Laboratori Nazionali di Frascati

G. De Franceschi, A. Reale and G. Salvini: THE η AND η'
PARTICLES IN THE PSEUDOSCALAR NONET

# THE η AND η′ PARTICLES IN THE PSEUDOSCALAR    5513
## NONET

### G. DE FRANCESCHI, A. REALE

*Laboratori Nazionali di Frascati del C.N.E.N.*

### G. SALVINI

*Istituto di Fisica dell' Universita di Roma and Istituto Nazionale de Fisica Nucleare*

**CONTENTS**

## 1. INTRODUCTION

This review is concerned with the present state of our knowledge on the new pseudoscalar mesons $\eta$ and $\eta'$, which were discovered in the last 10 years, completing the pseudoscalar nonet.

The first proposal of classifying the $0^-$ mesons (zero spin and negative parity) in a pseudoscalar octet and a pseudoscalar singlet belonging to higher symmetries than those already established by isospin and strangeness conservation was made 10 years ago by Gell-Mann (1). Seven places of the octet were already occupied by $\pi^+$, $\pi^-$, $\pi^0$, $K^+$, $K^0$, $\overline{K}^0$, $K^-$ mesons, while an eighth particle, the present $\eta$, was predicted by Gell-Mann with its definite quantum numbers and was discovered at about the same time. The problem of the singlet $\eta'$, which forms with the octet the $0^-$ nonet, is not completely resolved and the situation is largely discussed in our review. We shall see that we could have more than one candidate for this role. Moreover, we shall see that it is not exact, due to the singlet-octet mixing, to consider the $\eta$ and the $\eta'$ as clearly separated particles.

Of course the pseudoscalar mesons are not all equally well known: the pions and the kaons have a very extended literature at low and high energy. The $\eta$ particle is by now rather well studied, but there are still some uncertainties in the production rate at high energies and in the decay modes. The situation with the $\eta'$ is still rather complicated, and the quantum numbers of the candidates ($X^0$ and $E$ mesons) have not been definitely established as yet. Therefore, we have written the present article with two objectives in mind. One is to establish the extent of our present knowledge on the production rate, specific properties, and decay modes of the $\eta$ and $\eta'$ particles. The second is to discuss the foundation of the pseudoscalar nonet and its position in the unitary symmetries today.

In Reference (2) we have suggested some books or reviews where one may find an introduction to conservation laws, quantum numbers, and general theories.

The properties of the $\eta$ particle are presented in Section 2. Section 3 is concerned with the contribution of the eta decay to the verification of $C$ conservation. Section 4 is devoted to the $\eta'$ mesons. In Section 5 we give a short introduction to the position of the $0^-$ octets and singlets in the present

unitary symmetries, and we discuss the mixing angles of $\eta$ and $\eta'$ and other theoretical aspects connected with these particles.

Section 6 is devoted to the present theories on the decay modes of $\eta$ and $\eta'$ particles. We decided to give some space to this argument, considering that some of the most interesting and general problems of present-day theoretical physics are connected with the e.m. decays of the $\eta$. Section 7 deals with the production at high energy of $\eta$ particles, and the Primakoff effect.

In Section 8 we have tried to summarize the present situation in some logical way, indicating the open theoretical problems and the experimental measurements which are more specifically needed to put the present knowledge of this subject on a firmer basis.

## 2. THE ETA PARTICLE. PRODUCTION AT LOW ENERGY AND DECAY MODES

### 2.1 DISCOVERY OF THE ETA

The $\eta$ particle was discovered in 1961 by Pevsner et al (3) using the 72-inch Alvarez bubble chamber to investigate the reaction

$$\pi^+ + d \rightarrow p + p_{\text{spect}} + \pi^+ + \pi^- + \pi^0 \qquad 1.1$$
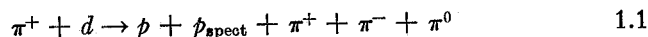
at a momentum of the incident pion beam of 1.23 GeV/$c$.

Figure 1 shows the histogram of the invariant mass $M$ of the $3\pi$ system, for a total of 233 events. Besides the expected $\omega$ peak, another peak in the $3\pi$ mass plot of Figure 1 is seen near 550 MeV. This suggested the existence of a second $3\pi$ resonance (or particle). As the authors (3) said, "we shall hereafter refer to this particle as $\eta$."

The existence and the properties of this new particle had been predicted a few months before its discovery in a now famous paper by Gell-Mann (1), where he proposed the octet of pseudoscalar mesons (see Section 5) formed by pions, kaons, and the $\eta$.

The study of reactions

$$\pi^\pm + p \rightarrow p + (\pi^\pm + \pi^0 + \pi^0) \qquad 1.2$$
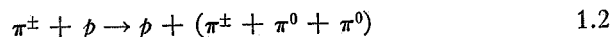
by Carmony et al (4), and of

$$p + p \rightarrow p + n + \pi^+ + \pi^+ + \pi^- \qquad 1.3$$
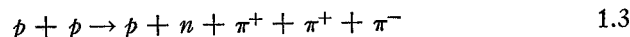
by Pickup et al (5) did not show any peak of the 3-pion system rest mass around 550 MeV. This was convincing evidence that the $\eta$ meson has isospin $I = 0$.

The existence of the $\eta$ was soon confirmed by the observation of reactions

$$K^- + p \rightarrow \Lambda^0 + (\pi^+ + \pi^- + \pi^0) \qquad 1.4$$

$$K^- + p \rightarrow \Lambda^0 + MM \text{ (missing mass)} \qquad 1.5$$

by Bastien et al (6).
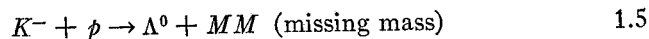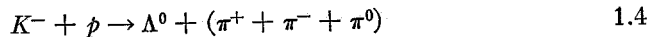
**Effective Mass in the 3π System (MeV)**

FIGURE 1. Hystogram of the effective mass of the 3π system in the reaction
$\pi^+ + d \rightarrow p + p + \pi^+ + \pi^- + \pi^0$. Results by Pevsner et al (3). The plotted curve is the
phase space. The peak of the eta particle is at left; the peak at right is the already
known ω particle.

The momentum of the impinging kaons was close to 760 MeV/$c$. The width of the peak in the $\pi^+\pi^-\pi^0$ and $MM$ distribution was equal to the experimental resolution. Using $K^-$ mesons of the same momentum, Prowse (7) looked for a peak in the charged $3\pi$ combination in the reaction

$$K^- + d \rightarrow \Lambda^0 + \pi^- + \pi^- + \pi^+ + P_{\text{spect}} \qquad 1.6$$

No evidence of a peak was found in reaction 1.6, so again one had a strong indication of $I = 0$ for the $\eta$.

The work of Bastien et al (6) succeeded in fixing the mass and full width of the $\eta$ within the limits

$$m_\eta = (549 \pm 1.5) \text{ MeV} \qquad \Gamma_\eta \leq 7 \text{ MeV} \qquad 1.7$$

In both reactions 1.5 and 1.6 the peak around the mass 550 MeV appeared to indicate, through the measurement of the missing mass in reaction 1.6 ("the neutrals"), that the $\eta$ could decay in the $\pi^+\pi^-\pi^0$ charged mode and in pure neutral modes. A branching ratio

$$R_{c,n} = \frac{\eta \rightarrow \pi^+\pi^-\pi^0(\text{or } \pi^+\pi^-\gamma)}{\eta \rightarrow \text{neutrals}} = 0.33 \pm 0.11 \qquad 1.8$$

was given (6).

Evidence of the $\eta$ particle decaying into all neutrals was found also in many other channels. We recall as a nice example the missing-mass distribution obtained by Kraemer et al (8), again in the channel $\pi^+ + d \rightarrow p + p$ +neutrals (reaction 1.1).
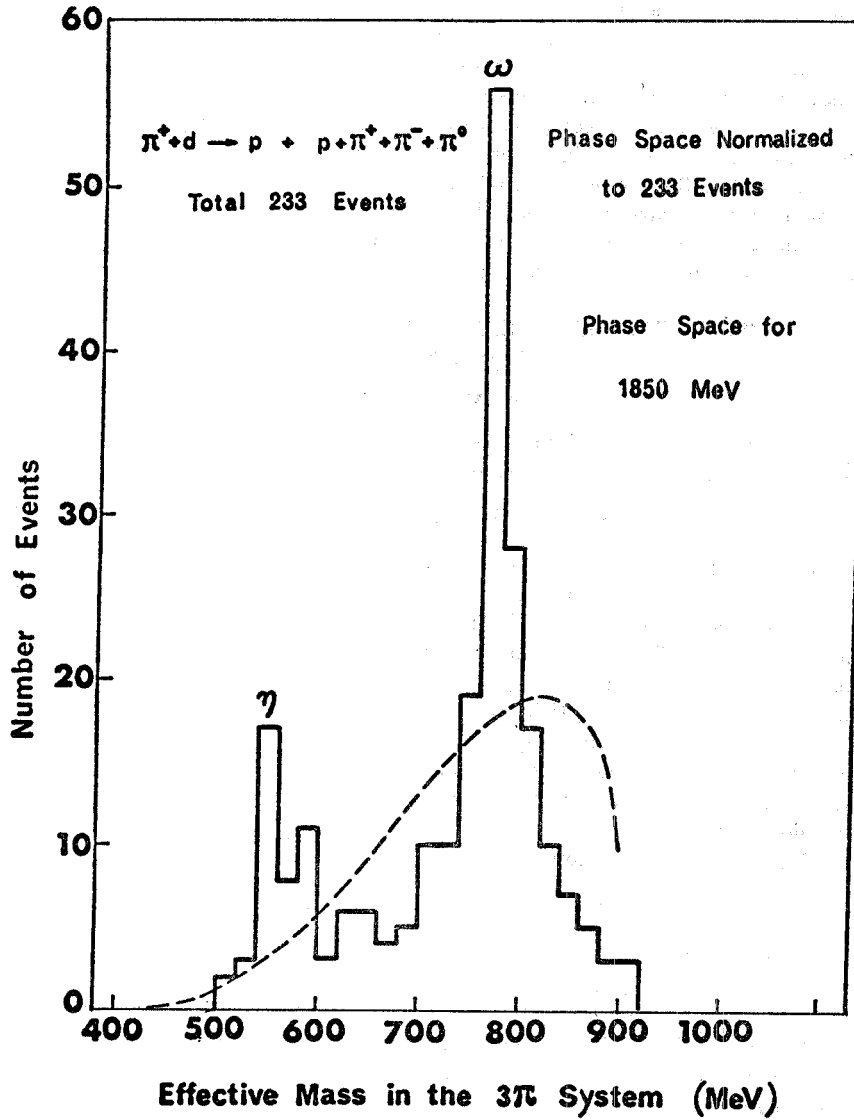
### 2.2 DETERMINATION OF THE $\eta$ QUANTUM NUMBERS

A survey of the $\eta$ particle has been given by Salvini (9).

The quantum numbers of the $\eta(I^G, J^P = 0^+0^-)$ were soon established in the work of Bastien et al (6). In order to establish the value of the spin $J$ and the parity $P$, these authors assumed that $\eta$ has isospin $I = 0$, and considered the two cases $C = G = -1$ and $C = G = +1$.

$C = G = -1$.—This allows the $\eta$ to decay strongly into $\pi^+\pi^-\pi^0$. The complete spatial antisymmetry of the $I = 0$ three-pion system would then forbid the decay into $3\pi^0$ and imply a sextant symmetry in the $\pi^+\pi^-\pi^0$ Dalitz-Fabri plot (D.F. plot). This symmetry, however, as well as the other characteristic features expected for each of the simplest $J^P = 0^-$, $1^-$, $1^+$ assignments, were not consistent with the actual density of points on the D.F. plot of Bastien et al (6). (See Figure 2.)

$C = G = +1$.—The $\eta$ decay into $3\pi$ is still possible via $G$ nonconserving e.m. interactions. With $I = 1$ for the three-pion system, decay into $3\pi^0$ is allowed with the branching ratio $\Gamma(3\pi^0)/\Gamma(\pi^+\pi^-\pi^0) \leq 3/2$. The measured (6) branching ratio $\Gamma(\eta \text{ neutr})/\Gamma(\eta \text{ ch}) \sim 3$ would then indicate that radiative de-

**a )**
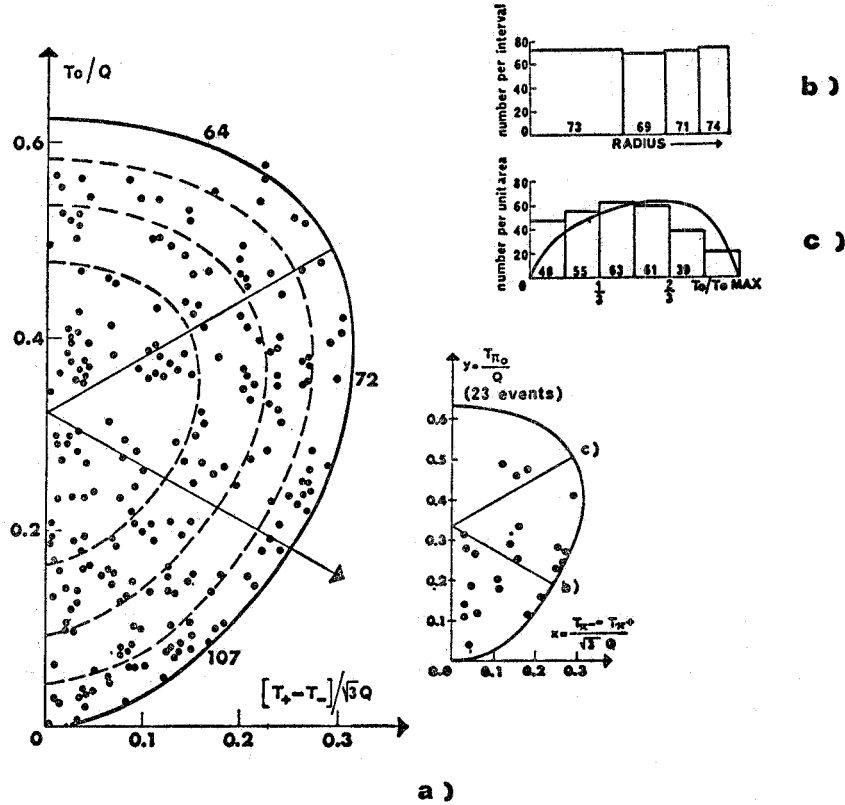
Figure 2. a) Dalitz-Fabri plot for $\eta$ decay, obtained through collection of all $\eta$ decays by 1962 (25). The smaller D.F. plot refers to the 23 events of Bastien et al (6), which first allowed the assignment of the proper quantum numbers to the eta particles. $x = |\,T_{\pi^-} - T_{\pi^+}\,| \,/\sqrt{3}Q$; $y = T_{\pi^0}/Q$. b) The radial density. c) The projection on the $\pi^0$ axis.

cays are present (e.g. $\eta \rightarrow \gamma\gamma$). When assuming $I = 0$, $G = +1$, the simplest possible $J^P$ values are $1^-$, $1^+$, $0^-$. ($J^P = 0^+$ would allow strong $2\pi$ decay, which has never been observed.) However, the $1^-$ and $1^+$ states are not favored by the experimental density on the D.F. plot. Hence we are left only with the possibility $J^P = 0^-$. The simplest matrix element in this case is a constant, whereas the data (see Figure 2) seem to favor low values of $T_{\pi^0}$. However, this result is not necessarily inconsistent with the $J^P = 0^-$ assignment. It only implies that the approximation of a constant matrix element is bad, due e.g. to final state interactions.

The conclusion of the authors was therefore that the eta decays through an e.m. transition from an $I^G J^P = 0^+ 0^-$ state to a $3\pi$ $1^- 0^-$ final state.

An almost direct verification of $G$ violation in $\eta$ decay came from the work of Rosenfeld et al (10). These authors, following an indication by Feinberg (11), made a search for the decay:

$$\rho \rightarrow \eta + \pi \qquad\qquad 2.1$$

This mode is allowed only if the $G$ parity of the $\eta$ is negative as for the $\pi$, considering that it is a strong decay. Rosenfeld et al found experimentally that reaction 2.1 has a branching ratio $<0.6\%$. This was good evidence again that $\eta$ has even $G$ parity.

In Table 1 we synthesize the logic that first allowed the attribution to the eta of the correct quantum numbers. Confirmation that the $\eta$ decays through e.m. interactions, and that among the neutral modes radiative processes are present in a substantial amount, came from other experiments around 1962.

