell-Mann-Okubo mass formula. In the framework of the quark model however we have some arguments favoring the linear one. In fact, Butler et al.[19] have measured the production ratios φ/ω and η/η' in the reaction $\pi^+ p \rightarrow \Delta^{++} + $ Boson and have analyzed their results in light of the quark model predictions of Alexander et. al.[20]. They obtain $\theta_V^{exp} = 37.5^{o+1.3}_{-2.2}$ and $\theta^{exp} = -29.0^o + 3.3^o$ in agreement with the linear mass formula predictions

$$(16) \qquad \theta_V = 37.7^o, \qquad \theta_P = -23.4^o$$

Furthermore, from an analysis[21] of the data on the process $\pi^+ p \rightarrow \Delta^{++} \eta(\eta')$ one obtains the ratio

$$r \cong \frac{g_{A_2\pi\eta'}^2}{g_{A_2\pi\eta}^2} = 0.38 \pm 0.07$$

which agrees with the result $r = tg^2(\theta_P - \theta_o) = 0.37$ derived from simple quark model arguments and eq. (16).

With all this in mind we turn now to a determination of the free parameters $g_{\rho\omega\pi}$ and λ and a comparison of our predictions with experiments. The decay widths and the branching ratios for processes with a clear experimental situation are shown in Table 1. The experimental data are taken from ref. (11) except the values of R_φ and R_η, which can be deduced from the recent results quoted in references (14) and (22). Using equations (3), (8), (12) and (15) and the least squares method to fit these experimental data we obtain

$$(17) \qquad \begin{aligned} g_{\rho\omega\pi}^2 &= (370 \pm 32)\,\text{GeV}^{-2} \\ \lambda &= -0.16 \pm 0.03, \end{aligned}$$

the corresponding χ^2 being 3.1 for 6 degrees of freedom. The predictions

TABLE 1

Process	Experimental data	EVMD	VMD
$\Gamma(\omega \to \pi^+ \pi^- \pi^0)$ (MeV)	10.2 ± 0.9	10.4 ± 1.2	6.5 ± 0.7
$R_\omega \equiv \dfrac{\omega \to \pi^0 \gamma}{\omega \to 3\pi}$ (%)	10.4 ± 1.4	10.6 ± 1.4	15 ± 2
$\Gamma(\varphi \to \pi^+ \pi^- \pi^0)$ (MeV)	0.73 ± 0.22	0.73 ± 0.08	0.45 ± 0.05
$R_\varphi \equiv \dfrac{\varphi \to \pi^0 \gamma}{\varphi \to 3\pi}$ (%)	1.37 ± 0.65	0.65 ± 0.09	0.92 ± 0.11
$\Gamma(\pi^0 \to \gamma\gamma)$ (eV)	7.8 ± 1.0	8.2 ± 1.6	11.1 ± 1.7
$\Gamma(\eta \to \gamma\gamma)$ (eV)	990 ± 220	730 ± 150	980 ± 150
$R_\eta \equiv \dfrac{\eta \to \pi^+ \pi^- \gamma}{\eta \to \gamma\gamma}$ (%)	12.2 ± 0.6	10.5 ± 2.2	6.9 ± 1.4
$R_{\eta'} \equiv \dfrac{\eta' \to \gamma\gamma}{\eta' \to \text{all}}$ (%)	1.8 ± 0.5	1.53 ± 0.20	2.34 ± 0.27

of our model for these values are then reported in the third column of Table 1.

A similar analysis has also been performed with $\lambda = 0$, i.e., within the restricted VMD model. We obtain

$$g^2_{\omega\varrho\pi} = (230 \pm 20) \text{ GeV}^{-2}$$

and the results quoted in the last column of Table 1. The χ^2 has increased to 36.8 for 7 degrees of freedom.

Similar results concerning radiative decays with a less definite experimental situation are shown in Table 2. The experimental data are taken again from reference (11) except the quoted values for the $\varphi \to \eta\gamma$ decay width deduced from references (14, 23).

TABLE 2

Process	Experimental information	EVMD	VMD
$\Gamma(\eta' \to \gamma\gamma)$	< 72	4.3 ± 0.9	5.7 ± 0.9
$\Gamma(\eta' \to \pi^+ \pi^- \gamma)$	< 1200	83 ± 13	73 ± 11
$\Gamma(\eta' \to \omega\gamma)$	---	9.2 ± 1.3	7.9 ± 1.0
$\Gamma(\varrho \to \pi\gamma)$	< 625	130 ± 18	116 ± 15
$\Gamma(\varrho \to \eta\gamma)$	---	98 ± 14	83 ± 10
$\Gamma(\omega \to \eta\gamma)$	< 180	11.5 ± 1.6	9.8 ± 1.2
$\Gamma(\varphi \to \eta\gamma)$	84 ± 30 292 ± 80	110 ± 16	91 ± 12

All rates are expressed in KeV.

5. - CONCLUSIONS. -

We have analyzed the radiative and strong decays (1) of pseudo-scalar and vector mesons. Decay amplitudes with three particles final state have been described by means of finite dispersion relations, whose reliability has been tested through a FESR. This implies an important modification to the pole model result for the $\eta \to \pi^+ \pi^- \gamma$ decay and a non-negligible one for $\omega \to \pi^+ \pi^- \pi^0$. The coupling constants are then related by the simple quark model. This allows a description of all pro cesses in terms of a reduced number of unknown parameters: only one, if we restrict ourselves to the known VMD with ρ, ω and φ mesons, or two, if further vector mesons are assumed to exist.

In the first case only a semi-quantitative agreement between theo ry and experiments is achieved.

The extension of the simple VMD model, by the inclusion of further vector mesons, leads on the contrary to a significant improvement in the agreement between theory and experiments. This extension is also favoured by recent experimental indications on the existence of new vector mesons[16]. The implications of our model in the e^+e^- annihilation

into hadrons, as well as in proton Compton scattering and vector meson photoproduction have been previously discussed[17].

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