A first evidence of the radiative decay of the $\eta$, as well as of photoproduction of the $\eta$ in the $\gamma+p$ channel, came from a Frascati group (12). These authors found the process

$$\gamma + p \rightarrow p + \eta \qquad \text{with } \eta \rightarrow \gamma\gamma \quad \text{or} \quad \pi^0\gamma \qquad 2.2$$

This was the first counter-experiment on the eta. The protons of reaction 2.2 were detected by spark chambers, and the authors looked at the spectrum of the $\gamma$ rays produced in coincidence. This spectrum appeared to have the shape expected from the decay process $\eta \rightarrow \gamma+\gamma$ or from the decay $\eta \rightarrow \pi^0+\gamma$, plus a contribution from $\eta \rightarrow 3\pi^0$. The authors could not decide

**TABLE 1. Quantum numbers of the eta**[a]
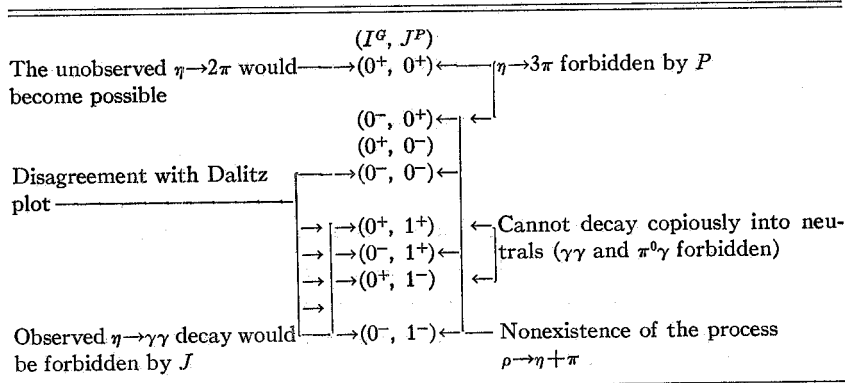


$^a$ This is a synthesis of the discussion which led to establishing the quantum numbers of the eta. In the center is the list of the simplest $I^G$, $J^P$ values ($J \leq 1$), having already assumed $I=0$ (see text). The arrows indicate the quantum numbers which are forbidden by the reasons written aside. Only $I^G$, $J^P = 0^+$, $0^-$ remains possible.

## TABLE 2. Main Data for the $\eta$ Particle[a]

|  | Value | Notes |
|---|---|---|
| Mass (MeV) | 548.8±0.56 | $4m_{\pi^0} < M_\eta < 2m_{\pi^0} + m_{\pi^+} + m_{\pi^-}$ |
| Width (eV) | 2700±670 | Ref. (168) |
| $I^G$, $J^P$, $C$ | $0^+$, $0^-$, $+$ | |
| Branching ratios (%): | | |
| $\gamma\gamma$ | 37.5±1.6 | |
| $\pi^0\gamma\gamma$ | 4.7±2.4 | |
| $3\pi^0$ | 30.0±2.2 | |
| $\pi^+\pi^-\pi^0$ | 23.1±1.0 | |
| $\pi^+\pi^-\gamma$ | 4.7±0.2 | |
| $\pi^0e^+e^-$ | $2.10^{-2}$ | 90% confidence level |

[a] See Reference (15).

between the mode $\gamma\gamma$ (that is, $J=0$ for the $\eta$, or at least even $J$) and the mode $\gamma+\pi^0$ (that is, $J=1$).

The definite choice between the two modes and the consequent definite confirmation of the right spin value ($J=0$) to the $\eta$ came from an experiment of Chretien et al (13). Their experiment makes use of the reaction

$$\pi^- + p \rightarrow n + \eta \qquad\qquad 2.3$$

which, from charge symmetry, must have the same cross section as reaction 1.1, $\pi^+ + n \rightarrow p + \eta$.

The events of reaction 2.3 were detected in a heavy liquid bubble chamber with an incident negative-pion beam of incoming momentum 1140 MeV/c. The $\eta \rightarrow 2\gamma$ could be identified by the conversion of the $\gamma$s into electron pairs. The opening angle distribution among the two $\gamma$s exhibits two peaks. The peak at $\sim 35°$ corresponds to the $\pi^0 \rightarrow \gamma\gamma$ decay. The peak at $\sim 100°$ corresponds to a mass of (545 ± 30 MeV), in good agreement with the known mass of the $\eta$. In Table 2 we have reported the main data on the $\eta$ particle.

### 2.3 THE $\eta$ DECAY MODES: EXPERIMENTAL RESULTS

Having established that eta has quantum numbers $I^G$, $J^P = 0^+$, $0^-$, it follows that the $\eta$ can decay in at least the following ways:

(a) Through strong interactions, in the mode $\eta \rightarrow 4\pi^0$. The $Q$-value of this decay is $m_\eta - 4m_{\pi^0} = 9$ MeV. Because of the low $Q$ value and phase space effects, the relative probability for this (unobserved) decay mode must be very small (14). The strong decays will not be discussed further in this article.

(b) Through electromagnetic interactions. This is a rather long list and we recall now the most important decays:

| decay: | transition rate: |
|---|---|
| $\eta \to \pi^+\pi^-\gamma$ | (first e.m. order, $\alpha$); |
| $\to \pi^+\pi^-\pi^0$, $\pi^0\pi^0\pi^0$, $\gamma\gamma$, $\pi^0\gamma\gamma$ | (second e.m. order, $\alpha^2$); |
| $\to \gamma e^+e^-$, $\gamma\mu^+\mu^-$ | (second e.m. order, $\alpha^2$), |
| $\to \pi^0 e^+e^-$, $\pi^0\mu^+\mu^-$, $\mu^+\mu^-$, $\pi^+\pi^-\pi^0\gamma$ | |

It is difficult to recount the many experiments which have been published on the $\eta$ decays. We give in Table 3 only a synthesis of the most recent experimental results; for a more complete record, see (9) and (15). A table in matrix form has been chosen in order to allow an easier comparison among various experimental results.

A few facts emerge from Table 3:

The situation for the charged modes ($\eta \to \pi^+\pi^-\pi^0$; $\pi^+\pi^-\gamma$) is satisfactory, without inconsistencies among the different experiments.

The branching ratios among the neutral modes have some uncertainties. One controversial point, in particular, is the $\pi^0\gamma\gamma$ mode.

### TABLE 3. Branching ratios for the $\eta$ decay modes[a]

| | $\pi^+\pi^-\pi^0$ ($l=1$) | $\pi^+\pi^-\gamma$ (2) | $3\pi^0$ (3) | $\gamma\gamma$ (4) | $\pi^0\gamma\gamma$ (5) | All neutrals (6) |
|---|---|---|---|---|---|---|
| All modes ($m=1$) | 23.1±1.0 | 4.7±0.2 | 30.0±2.2 | 37.5±1.6 | 4.7±2.4 | 72.2±0.01 |
| All neutrals (2) | (32.0) | (6.5) | (41.6) <br> 32±9[b] <br> 39.2±4.2[c] <br> 43.9±2.4[d] | (52.0) <br> 57±9[b] <br> 48.6±3.6[c] <br> 53.5±1.8[d] | (6.5) <br> 1.6±4.7[f] <br> 2.6±1.9[d] <br> $12.2^{+5.2c}_{-4.4}$ <br> 11.±3 | (100) |
| $\pi^+\pi^-\pi^0$ (3) | (100) | (20.4) | (130) <br> +15[e] <br> 150 <br> −29 | (162) <br> 172±25[e] | (20.4) <br> 0[e] | (312) |
| $\gamma\gamma$ (4) | (61.6) | (12.5) | (80.0) | (100) | (12.5) | (187) |
| All charged modes (5) | (83.2) | (16.9) | (108) | (135) | (16.9) | (260) |

[a] The values in brackets are those calculated by the Berkeley report (15). Each number in column $l$, row $m$ is the value of the branching ratio $R_{l,m} = \Gamma(\eta \to \text{mode } l)/\Gamma(\eta \to \text{mode } m)$ multiplied by 100. The reference number is the same as that in the text. For instance, 100 $R_{l,m} = 100$ $R_{3,2} = 100\Gamma(\eta \to 3\pi^0)$ $\Gamma(\eta \to \text{all neutrals}) = (32 \pm 9)$ is the measurement by Strugalski et al (16).

[b] Strugalski et al (16).

[c] Cox, Fortney & Golson (17).

[d] Buttram, Kreisler & Mischke (18).

[e] Baglin et al (19).

[f] Schmitt et al (20).

The branching ratios of the $\eta$ have been measured with a variety of techniques: bubble chambers, spark chambers controlled by scintillation and (or) Cerenkov counters, and the pure counter technique.

The bubble chambers with $H_2$ and $D_2$ are less efficient when one measures the branching ratios among the different neutral decays, for all neutral decay modes of the eta result in a given number of $\gamma$ rays. The experiments with electronics were dedicated mostly to the study of neutral modes.

The theoretical analysis of these decay values has been developed in Section 6, following a discussion in Section 5 on the general properties of the $0^-$ nonet.

### 2.4 PION PRODUCTION OF THE $\eta$ AT LOW AND INTERMEDIATE ENERGIES. OTHER PRODUCTION CHANNELS

We report in this section the experimental results for the $\eta$ production published up to the beginning of 1971.

*Reaction* $\pi^-p \rightarrow \eta n$ *at low and intermediate energies.*—The reaction:

$$\pi^- p \rightarrow \eta n \qquad\qquad 4.1$$

is the most thoroughly explored. The $\eta$s are produced in this case in a pure $T = \frac{1}{2}$ two-body final state.

The first results came from Bulos et al (21). These authors measured the cross-section and angular distribution for process 4.1; the eta was detected through decay into $2\gamma$ rays.

The cross section $\sigma$ was found to rise rapidly above threshold, to reach a maximum of $(0.98 \pm 0.08)$ millibarn at 659 MeV, and then to decrease slowly. These results are reported in Figure 3 as a function of the kinetic energy $T_{\pi^-}$ of the $\pi^-$.

As a result of their analysis the authors found that the angular distributions $d\sigma/d\cos\Theta_\eta^*$ are isotropic up to 950 MeV of the $\pi^-$. The 1003 MeV and 1151 MeV distributions require at least a $\cos\Theta_\eta^*$ term. The isotropic behavior is consistent with $S_{\frac{1}{2}}$ or $P_{\frac{1}{2}}$ production at threshold, with some higher states entering at higher energies.

The same energy region of reaction 4.1 also has been explored by other experimental groups. Richards et al (22) have measured reaction 4.1 at seven different pion energies from thresholds to 1300 MeV. These results also are plotted in Figure 3. The angular distribution of Richards et al shows departure from isotropy somewhat earlier than in the previous case.

When writing the usual expression

$$\frac{d\sigma}{d\cos\theta_\eta^*} = \sum_{l=0}^{l_{max}} A_l(\cos\theta_\eta^*)^l$$

the analysis of the authors (22) includes coefficient $A_4$; coefficients $A_1$, $A_2$, $A_3$ are already introduced beyond $T_{\pi^-} = 650$ MeV.
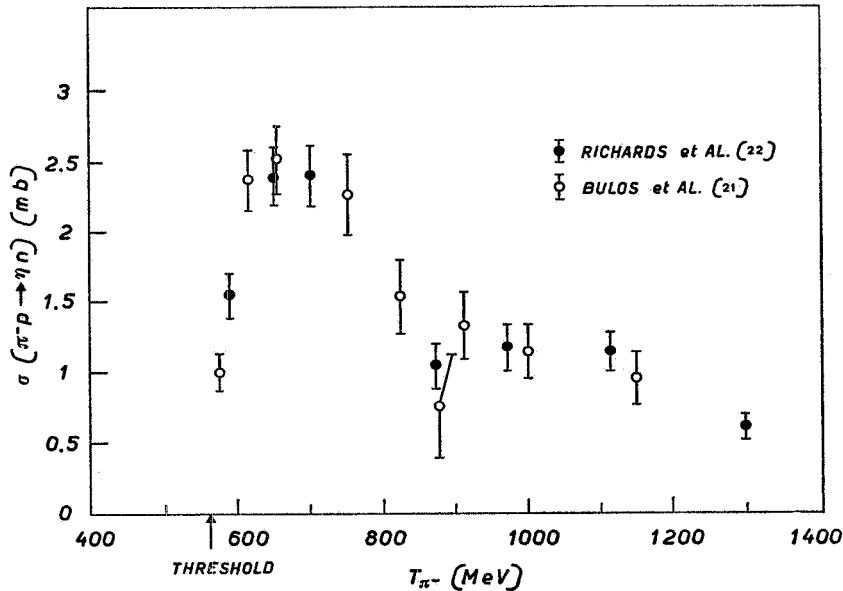
FIGURE 3. Cross section for $\eta$ production in reaction $\pi^- p \to \eta n$, all decay modes. $T_{\pi^-}$ is the $\pi^-$ kinetic energy in the laboratory system. Results of Bulos et al (21) ($\bigcirc$) and results of Richards et al (22) ($\bullet$).

The angular distributions for $\eta$ production in reaction $\pi^- p \to \eta n$ have been measured also by Buniatov et al (23) at $\pi^-$ beam momenta from 718 to 1050 MeV/c. This experiment is a part of a general study of the reactions

$$\pi^- + p \to \text{neutron} + \text{neutrals} \qquad 4.2$$
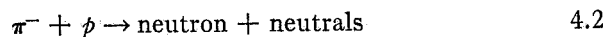
and the results agree with those already reported.

Jones et al (24) have carefully studied the behavior of the $\eta$ production in reaction 4.1 near threshold. In this experiment the negative pions were momentum-analyzed into narrow bands by two scintillation counter hodo-scopes. By measuring the time of flight of the neutrons, one can identify reaction 4.1 and verify that it is isotropic near threshold. The cross section is given in Figure 4 as a function of the c.m. momentum $P_\eta^*$.

The linear dependence of $\sigma$ on $P_\eta^*$ is strong evidence for $S$-wave dominance near threshold. The interest of these results lies in their remarkable precision near threshold.

*Channels $\pi^+ d$ and $\pi^+ p$.*—Kraemer et al (8) studied the production and decay properties of $\eta$, $\omega$, $\rho$, and $\pi^0$ mesons produced in $\pi + d$ collision at 1.23 GeV/c with the Berkeley 72-inch bubble chamber. The experiment studied the channel [which led to the discovery of the $\eta$ particle (3)]:
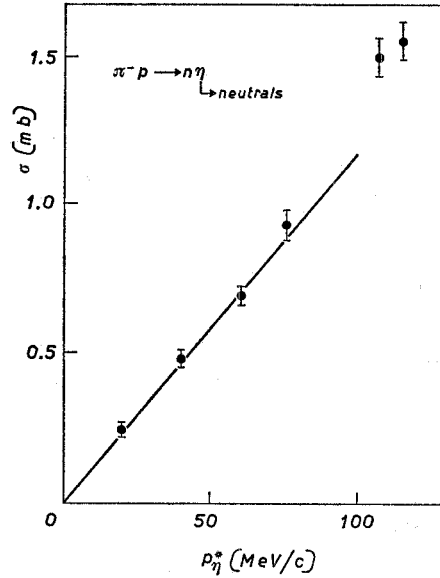
FIGURE 4. Results of Jones et al (24) for reaction $\pi^- p \rightarrow \eta n$ near threshold. The cross section for $\eta$ production is given as a function of c.m. momentum $p_\eta^*$, and shows the evidence of $S$-wave dominance near threshold.

$$\pi^+ + n \rightarrow \eta^- + p \qquad\qquad 4.3$$

The center-of-mass angular distribution is close to that observed in channel 4.1 and reported in the previous section. This agreement is consistent with the predictions of isotopic-spin conservation. Both reactions are pure $I = \frac{1}{2}$ states.

Reactions $\pi^\pm + p \rightarrow \pi^\pm + \eta + p$ have been studied by a few authors (25) since 1962. These results can help to fix the frequency of production of the $\eta$ at intermediate energies of the $\pi^+$ with respect to other channels: $\rho$, $\omega$, and multipion production.

The general feature of the interactions $\pi^+ p$ at these energies is the dominance of pionic resonant states, and this is, in fact, the relevant point of Alff et al (25).

According to Ascoli et al (25), the $\pi^- p$ interaction has given evidence of the chain $\pi^- p \rightarrow A_2^- p \rightarrow \pi^- \eta p$.

*Other production channels.*—The $\eta$ particle has been observed in proton-antiproton $(p\bar{p})$ annihilation. Foster et al (26) analyzed reaction $p\bar{p} \rightarrow 2\pi^+ 2\pi^- \pi^0$, and found a rate $(p\bar{p} \rightarrow \eta \pi^+ \pi^-)/(p\bar{p}$ all modes with 5 pions in the final state) $= (1.5 \pm 0.2)\%$. The $\eta \pi \pi$ system was a twobody $\rho \eta$ system in 50% of the cases.

## 2.5 PHOTOPRODUCTION OF THE $\eta$

Photoproduction of the $\eta$ particle in the process

$$\gamma + p \rightarrow p + \eta \qquad\qquad 5.1$$

has been observed (See Section 2.2) by Mencuccini et al (12) and by Bacci et al (27) since 1963, using the 1100 MeV Frascati electron synchrotron.

The authors evaluated the differential production cross-section $d\sigma/d\Omega^*$ for process 5.1 at an angle $100° <\Theta_\eta^* <120°$. The quantity directly measured was $(d\sigma/d\Omega^*)(\Gamma_{\gamma\gamma}/\Gamma_{total})$. The results are given in Figure 5 together with other experimental results. Notice again the fast decrease of the cross section after the maximum.

The photoproduction 5.1 in the region from threshold to 900 MeV has been measured by Prepost et al (28), using counter techniques with the Stanford Mark III 1.1 GeV electron linear accelerator. The eta production
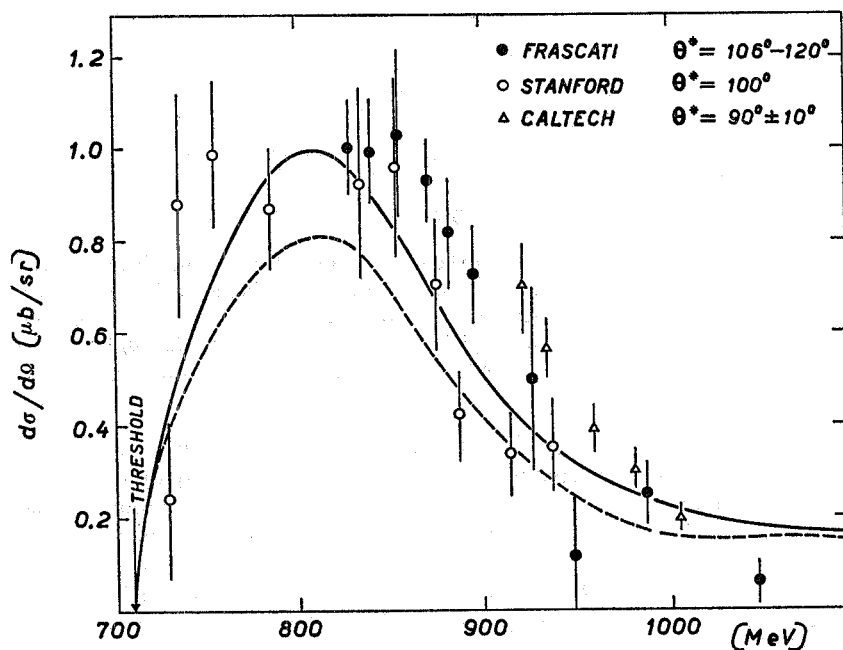


FIGURE 5. Differential cross sections (all modes) for the photoproduction process $\gamma p \rightarrow \eta p$. The experimental points are over the indicated range of c.m. $\theta_\eta^*$ angles. ($\bullet$) Frascati (27), $\theta_\eta^* = 106° - 120°$; (O) Stanford (28), $\theta_\eta^* = 100°$; ($\triangle$) Caltech (30), $\theta_\eta^* = 90° \pm 10°$. The lines (————— $\theta_\eta^* = 90°$; ————— $\theta_\eta^* = 45°$) correspond to the assumption of an $S_{11}$ (1570 MeV) and an $F_{15}$ (1688 MeV) resonance for the $\eta$-nucleon system. See Deans & Holladay (35).
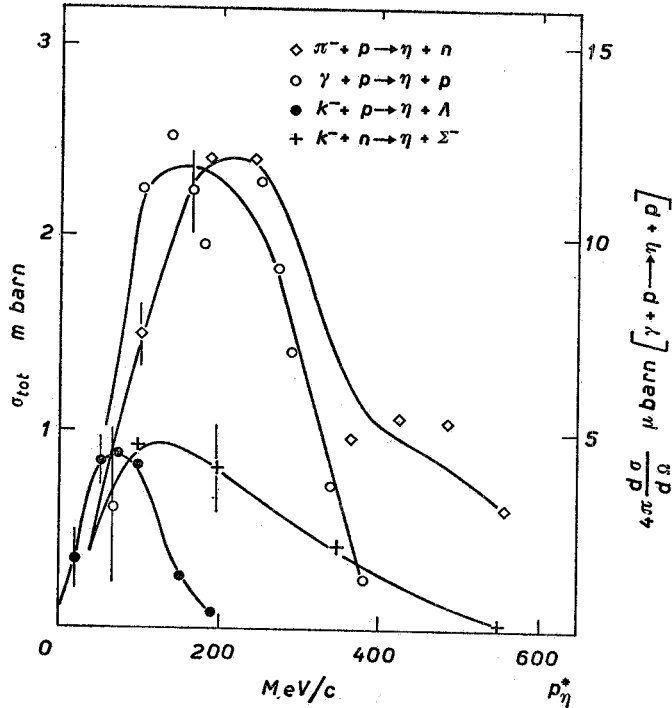
FIGURE 6. A comparison between the cross sections for $\eta$ production, all decay modes. In abscissa is $p_\eta{}^*$, the $\eta$ c.m. momentum. On the ordinates, at left, total cross section in millibarns, for processes: ($\Diamond$)$\pi^-p{\to}\eta n$ (21, 22); ($\bullet$) $K^-p{\to}\eta\Lambda^0$ (40); (+) $K^-n{\to}\eta\Sigma^-$ (42). At right, the differential cross section $\gamma p{\to}\eta p$ (27, 28, 30) for photoproduction around $\theta^*=90°$ c.m. multiplied by $4\pi$. Solid lines drawn by hand as a guide.

was seen as a step in the proton yield at the appropriate threshold energy. The results are also reported in Figure 5. In order to give a synoptic view of the situation, we report the results of the photoproduction reaction 5.1 and of the $\pi^- - p$ reaction 4.1 as a function of the center-of-mass momentum of the produced $\eta$, in Figure 6.

The same authors (28) have measured also the behavior of the differential cross section for three different angles at $K = 790$ MeV for the incident-photon energy. Within the limits of their statistics there is an indication that the cross section is isotropic in the center of mass at this energy. These results are reported in Figure 7, together with the results of Bacci et al (29). These authors have recently measured the differential cross section of reaction 5.1 at the three energies $K = 775$, 800, 850 MeV of the incident photons for the different eta c.m. angle $\theta_\eta{}^*$. The purpose again was to investigate the presence of higher partial waves other than the dominant $\eta$-nucleon $S$-wave
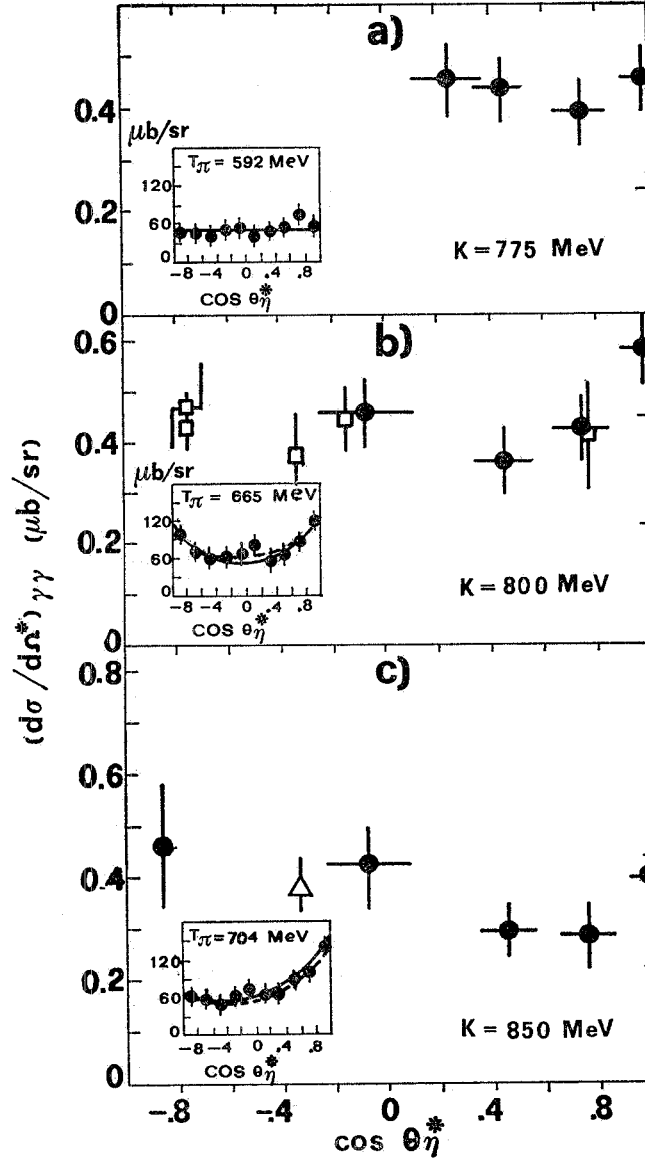
FIGURE 7. Differential c.m.s. cross section $d\sigma(\eta \to \gamma\gamma)/d\Omega^*$ as a function of $\theta_\eta^*$ (c.m. angle of the eta) at three different laboratory energies $K$ of the incident photon. (●) Results by Bacci et al (29); (□) results by Prepost et al at 790 MeV (28); (△) results by Bacci et al (27). The angular distribution for the reaction $\pi^- p \to \eta n$ of approximately the same c.m. total energy (22) are reported in the small frames. a) $K = 775$ MeV, $T_\pi = 592$ MeV; b) $K = 800$ MeV, $T_\pi = 665$ MeV; c) $K = 850$ MeV, $T_\pi = 704$ MeV.

resonance. The results are shown in Figure 7 a, b, c. They may be compared with the angular distributions for reaction $\pi^- + p \rightarrow \eta + n$ at approximately the same c.m. total energy. As one can see, up to $K = 850$ MeV the angular distributions are almost isotropic and consequently quite different from those in reaction 4.1.

It may be that in the pure isotopic spin $I = \frac{1}{2}$ photoproduction amplitude, a destructive interference between the isoscalar and the isovector part eliminates some higher partial waves. If this is the case, the presence of waves higher than $S_{11}$ could appear, as we shall see in the photoproduction on neutrons. In this respect the study of the reaction $\gamma + n \rightarrow \eta + n$ in deuterium is of high interest.

Reaction 5.1 has been measured by Bloom et al (30) at energies from 0.8 to 1.4 GeV and at different angles (50°, 70°, 90°) using the 1.5 electron synchrotron of the California Institute of Technology. These results are important in the study of $I = \frac{1}{2}$ nucleon isobars. In particular it is confirmed that the $F_{15}$ (1688) isobar does not show up appreciably in $\eta$ production.

Heusch et al (31) have measured recoil proton polarization in hydrogen at various energies from 0.8 to 1.1 GeV. They observe appreciable (0–50%) polarization values for the protons, indicating $S - P$ interference in the production process.

Process:

$$\gamma + n \rightarrow \eta + n \qquad\qquad 5.2$$

has been measured by a Frascati group (32) for different c.m. angles at $\kappa \simeq 850$ MeV. They find that the neutron and proton cross sections are quite similar and consistent with an isotropic c.m. angular distribution. The conclusion of Bacci and co-workers (32) was that at this energy either the isovector or the isoscalar part of the $\eta$ $S$-wave photoproduction amplitude, is small.

Anderson et al (32) have studied coherent photoproduction in deuterium of the $\eta$ meson, $\gamma + d \rightarrow \eta + d$. By comparing their results with those of Frascati, they conclude that the $\eta$ photoproduction near threshold is dominated by the isoscalar amplitude, up to $\sim 80$ MeV above threshold.

## 2.6 Theoretical Views on the $\eta$-Nucleon Final State (Reactions $\pi^- + p \rightarrow \eta + n$ and $\gamma + p \rightarrow \eta + p$)

The analysis of reaction 4.1 $\pi^- + p \rightarrow \eta + n$ can start from a paper by Hendry & Moorhouse (33) of 1965, which examines in a simplified way (two channels only) the possible explanation of the $\eta$ production in reaction 4.1 using the results of Bulos et al (21); already it seems to point to the right conclusions.

Considering, as we said, that the cross section rises linearly with the $\eta$ center-of-mass momentum, suggesting $S$-wave production, the cross section near threshold can be described by the formula

$$\sigma(\pi N \to \eta N) = 4\pi(k_2/k_1) \mid T_{12} \mid^2 \qquad\qquad 6.1$$

where $k_1$ is the momentum in the $j$th channel and $T_{ij}$ the $T$-matrix element for the process $i \to j$. In this case one takes $\pi + N$ as channel 1 and $\eta + N$ as channel 2. The fast decrease of the cross section requires a strong energy variation in the $T_{12}$-matrix element.

To stick to the two-channel hypothesis is an oversimplification which corresponds to the assumption that the $S_{11}$ pion-nucleon scattering below $\eta$ threshold is elastic, while above threshold its inelasticity is caused solely by the $\eta$ production. Nevertheless, the approach of Hendry & Moorhouse with only two channels can give the essentials of our problem. They found that all the best fits obtained manifest a resonance behavior, while nonresonant solutions had a much larger $\chi^2$.

This approximation does not yet allow the conclusion of the existence of a resonance, and to get better fits the authors (33) took into account a background from one or more channels. For this more detailed analysis, all best fits corresponded to resonant $S$-wave solutions for the $\eta$-nucleon system.

Other authors introduced in reaction 4.1 the necessary complications of other levels and channels. Minami (34) analyzes reaction 4.1 and photoproduction 5.1 assuming that the cross section rather near threshold can be explained in terms of strong interactions in $S_{11}$ and $D_{13}$ states. In his opinion, the analysis indicates that the effects of the $D_{13}$ resonance are comparable to or larger than those of $S_{11}$ even in the neighborhood of the observed peak of $\sigma(\pi^- + p \to \eta + n)$.

An analysis on similar lines has been done by Deans & Holladay (35), who have particularly taken into consideration the $\eta$ photoproduction experiments. In each combination the best values of the parameters have been obtained by minimizing $\chi^2$. The authors found excellent fits for certain combinations of poles and resonances. Among the more successful combinations is one including the states $S_{11}$ and $F_{15}$ reported in Figure 5.

It is relevant to observe that evidence is also obtained by Deans et al (35) for classifying the $F_{15}$ (1688) as a member of an octet rather than a 27-plet. This point had been demonstrated already in a paper by Heusch et al (36). More precise conclusions, not in disagreement with those of (34) and (35), have been obtained in a more recent analysis by Davies & Moorhouse (37).

The main point of this work is to review the present status of the possible $S_{11}$ and $D_{13}$ resonant states observed in the process $\pi^- p \to \eta n$. The conclusion again is that the present experiments sustain the $\eta N$ resonance hypothesis. Using all the recent results on reaction 4.1, an estimate of the branching ratios of these resonances into $\pi N$ and $\eta N$ also can be given, as we shall see now.

Davies & Moorhouse (37) approach the $\pi^- p \to \eta n$ reactions with an attitude similar to that of Deans & Holloday: a possible consequence of a large $D$-wave contribution at the peak of the $\eta$ cross section is that the $S$-wave is

**TABLE 4. Parameters of the resonances $S_{11}$ and $D_{13}$[a]**

|          | $S_{11}$(MeV) | $D_{13}$(MeV) |
|----------|---------------|---------------|
| $E_r$    | 1534          | 1530          |
| $\Gamma$ | 168           | 65            |
| $\Gamma_\pi$ | 72        | 26            |
| $\Gamma_\eta$ | 76       | 0.4           |

[a] Parameters resulting from the best fit of the differential and total cross sections of Bulos et al (21) and Richards et al (22). This fit has been obtained using the phase-shift analysis of Bareyre et al (39). See also (37).

not resonant. It is important, therefore, to make a careful search for solutions of this type. The relevant point again is that the authors found no acceptable solution without an $S$-wave $\eta N$ resonance: all their acceptable fits to experiment, with small or large $D$-wave, show this resonance. These conclusions agree with those obtained with photoproduction which we have already reported.

The analysis has been made by using the multichannel effective-range formalism (38), and in case the data cannot be fitted by two channels only, a third channel $\pi + N \rightarrow \sigma + N$ is added, $\sigma$ being a meson of mass chosen somewhat arbitrarily to be 390 MeV.

We report in Table 4 the results of one of the best fits obtained (37). It was found under the assumption of an $S_{11}$ and a $D_{13}$ resonance with all three channels open. The conclusion is, therefore, that the $S_{11}$ and $D_{13}$ resonances both couple (see the values of $\Gamma_\pi$, $\Gamma_\eta$) to the $\eta N$ channel and to the $\pi N$ channel; but in $D$ wave $\Gamma_\pi \gg \Gamma_\eta$; in $S$-wave $\Gamma_\pi \sim \Gamma_\eta$.

As a summary of this theoretical examination, we conclude that the analysis of the $\eta$ production by $\pi$s and $\gamma$s establishes the existence of a new $S_{11}$ resonance at the energy $E_r \sim 1520$ MeV with a good degree of confidence. This resonance decays into at least $\eta + N$ and $\pi + N$, but other modes ($\pi\pi N$?) cannot be excluded. The two main decay modes seem to be comparable. The well-known $D_{13}$ resonance seems to participate to a lower extent in the $\eta$ production.

### 2.7 PRODUCTION OF THE $\eta$ IN STRANGE CHANNELS: $K^- + p \rightarrow \Lambda^0 + \eta$<br>AND $K^- + n(p) \rightarrow \Sigma^-(\Sigma^0) + \eta$

Reaction

$$K^- + p \rightarrow \Lambda^0 + \eta \qquad\qquad 7.1$$

was discovered in the experiments of Bastien et al (6), which we have already reported. After this, Berley et al (40) measured reaction 7.1 at a number of $K^-$ momenta near the threshold. The resulting cross sections are plotted in Figure 8 as a function of the momentum of the $\eta$ in the $\Lambda^0\eta$ center-of-mass system. A sharp peak is observed near the threshold, and the decrease is also

fairly steep. The isotropic behavior, at least at low momenta, was confirmed also in this reaction.

More recent results by Fowler et al (41) on reaction 7.1 exhibit a similar behavior, but perhaps with an easier slope to the maximum. They are reported also in Figure 8.

All these data seem to favor the hypothesis that the $\eta$ particle is produced in an $S_{\frac{1}{2}}(J^P = \frac{1}{2}^-)$ resonant state.*

Let us look now to reaction

$$K^- + n \rightarrow \Sigma^- + \eta \qquad\qquad 7.2$$

This is a reaction with isospin $I = 1$ and strangeness $-1$.

Cline & Olsson (42) report experimental evidence for an enhancement in the $\Sigma^- + \eta$ cross section (sharp rise and a fast descent) that suggests a phenomenon similar to that observed in the $\Lambda^0 + \eta$ and $N + \eta$ channels.

The experiment has been performed using film from a bubble chamber filled with deuterium; the reaction studied was

$$K^- + d \rightarrow p_s + X + \Sigma^- \qquad\qquad 7.3$$
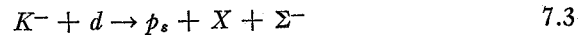
where $X$ is any neutral system ($\pi^0$, $\pi^0\pi^0$, $\eta$, $\cdot\,\cdot$) and $p_s$ is the spectator proton from the deuteron.

Reaction 7.3 has been studied also by the CERN-Saclay group (42). These results are plotted in Figure 9 as given by the authors. In the same Figure 9 are also plotted the results of the reaction

$$K^- + p \rightarrow \Sigma^0 + \eta \qquad\qquad 7.4$$

as reported by the CERN-Heidelberg-Saclay collaboration (43). Notice that reaction 7.4 is almost one-half of reaction 7.3, which can be foreseen by using simple isospin arguments.

The precision of the present experiments is not sufficient to allow a definite statement about a possible anisotropy in the angular distribution, but it has been established that an unambiguous bump exists in the cross section near threshold.

### 2.8 THEORETICAL VIEWS ON THE $\eta$-BARYON FINAL STATES $\eta\Lambda^0$ AND $\eta\Sigma$ ($S \neq 0$)

Several possible interpretations of the peak of reaction 7.1, $K^- + p$

* Note added in proof: Unpublished results presented at the Lund International Conference (1969) by a CERN (R. Armenteros et al)–Heidelberg (E. Burchardt et al)–CEN Saclay (R. Barloutaud et al) collaboration give for reaction 7.1 results very similar to those of Berley et al (Figure 8). These results are not incompatible with resonant state $S_{\frac{1}{2}}(J^P = \frac{1}{2}^-)$. However, they could suggest, as well as those of Berley, some structure of the cross section of process 7.1 when very near to the threshold. New results with higher statistics are needed to clarify this point. We are grateful to Dr. M. Ferro-Luzzi for the information.
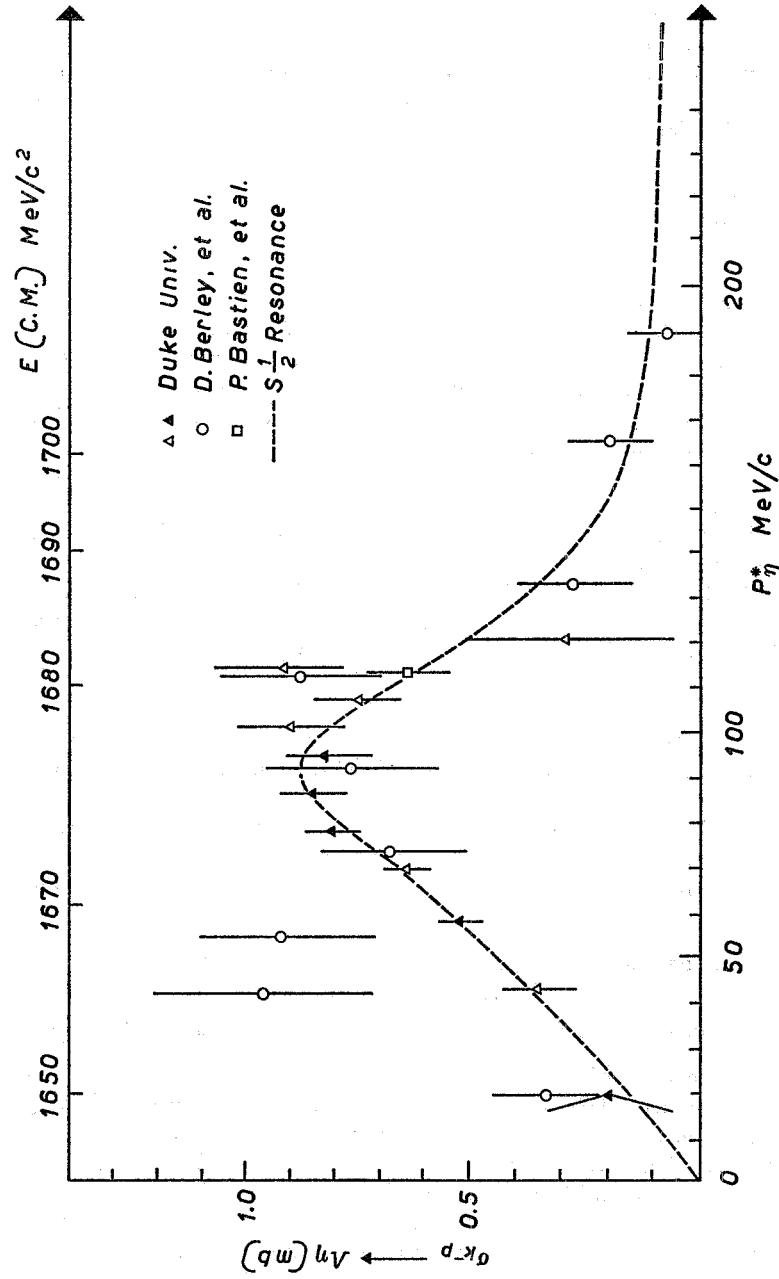
FIGURE 8. Total cross section (all modes) $K^- p \to \eta \Lambda$ as a function of the momentum of the $\eta$ in the c.m. Data from Duke University (41); Berley et al (40); Bastien et al (6). The dotted line as given by the authors from Duke University is a fit of the $S \frac{1}{2}$ resonance to their data.

$\rightarrow\Lambda^0+\eta$, have been considered since the results by Berley et al (40): an effect arising from a large scattering length in $S_{\frac{1}{2}}$ or in $P_{\frac{1}{2}}$ state, and an $S_{\frac{1}{2}}$ or a $P_{\frac{1}{2}}$ resonance. The scattering length in $S_{\frac{1}{2}}$ or $P_{\frac{1}{2}}$ did not give a good fit to the data; neither did a $P_{\frac{1}{2}}$ resonance.

An $I=0$, $S_{\frac{1}{2}}$ Breit-Wigner resonance formula, fitted to their own data by Fowler et al (41), is reported in Figure 8 and gives a rather good fit. So it seems to be a good working hypothesis to assume also in the $\Lambda^0\eta$ channel (as in the $\eta N$ channel) the existence of an $S_{\frac{1}{2}}$ resonance.

It is not possible yet to get a similar analysis for the reactions 7.3 or 7.4, which open the $\eta\Sigma$ channels. The analogy with what is known from the $N\eta$ and the $\Lambda^0\eta$ system near threshold makes the hypothesis of an $S$-wave resonance near threshold attractive also in this case.

A first analysis of the results (42, 43) has been made by Meyer (44), and some results are reported in Figure 9. The bump appearing in Figure 9 for the $\eta\Sigma$ cross section indicates that the results are compatible with an $I=1$ resonance having $E^*=1750$ MeV, $\Gamma=50$ MeV, $J^P=\frac{1}{2}^-$, in $S$ wave.

In these last years the $K^-p$ channel at energies of interest for the
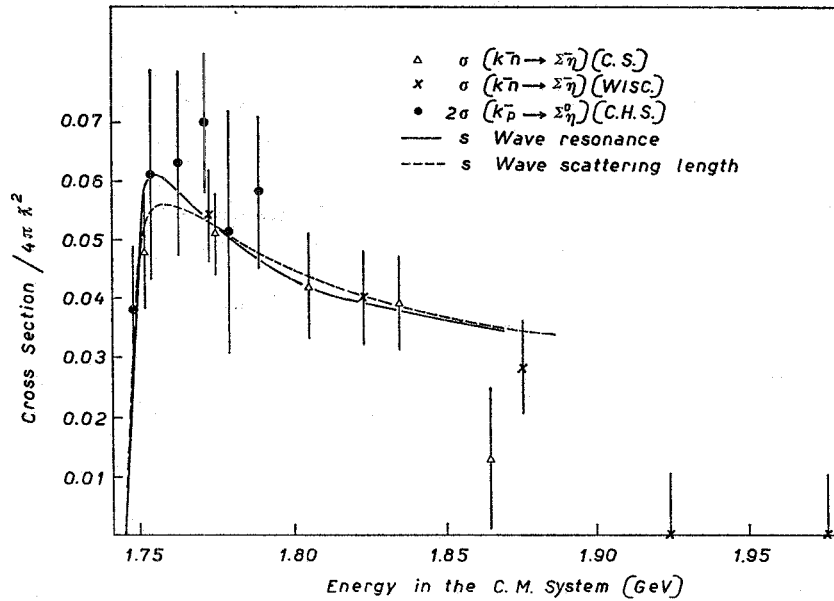


FIGURE 9. Total cross sections $/4\pi\lambda^2$ for reactions $K^-n\rightarrow\Sigma^-\eta$ and $K^-p\rightarrow\Sigma^0\eta$ near threshold. ($\Delta$) Results by Armenteros et al (46); (x) by Cline & Olsson (42); ($\bullet$) by C. H. S. collaboration (43). Notice that reaction $K^-p\rightarrow\Sigma^0\eta$ is almost one-half of reaction $K^-n\rightarrow\Sigma^-\eta$, in agreement with simple isospin arguments. The $\lambda$ is the $\hbar/p$ value of the $\Sigma\eta$ system in the center of mass.

$\eta$-Baryon resonances has been thoroughly investigated (45, 46), and without entering into the details of the analysis, we summarize the main conclusions:

(a) Reaction $K^-p \to \eta\Lambda^0$ has a well-established resonance with a maximum between 1665 and 1675 MeV (nominally 1670), with quantum numbers $I(J^P) = 0(\frac{1}{2}^-)$, the $S'_{01}$. This resonance decays into $N\overline{K}$ (20%); $\Lambda^0\eta$ (35%); $\Sigma\pi$ (45%). The total width of $S'_{01}$ is between 15 and 30 MeV.

(b) Reaction $K^-N \to \Sigma\eta$ probably goes through a resonance at an energy around 1750 MeV, this evidence coming also from other channels (44, 47), not including the $\eta$.

This resonance has quantum numbers $1(\frac{1}{2}^-)$, and we call it $S_{21}$. The width of $S_{21}$ is between 50 and 80 MeV; the resonance decays into $\overline{K}N$ in about 15% of the cases. It is difficult to estimate the branching ratio into $\Sigma\eta$ and $\Lambda^0\pi$.

In Table 5 we have summarized the situation for the $\eta$ resonances, strange and not, in view of a possible $\frac{1}{2}^-$ $\eta$-baryon octet.

### 2.9 Conclusions on the $\eta$-Baryon System. The $\frac{1}{2}^-$ Octet

By summarizing the properties of reactions

| | | |
|---|---|---|
| $\pi^-p \to \eta n$   4.1 | $\gamma p \to \eta p$   5.1 | $K^-p \to \eta\Lambda^0$   7.1 |
| $\pi^+n \to \eta p$   4.3 | $\gamma n \to \eta n$   5.2 | $K^-n \to \eta\Sigma^-$   7.2 |
| | | $K^-p \to \eta\Sigma^0$   7.4 |

**TABLE 5. Possible values for the $\eta$-baryon $J^P = \frac{1}{2}^-$ resonances ($\frac{1}{2}^-$ octet)[a]**

| $I(J^P)$ | Mass (MeV) | Total width (MeV) | Comments |
|---|---|---|---|
| $N^*(1535) = \frac{1}{2}(\frac{1}{2}^-)S_{11}'$ | 1535 | $100 \pm 50$ | The width is still uncertain for possible presence of $D_{3/2}$ wave. Decay modes: $N\pi$ (35%); $N\eta$ ($\sim$55%); $N\pi\pi$ probably small |
| $N^*(1670) = 0(\frac{1}{2}^-)S'_{01}$ | 1670 | $\sim$20 | Well established. Decay modes $\Lambda^0\pi$ (35%); $N\overline{K}$ (20); $\Sigma\pi$ (45) |
| $\Sigma(1750) = 1(\frac{1}{2}^-)S_{21}$ | 1750 | 50 | Very probable. Decay modes $N\overline{K}$; $\Lambda\pi$; $\Sigma\eta$ in uncertain fractions |
| $\Xi = \frac{1}{2}(\frac{1}{2}^-)$ | <1866 | | Unobserved. Should be a bound state. |

[a] We use the nomenclature of (15).

**TABLE 6. Angular dependence of the $X^0$ decay matrix element**[a]

| $J^P$ | $I=0$ | $I=1$ |
|---|---|---|
| $0^-$ | $1$ | $p^2q^2 \cos\theta$ |
| $1^+$ | $p^2$ | $q^2$ |
| $1^-$ | $p^4q^4 \sin^2\theta \cos^2\theta$ | $p^2q^2 \sin^2\theta$ |
| $2^+$ | $p^2q^4 \sin^2\theta$ | $p^4q^2 \sin^2\theta$ |
| $2^-$ | $\lvert a\rvert^2 p^4 + \lvert b\rvert^2 q^4 +$ $+3Re(ab^*)p^2q^2(\cos^2\theta - \frac{1}{3})$ | $p^2q^2(1+\frac{1}{3}\cos^2\theta)$ |

[a] See Dufey et al (78).

the following conclusions have been drawn:

(a) All experimental results indicate the existence of a $J^P = \frac{1}{2}^-$ ($S$-wave) resonance for the $\eta$-baryon system near threshold.

(b) The relative widths $\Gamma_\pi$ and $\Gamma_\eta$ for decay of these $\frac{1}{2}^-$ resonances into $\pi$+baryon and $\eta$+baryon are of comparable order.

(c) These $\frac{1}{2}^-$ resonances can form an $S_{\frac{1}{2}}$ parity minus ($\frac{1}{2}^-$) octet of baryon states (48).

Let us examine this last point. In the framework of their general approach by current algebra, Dashen & Gell-Mann (49) proposed to classify the hadrons states in representations of the group $SU_6 \otimes O_3$. From the analysis of the experimental situation, they suggested for the lower negative-parity baryon states the ($70^-$, $L=1$) representation, which contains, among the others, two $J^P = \frac{1}{2}^-$ octets. The same proposal has been advanced by Dalitz (50).

If we accept the existence of the $\frac{1}{2}^-$ octet, then the masses should satisfy the octet-type mass formula (48):

$$\tilde{N} + \tilde{\Xi} = \frac{1}{2}(3\tilde{\Lambda} + \tilde{\Sigma})$$

where by $\tilde{N}$, $\tilde{\Xi}$, etc. we indicate the total energy of the resonance $\eta$-nucleon, $\eta-\Xi$, etc.

Using (42) the resonance positions of our Table 6 for the known numbers $\tilde{N}$, $\tilde{\Lambda}$, $\tilde{\Sigma}$, one finds that $\tilde{\Xi}$ falls below the $\Xi\eta$ threshold ($m_\Xi + m_\eta = 1864$ MeV); in fact, $\tilde{\Xi} \sim 1845$ MeV. In such a case the $\tilde{\Xi}$ system should be a bound state: a fact that does not yet contradict the possibility of a meson-baryon octet.

An interesting analysis of the positions of the $\frac{1}{2}$-nucleon resonances in the nonrelativistic quark model has been done by Heusch et al (51). The analysis of the authors confirms or enlarges our conclusions. These authors remark that the resonance $S_{11}$ (1700), which may perhaps decay into $N\eta$ (15), may be attributed to the (8, 3/2) $\in 70(1^-)$ octet. The analysis of these results suggests that the two $\frac{1}{2}^-$ octets may mix, the mixing angle being $\sim 45°$ (52).

### 2.10 PHOTOPRODUCTION OF THE $\eta$ IN COMPLEX NUCLEI

Processes $\gamma + p(n) \rightarrow \eta + p(n)$ must also happen inside a nucleus. These

processes should be rather independent of the surrounding nuclear matter if the transferred momenta are high enough to avoid cooperative effects among the nucleons. When this condition is reached, the measurements of the production cross sections

$$\gamma + \text{nucleus} \to \text{nucleus} + \eta \qquad\qquad 10.1$$

could provide a unique method to evaluate the interaction properties of the $\eta$-nucleon system.

Reaction 10.1 has been studied at low energies ($\sim$1 GeV of the photons) by Bacci et al (53) at the Frascati electron synchrotron, by detecting the $\eta$ through $\eta \to 2\,\gamma$ decay. These authors find a large $\eta$-nucleon interaction cross section $\sigma$ at an $\eta$-nucleon c.m.s. momentum of $\sim$250 MeV/$c$, that is

$$\sigma \equiv \sigma_{sc} + \sigma_r \geq 65\,mb$$

where by $\sigma_r$ we mean the sum of all inelastic reaction cross sections.

Process 10.1, as well as the process $\gamma + \text{nucleus} \to \pi^0 + \text{nucleus}$, has been recently measured by Guisan et al (54) at 7.82 GeV/$c$, with different nuclei. These authors find that the total cross-section $\pi^0$ nucleon is in good agreement with the value deduced from $\pi \pm p$ interaction and isospin conservation, and that the total $\eta$-nucleon cross section is equal to the total $\pi^0$-nucleon cross section, within 20% error.

## 3. TEST OF $C$ INVARIANCE IN $\eta$ DECAYS

### 3.1 $C$ Invariance in $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-\gamma$ Decays

By the study of the asymmetry between $\pi^+$ and $\pi^-$ in the decays $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to \pi^+\pi^-\gamma$, one can verify whether the electromagnetic interactions among hadrons proceed with the conservation of charge conjugation. This possibility was suggested by Bernstein et al (55).

Immediately after this suggestion an effect of nearly two standard deviations was seen in a compilation work (56). The result was $A = (N^+ - N^-)/(N^+ + N^-) = 0.058 \pm 0.034$, where $N^+(N^-)$ is the number of events, with the energy of the $\pi^-$ greater than that of the $\pi^+$. Another result in the same direction was found by a Columbia-Stony Brook collaboration (57). After correction for the background these authors found $A = 0.072 \pm 0.028$.

A bubble chamber experiment similar to the one of the Columbia-Stony Brook collaboration, and performed by a Rutherford Laboratory-Saclay Group (58), reported the result $A = -0.06 \pm 0.04$.

Fowler et al (59) reported the value $A = 0.041 \pm 0.041$. The first result of great statistical significance was from a counter-spark chamber experiment performed at CERN in Geneva (60, 61) which gave $A = 0.003 \pm 0.01$.

In the CERN experiment the $\eta$s are produced in the reaction $\pi^\mp p \to \eta + n$. The incident $\pi^-$ beam has a momentum of 713 MeV/$c$ with a spread of $\pm 1\%$ and typical intensity of $10^5$ particles/burst. This result indicates no violation of $C$ invariance in $\eta$ decay within $\sim 1\%$.

When comparing all the results on the value of the asymmetry parameter $A$, one can notice that most of the experimental values of $A$ are positive.

As we said, another way to test $C$ conservation in the electromagnetic interactions of the hadrons is the investigation of $\eta \to \pi^+\pi^-\gamma$. The result of the same CERN group for the $\pi^+\pi^-\gamma$ decay is (62, 63) $A = 0.015 \pm 0.025$, a result which is again compatible with zero asymmetry.

More recently, a Brookhaven-Columbia collaboration has done a new experiment (64) based on 6710 events of $\eta \to \pi^+\pi^-\gamma$ and 36,800 events of $\eta \to \pi^+\pi^-\pi^0$. Eta mesons were produced in the reaction $\pi^- p \to \eta n$. The $\pi^-$ beam had a momentum of 720 MeV/$c$ and the neutron momentum was measured by time of flight; the momenta of the charged pions from the eta were measured by a set of sonic spark chambers in a magnetic field.

For the decay $\eta \to \pi^+\pi^-\gamma$ the authors (64) found $A = 0.024 \pm 0.014$. For the decay $\eta \to \pi^+\pi^-\pi^0$ they found $A = 0.015 \pm 0.005$.

These authors make a best fit of the Dalitz plot relative to their $\eta \to \pi^+\pi^-\pi^0$ decays using a squared matrix element $|M(xy)|^2 = 1 + 2ay + a^2y^2 + 2bx + cxy + \cdots$ corresponding to first order for mixed $I = 1$ and $I = 2$ final states. [The $x$, $y$ coordinates on the D.F. plot are defined as $x = 3(T^+ - T^-)/Q$; $y = (3T^0/Q) - 1$, where $Q = M_\eta - (M_{\pi^+} + M_{\pi^-} + M_{\pi^0})$ and $T^{0,+,-}$ are the kinetic energies of the $\pi$s.]

They obtain $b = 0.025 \pm 0.008$ and $c = -0.015 \pm 0.004$ with $\chi^2$ of 72.5 for 76 degrees of freedom.

These data, as well as those relative to $\pi^+\pi^-\gamma$ decay, are suggestive of $C$ violation. If there is a $C$ violation, it is likely to be in the $I = 2$ (even) final state of the three pions.

We know that other measurements on this fundamental problem are being conducted. However, Yuta & Okubo (65) cast some doubt on the convenience of pushing to the highest statistics the experimental study of the asymmetry $\eta \to \pi^+\pi^-\pi^0$ in order to verify $C$ invariance. In fact, these authors find that interference with the $3\pi$ background may give a nonzero charge asymmetry up to about 2%. Gormley et al (66) do not agree with this conclusion, estimating that only $\leq 0.23\%$ asymmetry could be obtained through this mechanism.

### 3.2 Rare Decay Modes of the $\eta$ Particle

We recall some decay modes which are lower than the common decays of the eta by a factor $\geq 10^2$.

*Process* $\eta \to \pi^0 e^+ e^-$ (67), as observed by Bernstein et al (51): In case $C$ is conserved, this decay can proceed via two virtual $\gamma$ rays through $\eta \to \pi^0\gamma\gamma \to e^+e^-\pi^0$. In this case, $\Gamma(\eta \to \pi^0 e^+ e^-) \sim \alpha^2\Gamma(\eta \to \gamma\gamma)$. If $C$ is not conserved, $\eta \to \pi^0 e^+ e^-$ can proceed through one virtual $\gamma$ ray, via $\eta \to \pi^0\gamma \to \pi^0 e^+ e^-$, and therefore it could be $\Gamma(\eta \to \pi^0 e^+ e^-) \sim \Gamma(\eta \to \gamma\gamma)$. The best upper limit for this $\pi^0 e^+ e^-$ decay has been given by Billing et al (67): $r = \Gamma(\eta \to \pi^0 e^+ e^-)/\Gamma$ (all modes) $< 3.7 \times 10^{-4}$ (90% c.l.). By combining all the experimental results, we get the limit $r < 2 \cdot 10^{-4}$ (90% c.l.).

*Process $\eta \to \mu^+ \mu^-$*: Callan et al (68) have set a lower bound on the rate for this process by comparing it to the process $\eta \to \gamma\gamma$. They suppose that parity conservation holds for the strong and e.m. interactions. Then, assuming CPT invariance, they derive a unitarity relation for the imaginary part of the $\eta \to \mu^+ + \mu^-$ amplitude, and find $\Gamma(\eta \to \mu^+ \mu^-)/\Gamma(\eta \to 2\gamma) \geq 1.1 \times 10^{-5}$. It would be of considerable interest, according to the authors, to verify if this bound is violated or greatly exceeded.

The decay $\eta \to \mu^+ \mu^-$ has been observed by Hyams et al (69) in the reaction

$$\begin{aligned} \pi^- + p &\to \eta + n \\ &\phantom{\to \eta} \llcorner\!\!\to \mu^+ + \mu^- \end{aligned} \qquad\qquad 2.1$$

Their result is

$$B_\gamma = \frac{\eta \to \mu^+ + \mu^-}{\eta \to \gamma + \gamma} = (5.9 \pm 2.2)10^{-5} \qquad\qquad 2.2$$

Wehmann et al (70) gave an upper limit:

$$B = \frac{\eta \to \mu^+ + \mu^-}{\eta \to \text{all modes}} \leq 2 \cdot 10^{-5} \qquad\qquad 2.3$$

which is consistent with the previous value. Using a vector dominance model for the $\eta \to \mu^+ \mu^-$ decay, Quigg & Jackson (71) estimated $B_\gamma$ to be $1.13 \cdot 10^{-5}$, $1.17 \cdot 10^{-5}$, $1.29 \cdot 10^{-5}$ on the assumption that only $\rho$, $\omega$, $\phi$ contribute, respectively. Choosing proper $SU_3$ couplings, they find $B_\gamma = 1.08 \cdot 10^{-5}$.

It seems that the experimental value of $B_\gamma$ is somewhat higher than these theoretical expectations, unless one treats the contributions of $\rho$, $\omega$, $\phi$ as free parameters.

*Process $\eta \to \pi^+ \pi^- \pi^0 \gamma$*: Search for this decay mode has been done by Price et al (72), who fixed the experimental limits $(\pi^+ \pi^- \pi^0 \gamma)/(\pi^+ \pi^- \pi^0) < 0.9\%$; $(\pi^+ \pi^- \gamma\gamma)/(\pi^+ \pi^- \pi^0) < 0.9\%$. A prediction of Singer (73) gives $(\pi^+ \pi^- \pi^0 \gamma)/(\pi^0 \gamma\gamma) \sim 0.2\%$.

As a conclusion to this Section 3 we want to remark that the theoretical predictions considered here are rather soft in nature and basically model dependent. It is therefore difficult to give to the experimentalists quantitative limits beyond which the verification of $C$ conservation becomes really significant.

## 4. THE $\eta'$ PARTICLE

We go now to the other pseudoscalar mesons which could have $I^G$, $J^P$ = $0^+$, $0^-$ quantum numbers and therefore may be good candidates to be the singlet member of the $0^-$ nonet in the unitary symmetries. At present these candidates are the so-called $X^0$ (958) and $E$ (1420) mesons (15).

### 4.1 THE $X^0$ MESON

The first evidence of an unstable meson with a mass value around $\sim 1$

GeV came from the study of the missing-mass distribution (74) in the reaction

$$K^- + p \rightarrow \Lambda^0 + MM \qquad\qquad 1.1$$

A confirmation of this resonance came very soon (75, 76) in the reactions

$$K^- + p \rightarrow \Lambda^0 + \pi^+ + \pi^- + MM \qquad\qquad 1.2$$

$$K^- + p \rightarrow \Lambda^0 + 2\pi^+ + 2\pi^- + MM \qquad\qquad 1.3$$

The results of reaction 1.2 indicates a decay process of the type:

$$K^- p \rightarrow \Lambda^0 X^0 \quad \text{with} \qquad\qquad 1.4$$

$$X^0 \rightarrow \eta\pi^+\pi^-$$
$$\qquad \rightarrow \pi^+\pi^-\pi^0 \qquad\qquad 1.5$$

By analysis of the results of reaction 1.2, the authors established the values

$$M_{\chi^0} = (957 \pm 1) \text{ MeV} \quad \text{and} \quad \Gamma_{\chi^0} \leq 4 \text{ MeV}$$

In spite of the small width, the authors (76) did not believe that decay 1.5, $\chi^0 \rightarrow \eta\pi^+\pi^-$ was e.m. in nature and not strong. They assumed $G(\chi^0) = +1$ because a value $G = -1$ would have allowed a more abundant $3\pi$ decay, which was not the case.

An unambiguous determination of the correct $I$-spin assignment was carried out by searching for events

$$K^- d \rightarrow X^- + \Lambda^0 + p_{\text{sp}} \qquad\qquad 1.6$$

For $I = 1$ the cross sections of process 1.6 should be about twice that of reaction 1.4. Barbaro-Galtieri et al (77), combining all the data, were able to exclude the value $I = 1$.

The assignment of the quantum numbers $J^P$ of the $X^0$ is mainly founded on the D.F. plot associated to the decay 1.5:

$$X^0 \rightarrow \eta + \pi^+ + \pi^-$$

A detailed analysis is done by Dufey et al (78). In this case the reaction

$$\pi^- + p \rightarrow X^0 + n, \qquad X^0 \rightarrow \eta\pi\pi, \qquad \eta \rightarrow \text{neutrals} \qquad 1.7$$

was studied by a system of spark chambers.

The distribution of the missing mass to the $(n\pi^+\pi^-)$ system has shown a clear peak in correspondence of the $\eta$ mass.

In Figure 10 the angular dependence of the squared matrix element of the decay is shown, as expected on the basis of the theoretical predictions reported in Table 6. This table is taken from (78). In this table, **p** is the $\eta$
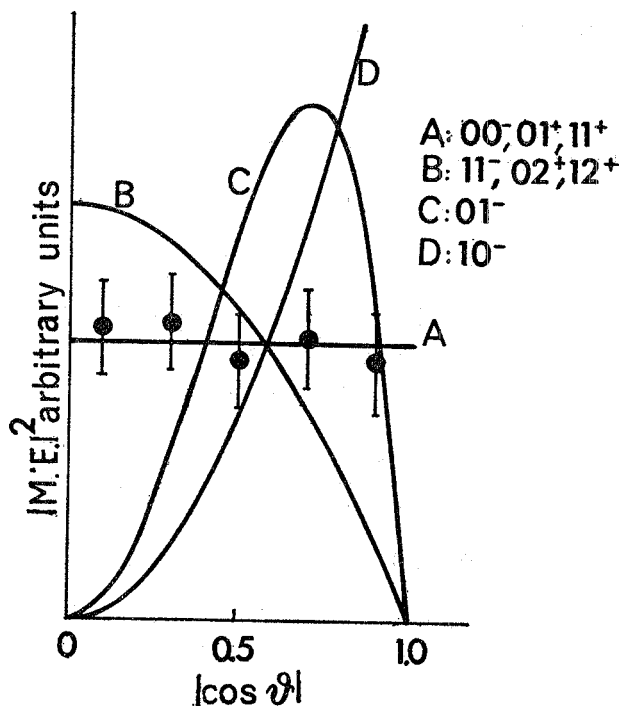
FIGURE 10. Angular dependence of the simplest square matrix elements compared with the experimental points. The different curves correspond to different $IJ^P$ quantum numbers for the $X^0$ meson (78).

momentum in the $X^0$ c.m.s.; q is the $\pi^+$ momentum, and $\theta$ is the angle between p and q in the dipion rest frame.

As one can see in Figure 10, the only assignments which are compatible with the experimental results are $IJ^P = 00^-$, $01^+$, $11^+$; the assignments $02^-$ and $12^-$ cannot be completely ruled out because of their small angular dependence. They are not reported in Figure 10.

A further selection can be done by studying the distribution of the $X^0$ events as a function of the $\eta$ kinetic energy (see Figure 11). At the end one is left with the possible assignments
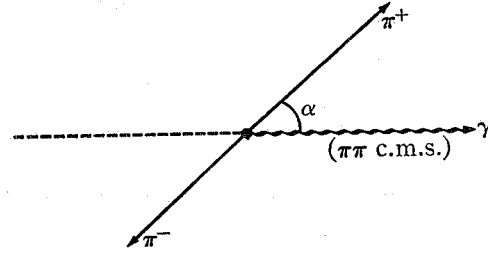
$$I^G J^P = 0^+0^-, \ 0^+2^-, \ 1^+1^+ \qquad\qquad 1.8$$

The $X^0$ meson has an e.m. decay

$$X^0 \rightarrow \pi^+\pi^-\gamma \qquad\qquad 1.9$$

and, according to a proposal by Kalbfleisch et al (79), one can try an assignment of the spin-parity of the $X^0$ meson by measuring the correlation be-

tween the relative pion momentum and the direction of the photon in the dipion rest frame.



In Table 7 we report (79, 80) the predictions in a first approximation for the $\alpha$ angular distribution. The experimental angular distribution (81) seems to follow the $\sin^2 \alpha$ behavior, and therefore to favor a $J^P = 0^-$ assignment; notwithstanding, the state $2^-$, which has a $(6 + \sin^2 \alpha)$ distribution, cannot be definitely ruled out.

The decay

$$X^0 \rightarrow \gamma\gamma \qquad\qquad 1.10$$

has been observed in a few experiments by Bollini et al (82) and by Bensinger et al (83). The most recent value of the first authors (84) for the branching ratio $B = \Gamma(X^0 \rightarrow \gamma\gamma)/\Gamma(X^0 \rightarrow \text{all modes})$ is $B \sim 2\%$.
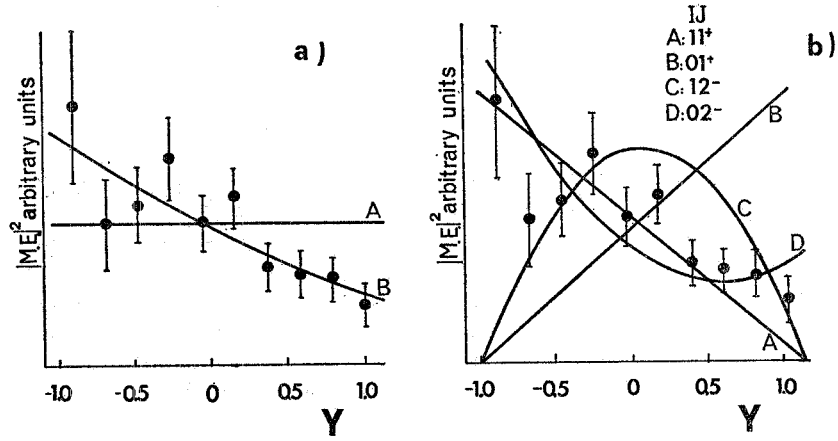


FIGURE 11. $T_\eta$ dependence in $X^0$ decay: a) $0^+0^-$ uniform (A) and linear (B) squared matrix elements; b) simplest squared matrix elements for the other assignments considered for the $X^0$ meson (78).

$$Y = \left(\frac{m_\eta + 2m_\pi}{m_\pi}\right) \frac{T_\eta}{Q} - 1$$

**TABLE 7. Distribution of the angle $\alpha$** [a]

| $I^G J^P$ | $\alpha$ dependence |
|-----------|---------------------|
| $0^+ 0^-$ | $\sin^2 \alpha$ |
| $0^+ 2^-$ | $6 + \sin^2 \alpha$ |
| $1^+ 1^+$ | $1 + \cos^2 \alpha$ |

[a] See sketch on page 29 for the $X^0 \rightarrow \pi^+ \pi^- \gamma$ decay. Also see (79) and (80).

In conclusion, the most probable attributions for the $X^0$ are:

$$I^G, J^P = 0^+, 0^- \qquad\qquad 1.11$$

but still the $0^+$, $2^-$ assignment cannot be definitely excluded.

## 4.2 THE $E$ MESON

The first evidence of the $E$ meson was given by Armenteros et al (85). By studying $\bar{p}p$ annihilation at rest, a bump in the $K\bar{K}\pi$ system was found in the process

$$\bar{p}p \rightarrow K_1^0 K^{\pm} \pi^{\mp} \pi^+ \pi^- \qquad\qquad 2.1$$

This bump corresponded to a mass of $\sim$1420 MeV with a width of $\sim$70 MeV.

A more detailed analysis was given later by Baillon et al (86), who analyzed the results of $1.4 \times 10^6$ $\bar{p}p$ annihilations at rest. The authors found:

$$M_E = (1425 \pm 7) \text{ MeV}; \qquad \Gamma = (80 \pm 10) \text{ MeV}$$

The main decay modes were found to be

$$\begin{aligned} E &\rightarrow K^* \bar{K} \qquad \text{(and } \bar{K}^* K) \quad (\sim 50\%) \\ &\rightarrow (K\bar{K})\pi \qquad\qquad\qquad (\sim 50\%) \end{aligned} \qquad 2.2$$

$K^*$ (892) being the $IJ^P = \frac{1}{2}$ $1^-$ meson resonance which decays into $K\pi$. The $(K\bar{K})$ system forms a resonance with a mass $M \sim$1000 MeV.

Comparison of the decay rates with the theoretical predictions for different sets of quantum numbers allowed the authors to conclude that only the values $C = G = 1$, $I = 0$ reproduced the experimental results in a satisfactory way. A value $J^P = 0^-$ was favored but $J^P = 1^+$ was not completely excluded.

A confirmation of the $E$ meson has been given by many authors (with a few exceptions). French et al (87) find evidence of the $E$ in the reaction $\bar{p}p \rightarrow K\bar{K} + m\pi$ at a momentum of 3, 3.6, and 4 GeV/$c$ of the antiprotons.

As a matter of fact, in the 5 bodies final state $K_1^0 K^{\pm} \pi^{\mp} \pi^+ \pi^-$ the neutral $K_1^0 K^{\pm} \pi^{\mp}$ system shows a peak at all incident energies. By fitting the experimental distribution by phase space plus a Breit-Wigner curve, a mass is found again at $(1423 \pm 10)$ MeV with a width of $(45 \pm 20)$ MeV.

The authors also report the $K\bar{K}\pi$ mass distribution when the $K\pi$ system has a mass close to the $K^*$ mass: the $E$ peak is then enhanced.

Dahl et al (88) have looked for the $E$ meson in the reaction $\pi^- p \rightarrow n K \bar{K} \pi$

at 2.9 and 4.2 GeV/$c$. By giving the frequency of the $E$ events as a function of the $K\overline{K}$ effective mass and as a function of the pion angle in the $K\overline{K}$ c.m.s., these authors find a distribution which, apart from the difficulty of a large background, seems to favor $J^P = 1^+$ with respect to the $0^-$ and $2^-$ assignments.

No evidence of the $E$ meson has been found in $\overline{p}p$ annihilation at 1.2 GeV/$c$ by d'Andlau et al (89), while confirmation of the $E$ has been reported by Loerstad et al (90) at 0.7 GeV/$c$. The absence of a $(K\overline{K}\pi)$ charged system was a confirmation of an $I = 0$ assignment for the $E$. An analysis by D.F. plot for the $E$ has been performed by the same authors; the results favor the possibilities

$$J^P = 0^- \quad \text{or} \quad J^P = 1^+ \quad \text{with } I^G = 0^+$$

but again no clear distinction can be made between the two assignments.

The result $I = 0$ has been confirmed by Bettini et al (91) in the $\overline{p}p$ annihilation at rest. A search for the decay $E \to \eta\pi^+\pi^-$ was done by Foster et al (92), who fixed an upper limit for the branching ratio

$$\frac{E \to \eta\pi^+\pi^-}{E \to K\overline{K}\pi} < 1$$

### 4.3 Concluding Remarks

In concluding this section we underline at least two clearly needed measurements concerning the $\eta'$ particles, $X^0$ and (or) $E$. The first is to know the absolute width $\Gamma(X^0 \to \gamma\gamma)$ and the total width of the $X^0$. The second is to look for any possible e.m. decay of the $E$ meson, in particular the $E \to \gamma\gamma$ decay. This mode would be quite important for deciding between $0^-$ and $1^+$ for the $E$ meson.

We recall that both measurements could be achieved in the near future by the new technique of the $e^+e^-$ storage rings. In fact, it is possible to produce the $X^0$ and the $E$ (if it has spin 0) through $\gamma - \gamma$ annihilation, and the cross section for this process is rather obviously proportional to $\Gamma(2\gamma)$ (93). More precisely, in the $e^+e^-$ storage rings, like the storage ring in Frascati of $2 \times 1.5$ GeV, one can produce the reaction

$$e^+ + e^- \to e^+ + e^- + X^0 \text{ (or } E).$$

This reaction goes through annihilation of two $\gamma$ rays emitted in a sort of double bremsstrahlung, as follows
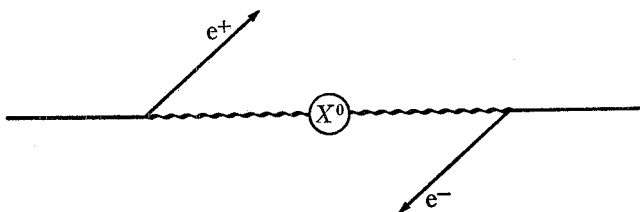
An estimate of the cross section in the case of the $X^0$ has been made and may be given in the form

$$\sigma_{X^0} = A \times 10^{-35} \times \Gamma_{\gamma\gamma} \text{ cm}^2$$

The absolute width $\Gamma_{\gamma\gamma}$ is given in keV. The value of $A$ is $\sim 2$ for a total energy of the electrons $E_{e^+} + E_{e^-} \sim 2.5$ GeV and increases with their energy. The present indications on $\Gamma_{\gamma\gamma}$ for the $X^0$ suggest that this value of $\sigma_{X^0}$ is within the present experimental possibilities of the $e^+e^-$ storage rings of a total energy beyond 2 GeV.

## 5. THE 0⁻ NONET

### 5.1 INTRODUCTION

Due to its quantum numbers, the $\eta$ particle appears as a natural candidate to fill up with $\pi$s and $K$s an octet of $J^P = 0^-$ mesons, as required by the unitary symmetry hypothesis proposed by Gell-Mann (94) and Ne'eman (95). In fact, the discovery of the eta was one of the first experimental evidences in favor of the idea that the strong interactions might exhibit an approximate symmetry higher than the well-known one leading to strangeness and isotopic spin conservation. This idea (96) is nowadays accepted as substantially correct, and a number of very important results have been derived by using it as a starting point (96).

The reader can easily find in the literature several exhaustive treatments of unitary symmetry (2, 97), but we shall limit ourselves here to a few relevant points.

In the exact symmetry limit the hadrons should appear in multiplets corresponding to irreducible representations of the group $SU_3$, all particles in a multiplet having the same $J^P$ and rest mass. Any such multiplet contains isospin submultiplets with a definite pattern of $T$, $Y$ values (remember that $Y = N_B + S$).

As far as the $T$, $Y$ quantum numbers are concerned, the $J^P = \frac{1}{2}^+$ particles can be assigned to a baryon octet and the $J^P = 3/2^+$ to a baryon decuplet. However, the large mass differences among isomultiplets are inconsistent with the prediction of the symmetrical theory.

When we try to build an octet with the 1⁻ meson states, we meet two $T = Y = 0$ states, the well-known $\omega$ and $\phi$ particles. We could assign one of them to an $SU_3$ singlet, and the remaining particle to an octet, still with the same difficulty of the mass differences. Clearly one should have a criterion, however, to decide whether the $\omega$ or the $\phi$ is the singlet.

A similar situation arises in the 0⁻ case, which is of course of particular interest to us. Here we have a possible triple occurrence of $T = Y = 0$ states ($\eta$, $X^0$ and $E$), should the 0⁻ spin parity assignments be confirmed for both $X^0$ and $E$ (see Section 4).

This leads us to the conclusion that (a) the $SU_3$ symmetry can hold only in an approximate way, and (b) the violation of it can be rather large. As a rough measure of the relative strength of the $SU_3$ violating forces to the

symmetric ones, we can assume the ratio of the mass differences between isomultiplets to the medium mass in the $\frac{1}{2}^+$ octet, $\Delta m/\bar{m} \simeq 0.1$.

Another problem is to find a justification of the fact that apparently only the singlet, octet, and decuplet, the so-called "nonexotic" $SU_3$ representations, are needed to explain the spectrum of the physical hadrons. The prediction of this fact is one of the successes of the quark model (98, 99). This model also indicates that, as a general rule, the bosons should appear as nonets, i.e. octet plus unitary singlet.

### 5.2 Mass Formulas and Mixing. The Nonet Structure of Bosons and the Quark Model

The most evident way in which the $SU_3$ violating interaction manifests itself is, as we said, in the mass differences among isomultiplets belonging to the same $SU_3$ representation. Remarkably enough, it is possible to describe such mass differences by the simple Gell-Mann/Okubo formula (94, 100). This can be derived under the assumption that the strong interaction Hamiltonian $H$ can be divided into two pieces, $H = H_0 + H_1$, where $H_0$ is invariant under $SU_3$, and $H_1$, representing the symmetry breaking interaction, has the same transformation properties under $SU_3$ as the hypercharge $Y$.

First-order perturbation treatment gives for the $\frac{1}{2}^+$ baryon octet the formula

$$m = m_0\{1 + aY + b[T(T+1) - \tfrac{1}{4}Y^2]\} \qquad 2.1$$

Eliminating the three unknown constants $m_0$, $a$, $b$, one gets

$$m_N + m_\Xi = \tfrac{3}{2}m_\Lambda + \tfrac{1}{2}m_\Sigma$$

which is satisfied by the experimental masses to better than 1%, with an accuracy which is rather unexpected in view of the fact that $H_1$ cannot be considered a "small" perturbation.

Similar peculiarities are found when studying other symmetry breaking effects, and the interesting theory of "octet enhancement" has been introduced to provide a unified explanation. (See Coleman 101.)

For an octet of bosons the term in 2.1 linear in $Y$ must vanish because of $C$ invariance. Application of 2.1 to the $\eta$, $\pi$, and $K$, assuming that they form an octet, gives

$$2m_K = \tfrac{1}{2}m_\pi + \tfrac{3}{2}m_\eta \qquad 2.2$$

$$(992 \text{ MeV}) \quad (891 \text{ MeV})$$

with far less accuracy than in the previous case. The agreement is better if we replace in formula 2.2 the masses by their squares. By inserting the experimental values of $m_K$ and $m_\pi$, one then can obtain a value which is within 3.5% equal to the experimental value $m_\eta = 549$ MeV. No conclusive arguments have been proposed in support of the quadratic mass formula or of the linear one. Detailed discussion may be found in several papers, e.g. (102).

For the vector mesons it is not possible to identify an octet among the

nine known particles $\rho$, $K^*$, $\omega$, $\phi$, in such a way that the physical masses, or their squares, satisfy the Gell-Mann & Okubo relation reasonably well.

This apparent failure was explained by Sakurai (103) and Glashow (104) by means of the theory of $\omega - \phi$ mixing. This rests on the simple observation that the symmetry breaking interaction can also connect isomultiplets with the same $T$, $Y$ values belonging to different $SU_3$ representations, in particular the two $T = Y = 0$ vector meson states.

In the limit of exact $SU_3$ we would have a degenerate octet whose $T = Y = 0$ member we call $\omega_8$, and a unitary singlet $\omega_1$. Writing the physical states $\omega$, $\phi$ as

$$\phi = \omega_8 \cos \theta + \omega_1 \sin \theta$$

$$\omega = - \omega_8 \sin \theta + \omega_1 \cos \theta$$

the value of the angle $\theta$ that diagonalizes the squared mass matrix referring to the $\omega_8$, $\omega_1$ states can be expressed as

$$\tan^2 \theta = \frac{m_\phi{}^2 - m_8{}^2}{m_8{}^2 - m_\omega{}^2}$$

where $m_8{}^2$ is the value given by the mass formula for the $T = Y = 0$ member of the octet.

Of course, with the introduction of the new parameter $\theta$, a test of the theory is no longer possible when using only the masses of the particles. This in particular implies that at this level a linear mass formula could be used as well. It turns out, however, that in the present case the value of the angle is practically insensitive to the use of one or the other mass formula.

Since we will later argue in favor of the idea that all bosons occur in nonets, we can apply the considerations made above to the $0^-$ mesons. We call $\eta'$ the ninth member, to be selected among $X^0$ and $E$ (Section 4), and define the pseudoscalar mixing angle $\alpha$ by

$$\eta = \eta_8 \cos \alpha - \eta_1 \sin \alpha$$

$$\eta' = \eta_8 \sin \alpha + \eta_1 \cos \alpha$$

As before

$$\tan^2 \alpha_{sq} = \frac{m_\eta{}^2 - m_8{}^2}{m_8{}^2 - m_{\eta'}{}^2}$$

or

$$\tan^2 \alpha_{lin} = \frac{m_\eta - m_8}{m_8 - m_{\eta'}}$$

When assuming the $X^0$ as the $\eta'$, the two angles are very different: $\alpha_{sq} \sim 10^0$ and $\alpha_{lin} \sim 23^0$. It follows that an independent measurement of $\alpha$ could serve to make a choice between the two formulas. Should instead $\eta' = E$, then $\alpha_{sq}$ and $\alpha_{lin}$ would have smaller values ($\sim 6^0$ and $\sim 13^0$).

Let us recall that $\eta - X^0$ mixing enters as an important ingredient in most of the treatments of the $\eta$ decays, and the results are usually sensitive to the use of a linear or a quadratic mass formula. In particular the $X^0$ decay modes, when better known, should offer a convenient test (105, 106).

The quarks, since their introduction by Gell-Mann (98) as a triplet of fundamental entities out of which all the strong interacting particles could be constructed, have been the starting point of a vast amount of original and interesting theoretical inquiry. For an up-to-date and comprehensive review of the field we refer the reader to Morpurgo (99) and to Dalitz (107).

Let us define quarks q to be a triplet of spin $\frac{1}{2}$ fermions belonging to the representation 3 of $SU_3$, and the antiquarks $\bar{q}$ to the 3* representation, which is, like 3, three-dimensional but not equivalent to it. With the same significance of the well-known formula $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$, we have

$$3 \otimes 3^* = 1 \oplus 8, \qquad 3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 \qquad 2.3$$

We see that composition of a quark with an antiquark gives the two $SU_3$ multiplets that appear in the boson spectrum and no others, and the composition of three quarks gives only the multiplets occurring in the fermion spectrum.

From now on let us focus our attention on the boson states made out of $q\bar{q}$. One finds (99) that these states always appear in nonets $3 \otimes 3^*$ as far as $SU_3$ properties are concerned. The $J^P$ values of these nonets are easily inferred from the known composition rules of spin and angular momentum. In this way one gets the nonets $J^{PC} = 0^{-+}$ and $1^{--}$ which can be identified with the pseudoscalar mesons and the vector mesons we have already considered.

It is easy to see that only one $0^-$ nonet can be obtained. Dalitz (107), however, suggested that the $q\bar{q}$ "molecule" could have radial excitations that would account for other $0^-$ nonets. In particular, were $X^0$ and $E$ both $0^-$ (Section 4), we could assign one of them to a first radial excitation of the usual $0^-$ nonet.

Assuming the $E$ to be the $T = Y = 0$ octet member of the new nonet, following Dalitz (107) one can estimate the mass $M$ of its $T = 1$ companion (which should correspond to the $\pi^0$ as the $E$ corresponds to the $\eta$) with the formula:

$$M^2 = m_E{}^2 - (m_\eta{}^2 - m_\pi{}^2) \simeq (1.3 \text{ GeV})^2.$$

No experimental evidence has been found either for or against this possibility. It should be noted, however, that the above estimate is much more model dependent than the idea of the radial excitations itself.

### 5.3 Theoretical Predictions Concerning the $\eta$, $\eta'$ Particles. Concluding Remarks

If we disregard the quark model, then at the $SU_3$ level there is essentially no problem; we could accommodate any unwanted $0^-$ particles in a singlet

or in a new octet, for example. In view of Section 5.2, however, this seems to us at the moment too drastic a solution. From the analysis of its decays, there are some indirect indications that the $\eta$ cannot be a pure octet state. We will see in Section 6 that mixing with the $X^0$ seems the cleanest way to explain the departure of $[\Gamma(\eta\rightarrow2\gamma)/\Gamma(\pi^0\rightarrow2\gamma)]_{\text{exp}}$ from the $SU_3$ prediction obtained with the hypothesis of no mixing.

Mixing with the $X^0$ gives substantial improvements in the calculation, via current algebra, of $\Gamma$ $(\eta\rightarrow\pi^+\pi^-\gamma)/\Gamma(\eta\rightarrow2\gamma)$ and also boosts in the right direction the low value of $\Gamma\eta\rightarrow3\pi^0$ obtained in the nonlinear Lagrangian approach (108). Note, however, that in none of these cases has the $\eta-X^0$ mixing assumption been confronted with the analogous $\eta-E$ possibility.

Direct predictions for the mass of the $\eta'$ particle are given by Pande (109) and Parisi et al (110). In both papers the structure of an assumed $0^-$ nonet is investigated in the framework of chiral $SU_3\otimes SU_3$ symmetry (111), broken in the way proposed by Gell-Mann et al (112). Apart from this common setting of general character, the techniques used in (109) and (110) are very different.

In (109) the mass of the $\eta'$ is predicted to be 958 MeV, which is coincident with the experimental value of the mass of the $X^0$. In (110) a relation is obtained between the mass of $\eta'$, $m_\eta$, and the mixing angle. The hypothesis $\eta'\equiv E$ gives $m_\eta=500$ MeV, whereas $\eta'\equiv X^0$ leads to $m_\eta=545$ MeV, which is a better prediction.

We can summarize our analysis as follows: there is strong evidence that the $\pi$, $K$, $\eta$ do not constitute a pure octet and that a new member $\eta'$ is needed, mixed with $\eta$ to form a $0^-$ nonet. Both direct and indirect indications exist that $\eta'\equiv X^0$, whereas $\eta'\equiv E$ is disfavored in at least one case.

Our analysis is also consistent with the simple quark model predictions. Even in the case that $E$ has $J^P=0^-$, the scheme might allow for a new $0^-$ nonet in which the $E$ meson could be placed. We already discussed in Section 4 the possible alternatives for the $J^P$ values for the $E$ and the $X^0$ mesons.

## 6. THEORY OF $\eta$ DECAY

### 6.1 INTRODUCTION

The variety and complexity of the decay modes of the $\eta$ particle (Section 2.3) has stimulated a large amount of theoretical work. This work has been dedicated in the overwhelming majority of cases to the five "main" modes $\eta\rightarrow2\gamma, \eta\rightarrow\pi^+\pi^-\gamma, \eta\rightarrow\pi^0\gamma\gamma, \eta\rightarrow3\pi^0, \eta\rightarrow\pi^+\pi^-\pi^0$.

None of these decays can occur via strong interactions alone, for obvious reasons, when photons are directly emitted; on the other hand, in the $3\pi$ modes $G$ parity is violated (Section 2). The commonly accepted interpretation is that in all of these processes e.m. interactions are taking part, as none of the invariance properties usually assumed for them are violated. If, however, we apply this hypothesis to the $3\pi$ decays, and we treat the e.m. interactions in the customary way, we get into serious trouble, as we will see in Section 6.3.

That things are not so easy can be illustrated here by the following naive argument. Both $\eta \rightarrow 2\gamma$ and $\eta \rightarrow 3\pi$ should be of order $\alpha$ in amplitude; however, we expect the $3\pi$ decay rate to be greatly depressed by unfavorable phase space. In fact, the two decays occur with comparable abundance. This example and others that we will discuss later indicate that the interplay between electromagnetic and strong interactions may have rather complicated features.

From this point of view, the $\eta$ phenomenology is a very useful source of information on the properties of the hadronic e.m. interactions themselves. As pointed out by Sutherland (113), the situation parallels that of the $K$ meson with respect to hadronic weak interactions. Similarities between $K$ and $\eta$ have been repeatedly noted: both are members of the same $SU_3$ multiplet in which they are the heaviest states with comparable rest mass (113); both have abundant $3\pi$ decays and exhibit nearly the same structure in the Dalitz plots for the charged modes (see, however, 114). This last point especially has prompted the application to $\eta \rightarrow 3\pi$ of methods applied successfully in the analogous $K \rightarrow 3\pi$.

We will discuss below the theoretical results, taking into account, for comparison, the data for the rates and branching ratios reported in Table 3 at the top of each column.

We should note that the experimental situation is not as clear as one might wish. Apart from the absolute width, the branching ratios among the neutral modes are still rather uncertain, depending on the presence or absence of a significant $\pi^0\gamma\gamma$ rate. We take the reported value $\Gamma(\pi^0\gamma\gamma)/\Gamma_{total}$ $=4.7\pm2.4\%$, in the light of the most recent experiments, as an indication that the $\pi$, $\gamma\gamma$ rate is very low, if not zero.

The following hypotheses are common to all the contributions with which we will have to deal: (a) the $\eta$ is a member of the pseudoscalar octet, with a small mixing with the $X^0$ (958); (b) there is no relevant $C$ violation in the $\eta$ decays (section 3).

## 6.2 The $\gamma\gamma$, $\pi^+\pi^-\gamma$, $\pi^0\gamma\gamma$ Modes

It is possible to relate these $\eta$ decays among themselves, as well as to the similar processes $\pi^0 \rightarrow 2\gamma$, $X^0 \rightarrow 2\gamma$, $X^0 \rightarrow \pi^+\pi^-\gamma$ and to the vector meson decays. A direct comparison with the $3\pi$ modes remains, however, beyond the possibility of present theoretical approaches. For this reason we analyze separately the two groups of decays, beginning with those directly involving photons.

*The vector meson dominance model.*—The vector meson dominance model (115), in the way originally applied by Gell-Mann, Sharp & Wagner (116), has been the guiding line in many papers on the $\eta$ decays. Its two essential features are the following:

(i) In the general decay process $A \rightarrow B + n\gamma$, where $A$ and $B$ are hadronic states, the photon emission occurs *only* through the (virtual) decay

$V \rightarrow \gamma$, where by $V$ we mean any vector meson which can couple to the photon.

Of course, one must add all the contributions of the various possible intermediate $V$ states. This assumption can be described by the following diagram expansion:
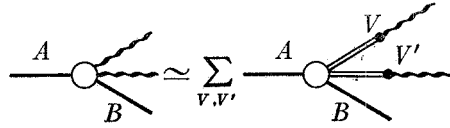


This approximation means that in the matrix element of the hadronic e.m. currents responsible for photon emission, one takes into account only the pole contributions due to the known vector mesons. See e.g. Kroll et al (117). In so doing one separates in a simple way the effects of e.m. interactions, and what remains is a purely strong interaction (virtual) process $A \rightarrow B + V + V'$.

The transition $V \rightarrow \gamma$ occurs for the $\rho$ and the $\omega_8$ which is the $T = Y = 0$ pure octet member of the vector meson nonet (the singlet $\omega_1$ cannot couple to the photon). The relevant coupling constants can be estimated, following Gell-Mann and Zachariasen (118) and using $SU_3$, as

$$g_{\rho\gamma} = \frac{em_\rho^2}{f_{\rho\pi\pi}}, \qquad g_{\omega_8\gamma} = \frac{em_{\omega_8}^2}{\sqrt{3}f_{\rho\pi\pi}}$$

in which $f_{\rho\pi\pi}$ is the coupling constant for $\rho \rightarrow 2\pi$, related to the $\rho$ width.

(ii) The strong amplitudes one needs to proceed are the form $P \rightarrow VV$, $V \rightarrow PV$, $V \rightarrow PP$, $P \rightarrow PVV$ and $V \rightarrow PPP$, where $V$, $P$ indicate a vector or a pseudoscalar meson. For the three-body amplitudes one assumes constant values which are related by $SU_3$. The last two four-body amplitudes can be expressed, again by vector meson exchange dominance (116), by means of the previously considered three-body coupling constants. As a clarification we give here the diagrams involved in the $\eta \rightarrow \pi^+\pi^-\gamma$ decay.
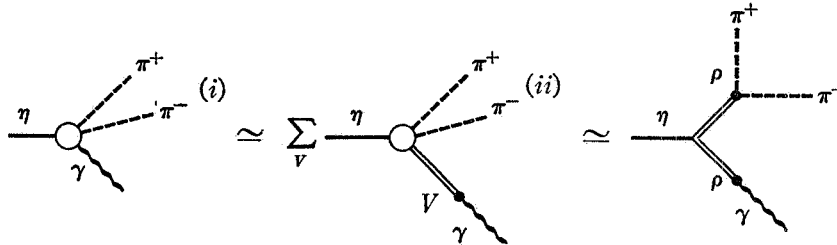
Summing up, we have a model which covers in a unified way all of the vector meson decays and all those involving photons of the pseudoscalar mesons. Let us now consider its predictive power and how it stands up when confronted with experiments.

Gell-Mann et al (116) obtained 0.25 for $\Gamma\eta\rightarrow\pi^+\pi^-\gamma/(\Gamma\eta\rightarrow2\gamma)$, a result which is rather different from the actual experimental value 0.13 but reproduces the important feature that although the $\eta\rightarrow2\gamma$ rate is of order $\alpha^2$, it is more abundant than $\eta\rightarrow\pi^+\pi^-\gamma$, which is of order $\alpha$. We note, however, that in the quoted paper the mixing of $\eta$ with $X^0$ had not been taken into account; in fact, at that time the $X^0$ was not known to exist.

Later on the hypothesis of $\eta - X^0$ mixing (Section 5) was suggested by Dalitz et al (119) to explain the large discrepancy between the measured $\eta\rightarrow2\gamma$ absolute width of about 1000 eV and the well-known $SU_3$ prediction (120): $\Gamma\eta\rightarrow2\gamma/(\Gamma\pi^0\rightarrow2\gamma) = \frac{1}{3}(m_\eta/m_{\pi^0})^3$. This relation, which was obtained without mixing, gives $\Gamma\eta\rightarrow2\gamma\sim160$ eV upon insertion of the experimental value $\Gamma\pi^0\rightarrow2\gamma = 7.2$ eV. The calculation by Dalitz et al (119) and others of a similar nature were based on the quark model and suffered from the ambiguities inherent in its formulation. These calculations, however, indicated that the mixing hypothesis produces substantial improvements on the pure $SU_3$ prediction without more extraordinary assumptions.

In the more general application [see Baracca & Bramòn (105), Cremmer (121), and references therein], inclusion of mixing in the pseudoscalar nonet enlarges the number of the unknown coupling constants which must be determined by fitting the experimental data from some selected processes.

One needs three independent values for the $PVV$ vertex, the $f_{\rho\pi\pi}$ constant (or the $\rho$ width), and the mixing angles in the vector and pseudoscalar nonets, for a total of six parameters. Hence the model can predict absolute rates (and consequently the relevant branching ratios) for a very large number of decays with essentially four conveniently selected processes as inputs (one of which is $\rho\rightarrow\pi\pi$); the values of the mixing angles can be calculated from the mass formula. In this connection, the use of a linear mass formula instead of the more fashionable quadratic one in the pseudoscalar nonet gives rise to appreciable differences. De Franceschi et al (106) pointed out how a better knowledge of the $X^0$ decay modes would permit a choice between the two, and the same conclusion has been reached by Baracca et al (105).

Let us go now to the comparison between the predictions of the pole model and experiments. Two general features should be noted at the start.

1. The large experimental errors in the input rates propagate to all the predicted values.

2. The input rates are those of the best known processes. As a consequence, the relevance in testing the theoretical results of the remaining poorly known processes is emphasized.

Baracca et al (105) take as inputs the rates for $\pi^0\rightarrow2\gamma$, $\eta\rightarrow2\gamma$ and $\omega\rightarrow\pi^0\gamma$ (or $\phi\rightarrow3\pi$), obtaining a satisfactory overall agreement with the data. For the decays we are considering in this section, they get the predictions $\Gamma\eta\rightarrow\pi^+\pi^-\gamma/(\Gamma\eta\rightarrow2\gamma) = 0.16\pm0.02$ and $\Gamma\eta\rightarrow\pi^0\gamma\gamma/(\Gamma\eta\rightarrow2\gamma) = (0.5\pm0.2)\times10^{-3}$.

A very low value for $\Gamma\eta\to\pi^0\gamma\gamma$ had already been given by Ferrari et al (122), Alles et al (123), and Van Royen et al (119).

We want to stress here that a precise measurement of the branching ratio $\Gamma\eta\to\pi^0\gamma\gamma/(\Gamma\eta\to2\gamma)$ would provide a severe test for the theory we have given.

*Critical tests of the pole model.*—The predictions of the pole model are critical in at least two other cases. The first is the $X^0\to2\gamma$ rate. To obtain the correct value of $\eta\to2\gamma$ via $\eta-X^0$ mixing, one calculates a value for $\Gamma X^0\to2\gamma/(\Gamma X^0\to\pi^+\pi^-\gamma)$ which turns out to be rather low. In (105) one finds the result $0.07\pm0.05$. The experimental situation is rather unclear, and a recent measurement by Bensinger et al (83) gives $\Gamma X^0\to2\gamma/(\Gamma X^0)=12\%\pm5\%$, which corresponds to $\Gamma X^0\to2\gamma/(\Gamma X^0\to\pi^+\pi^-\gamma)\sim(36\pm15)\%$. If confirmed, this would present a difficulty for the theory. However, unpublished results (84) seem to fit the theoretical value reported above (Section 4).

The other case is connected with a criticism to the pole model (113, 124): the relation between $\omega\to\pi^0\gamma$ and $\pi^0\to2\gamma$ implied by the model is not consistent with the data, if one assumes a negligible rate for $\phi\to\pi^0\gamma$.

To recover consistency between $\pi^0\to2\gamma$ and $\omega\to\pi^0\gamma$, Baracca et al (105), allowing for a nonzero $\phi\to\pi^0\gamma$ rate, predict $\Gamma\phi\to\pi^0\gamma=0.013\pm0.003$ MeV, which is on the border of the experimental limit $\Gamma\phi\to\pi^0\gamma<0.014$ MeV.

Cremmer (121), with the same inputs and general scheme as in (105), finds the value $\Gamma\phi\to\pi^0\gamma=0.26_{-0.24}^{+0.46}$ MeV. This difference between the two theoretical estimates should be better clarified. Even if any of these possible difficulties are confirmed, there could still be an escape by introducing a new intermediate vector meson as, e.g., the hypothesized $\rho'$ (125).

*Results from current algebra and related theories.*—The ratio $r=\Gamma\eta\to\pi^+\pi^-\gamma/(\Gamma\eta\to2\gamma)$ can be evaluated, as pointed out by Kawarabayashi et al (126), by relating $\eta\to\pi^+\pi^-\gamma$ to $\eta\to2\gamma$ in the two soft pion limit by PCAC (partially conserved axial vector current) and current algebra. Along essentially similar lines, Ademollo et al (127) and Pasupathy et al (128) obtain a value $r\simeq0.20$ which is decreased (127) to 0.14 (in excellent agreement with experiments) after inclusion of $\eta-X^0$ mixing. The careful analysis of Rubinstein & Veneziano (129) reveals, however, that these calculations are not so general and model independent as they may seem.

PCAC has also been used in connection with the problems of the absolute $\eta\to2\gamma$, $\eta\to\pi^0\gamma\gamma$ and $\eta\to\pi^+\pi^-\gamma$ widths, leading to the conclusion (Veltman 130) that in the soft pion limit the first two processes (as well as $\pi^0\to2\gamma$, $\omega\to\pi^0\gamma$ and $\omega\to\pi^+\pi^-\gamma$) are forbidden, i.e. have zero amplitude.

One can interpret this as an indication that the physical amplitudes, which of course are not zero, do not extrapolate smoothly to the soft pion limit. For a discussion of various proposals advanced along this line, see Sutherland (113, 131).

Alternatively, one can assume that PCAC must be modified in the

presence of e.m. interactions.[1] In the light of the general analysis presented by Adler (132), this appears to be a well-founded hypothesis enabling one to circumvent in a natural way the "forbiddenness" theorem.

This theorem has also been obtained, using normal PCAC, by Arnowitt et al (133) in the hard pion method introduced by Schnitzer & Weinberg (134). After modification of the PCAC condition, the authors (133) find generally satisfactory results for the radiative two-body decays $P\rightarrow\gamma\gamma$, $V\rightarrow P\gamma$. In particular, the value $\Gamma\eta\rightarrow2\gamma = (1.29 \pm 0.27)$ keV is found in good agreement with experiment. This result surprises us because no $\eta - X^0$ mixing is assumed, which, as we said before, appears crucial in other calculations to get agreement with the data.

Riazuddin & Sarker (135) also give an evaluation of $\Gamma\eta\rightarrow2\gamma$ by means of the hard pion method, including $\eta - X^0$ mixing. The result is lower than that quoted above, but it is still satisfactory: $\Gamma\eta\rightarrow2\gamma = (0.93\pm0.20)$ keV. The ratio $\Gamma\eta\rightarrow\pi^+\pi^-\gamma/(\Gamma\eta\rightarrow2\gamma)$ has been calculated by Goldberg et al (136), using the Veneziano model (137); they find $\Gamma\eta\rightarrow\pi^+\pi^-\gamma/(\Gamma\eta\rightarrow2\gamma) = 0.10$.

Finally, let us mention a calculation by Caprini et al (138) of $\Gamma\eta\rightarrow\pi^0\gamma\gamma/(\Gamma\pi^0\rightarrow2\gamma)^2$, leading to a very low value of $\Gamma\eta\rightarrow\pi^0\gamma\gamma$. They also obtain the value $\Gamma\eta\rightarrow2\gamma/(\Gamma\pi^0\rightarrow2\gamma)\simeq143$, in fair agreement with experiment.

### 6.3 THE $3\pi$ MODES

The experimental evidence available here can be summarized as follows:

1. $R_{3\pi}=\Gamma\eta\rightarrow3\pi^0/(\Gamma\eta\rightarrow\pi^+\pi^-\pi^0)\simeq1.5; \Gamma\eta\rightarrow3\pi^0\simeq800$ eV.

2. To a good approximation the Dalitz plot for the charged mode can be fitted with an amplitude linear in the $\pi^0$ energy: $A_{\pi^+\pi^-\pi^0}\propto 1+ay$, where $y=3(E_{\pi^0}-m_{\pi^0})/Q-1$ and $Q=m_\eta-2m_{\pi^+}-m_{\pi^0}$. For the slope parameter $a$ the value $-0.55\pm0.02$ is given by Cnops et al (63). The Dalitz plot for the neutral mode, on which there is very scanty information, is consistent with a constant amplitude (see Baglin et al 19).

We note that the result $R_{3\pi}\sim1.5$ can easily be obtained in any theory which allows only $\Delta T=1$ transitions in $\eta\rightarrow3\pi$.[2] ($C$ conservation allows $T=1$ but also $T=3$ for the total isospin of the three pions in the decay). A rough estimate, under the assumption of a constant matrix element, gives $R_{3\pi}\simeq1.7$, as discussed, e.g., by Salvini (9).

*Current algebra results with PCAC and "improved" PCAC in the hypothesis of conventional e.m. interactions.*—If we assume that the $\eta\rightarrow3\pi$ decay occurs via conventional e.m. interactions, the second order effective Hamiltonian density (113) is

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[1] This idea was first suggested by Veltman (130); his modification is different from that introduced by Adler.

[2] In 1966, values were reported of $R_{3\pi}<1$. The analysis made at that time, e.g. by Veltman & Yellin (124), indicated that it would be very difficult to get, without unusual assumptions, such a low value and at the same time agreement with the observed spectrum. Since all the recent values for $R_{3\pi}$ are close to 1.5, such speculations have not been pursued. We refer to the review by Salvini (9) for a survey.

$$H^{\text{e.m.}}(0) \propto e^2 \int d^4y D_{\mu\nu}(-y) T(J_\mu^{\text{e.m.}}(0) j_\nu^{\text{e.m.}}(y)) \qquad 3.1$$

where $D_{\mu\nu}(y)$ is the photon propagator and $j_\mu^{\text{e.m.}}(y)$ the e.m. current operator. Due to the transformation properties of $j_\mu^{\text{e.m.}}$ under $G$, only products of the isoscalar part of the current with the isovector are relevant in 3.1 for the $\eta \to 3\pi$ decay.

Hence this interaction satisfies $\Delta T = 1$ and, in accordance with the observation made above, one can see that all the available treatments based upon it reproduce the correct $R_{3\pi}$ value. The really difficult problem is that of explaining the high value of the absolute $\eta \to 3\pi$ width.

The ratio $\Gamma\eta \to 3\pi / (\Gamma\eta \to 2\gamma)$ is in itself not easy to understand, as already noted in 6.1. But so far we have essentially no theory for it, apart from qualitative estimates such as those one finds in the earlier works by Brown & Singer (139).

The width $\Gamma\eta \to 3\pi$ has been calculated by Möbius et al (140) using the static quark model. They found a value about $10^3$ times too low. More worrisome, however, is the result obtained using general assumptions by Sutherland (141), which we are now going to report.

Hara & Nambu (142) and Elias & Taylor (143) had treated the $K \to 3\pi$ decays with a local current $\times$ current weak Hamiltonian, using current algebra with PCAC and the soft pion techniques and obtaining strikingly good results. Now the same procedure, notwithstanding the similarities between $K$ and $\eta$ that we have already noted in 6.1, gives zero for the $\eta \to 3\pi$ amplitudes (141).

Let us display clearly the assumptions on which this conclusion depends: (i) use of the interaction 3.1 in which $j^{\text{e.m.}}$ has the usual properties; (ii) validity of PCAC in the normal form; (iii) a linear physical matrix element, as suggested by the data (see above) and as used in the $K \to 3\pi$ case (142, 143).

Retaining (i) and (iii), several authors (144) have used an "improved" form of PCAC, which in effect allows for a breakdown of its usual form in the case of the $\eta$ without destroying the success of the theory with $K$ decays. For a detailed discussion and an extensive bibliography up to 1967, see (113) and the last reference in (144).

In the papers referred to above, $R_{3\pi}$ and the slope are correctly given, but the amplitude, although different from zero, is still about one order of magnitude too low. This is confirmed also by Das et al (145).

The $\eta \to 3\pi$ problem has been considered anew by Brandt et al (146) in the context of the weak PCAC theory proposed by Brandt & Preparata (147). Although the calculations are not carried to the end, it is shown that in the new approach the assumptions (i) and (iii) do not imply a zero result for the amplitude.

Kellett (114) compares the results obtained from current algebra under hypotheses (i) and (ii) in $\eta \to 3\pi$ and $K \to 3\pi$ with extrapolations based on fits to the Dalitz plots including terms up to third order in the energies. He finds that this more complicated structure in contrast to a linear matrix element—

see (iii)—is consistent with the values in the soft pion limit. However, very large errors in the fitted parameters are implied.

Two other interesting conclusions are reached (114). First, all the data are consistent with only $\Delta T = 1$ transitions. Second, the nonlinear terms are especially pronounced in the $\eta$ Dalitz plot, indicating that perhaps the similarity with the $K$ is not as close as usually assumed.

*Theories assuming a nonconventional interaction and pole models.*—Several authors (148) have suggested, instead of the e.m. operator 3.1, the use of a scalar effective Hamiltonian density $u_3(x)$, which behaves like the third component of an isovector under $SU_2$. The $\eta \to 3\pi$ decay amplitude is assumed to be given by $(3\pi \mid u_3(0) \mid \eta)$. The forbiddenness theorem no longer applies because its derivation depends crucially on the explicit current $\times$ current structure of $H^{\text{e.m.}}(0)$.

By current algebra and soft pion techniques one can then relate the $\eta \to 3\pi$ rate to the pseudoscalar e.m. mass differences. The result depends in an essential way on whether this $u_3$ interaction is considered of purely e.m. origin or rather is a new, isospin violating, interaction. In either case, the calculations as a rule give a good $R_{3\pi}$ value but an absolute rate which is too low.

The rate $\eta \to 3\pi$ has been evaluated by Cabibbo et al (149) and Osborn et al (108) in the framework of $SU_3 \otimes SU_3$ chiral symmetry (111), broken in the way proposed by Gell-Mann et al (112). In both papers the same technique of nonlinear Lagrangians has been used, and the results obtained agree: $\Gamma \eta \to 3\pi^0 \sim 160$ eV. A low value previously had been found in a closely related approach by Kamazawa (150). In this connection the hypothesis of $\eta - X^0$ mixing can be of some help, as discussed in (108). By use of it one could enhance $\Gamma \eta \to 3\pi^0$ to about 450 eV, without disturbing too much the correct value calculated for the slope $a = -0.526$ with no mixing.

A suggestive proposal for the introduction of the $u_3$ interaction has been given by Oakes (151). The strength of $u_3$ is expressed by means of the Cabibbo angle and is numerically much higher than the estimate given for it in (108). In this way [see (108) again, however, for some criticism to Oakes] the experimental value of $\Gamma \eta \to 3\pi^0$ could be accounted for.

The idea of a direct $\eta - \pi^0$ transition, induced e.g. by $u_3$, allows one to relate $\eta \to 3\pi$ to the (off-shell) $\pi - \pi$ scattering amplitude:
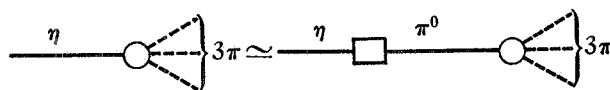


Pole approximations to the latter have been considered in several other models (152). Although the general scheme appears to be a possible one, we feel that the differences in these approaches and the consistency among the various contributions (152) should be reconsidered before we try to assess the relevance of these calculations in an unambiguous way.

Lovelace (153) uses for the $\pi - \pi$ amplitude the Veneziano representation (137) and is able to reproduce correctly $R_{3\pi}$ and the distribution of the D. F. plot. For a critical appraisal see Greenberg (154) and Sutherland (155).

## 6.4 Concluding Remarks

The theoretical understanding of the eta decays still appears to be far from satisfactory. The gamma modes are in better shape, however. The model of vector meson dominance allows one to relate all the processes to each other as well as to the vector meson decays, and the predictions are consistent with the data. However, some of the predictions refer to processes for which scant experimental information is available. Thus we must await further more refined results to assess properly the validity of the theory. There are also good calculations for some of the branching ratios, and even the high absolute width of the $\eta \to 2\gamma$ mode can be accounted for reasonably via $\eta - X^0$ mixing. Precise measurements of the $\pi^0\gamma\gamma$ mode would be crucial in several instances.

With regard to the $3\pi$ modes, a prominent and very serious problem is represented by the high experimental value of the absolute rate. It appears that, although the Sutherland theorem can be bypassed in some ways, the most reasonable theories still give results too far from the measured value. It may be that a better treatment of the final state interactions could provide a way out of the present difficulties without one's having to introduce untraditional hypotheses. In this connection a better knowledge of the nonlinearities in the D. F. plot would be of great importance.

## 7. HIGH ENERGY PRODUCTION OF THE ETA. THE PRIMAKOFF EFFECT

### 7.1 Production of the $\eta$ with Pion Beams

Guisan et al (156) have studied reaction $\pi^- p \to \eta n$ near the forward peak in a spark chamber experiment carried out at the CERN protron synchrotron through detection of both photons from the $\eta \to 2\gamma$ decay mode. The momenta of $\pi^-$ ranged from 2.91 to 18.2 GeV/c, and the squared momentum transfer $-t$ was measured between 0 and 1 (GeV/c)$^2$. The results of Guisan et al are summarized in part in Figure 12, in the form $d\sigma/dt$, as a function of the squared four-momentum transfer $(-t)$.

Regge pole theory has been applied (see Fig. 12) to the reaction $\pi^- p \to \eta n$ at high energy by Roger, Phillips & Rarita (157), taking into account the exchange in the $t$ channel of the $A_2$ (the $J^{PG} = 2^{+-}$ meson of mass 1310 MeV) trajectory, which is the only one allowed among those associated to existing particles. These authors have deduced the parameters of the trajectory from a fit to $\pi p$ and $Kp$ elastic and charge exchange scattering. As we can see, the agreement with the experiment is very good. Bonamy et al (158) have found small values (almost compatible with zero) for the polarization parameter in reaction $\pi^- p \to \eta n$. Note that single Regge pole exchange predicts a zero value of this parameter. The same formulae of Roger et al (157) have been extended to lower energies by Pugliese & Restignoli (9).

Reactions $\pi^- p \to \eta n$ and $\pi^+ n \to \eta p$ both have pure isospin $\frac{1}{2}$, and the cross section must be the same. This has been verified in a more recent paper of Danburg et al (159). Their data range from 1.1 to 2.37 GeV/c for the momentum of the incident pion. These authors have again used a Regge pole
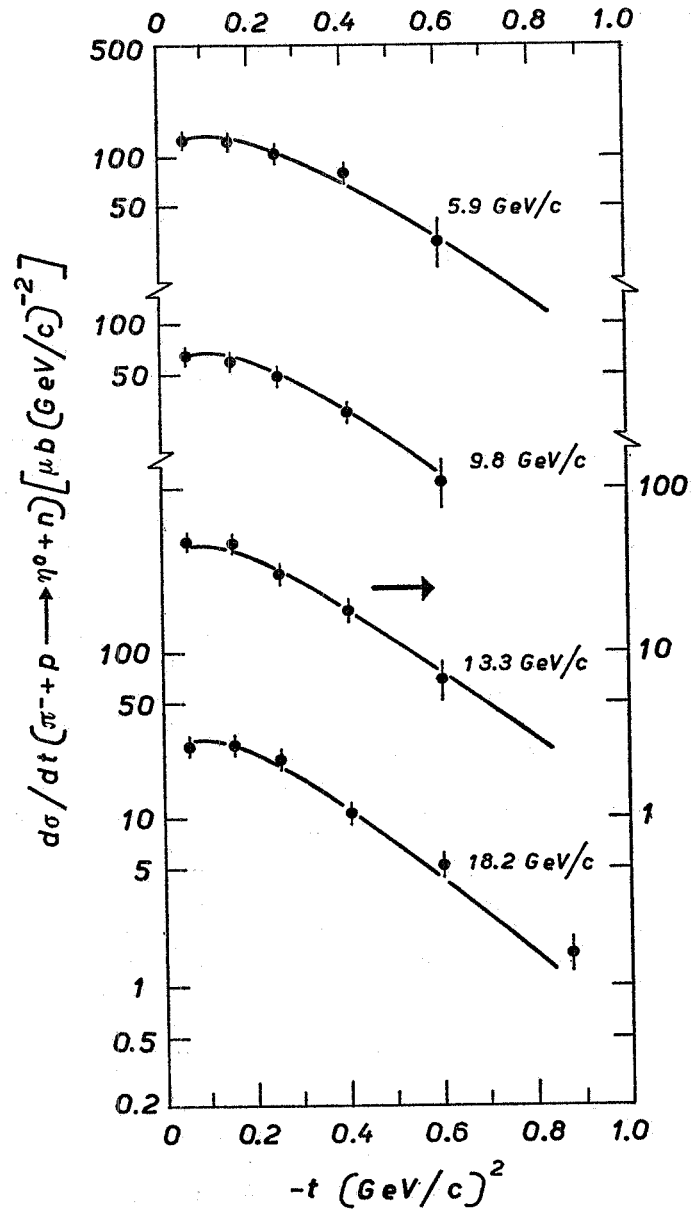
FIGURE 12. $\pi^- + p \rightarrow \eta + n$ differential cross sections, all decay modes, for different pion laboratory momenta in GeV/$c$. The experimental points come from the work of Guisan et al (156). The full lines are the fits to the data by Roger et al (157) where a single Regge trajectory is assumed.

exchange model involving only the $A_2$ trajectory, but with Veneziano-type residue functions. Agreement is very good for the total cross sections for an incident momentum range spanning from 1 to 18.2 GeV/$c$.

## 7.2 Photoproduction of the $\eta$ at Higher Energies

Good information on the total cross section of process $\gamma+p\rightarrow\eta+p$ comes from the results obtained with bubble chamber technique by the Aachen-Berlin-Bonn-Hamburg-Heidelberg-München collaboration (160).

Photoproduction of the eta in hydrogen has been studied up to 4 GeV by Bellenger et al (161), with the bremsstrahlung beam of the Cambridge electron accelerator. These authors—and this is one of the interesting problems in $\eta$ photoproduction—find no evidence for a dip or change of slope of $d\sigma/dt$ at $-t=0.6$ (GeV/$c$)$^2$ as seen in $\pi^0$ photoproduction at intermediate energies.

Reaction $\gamma+p\rightarrow\eta+p$ also has been studied by Anderson et al (162) and by a Desy-Bonn collaboration (163). Their recent results are reported in Figure 13. The $\eta$ cross sections $d\sigma/dt$, all modes, have been multiplied by $(S-M_p^2)^2$, so that the points at different energies fall together. This means that the energy dependence appears consistent with $E^{-2}$ for all the $t$ values covered by the experiments.

In the same figure (the scale of the ordinates being at the right), we have reported the result at 6 GeV for the process $\gamma p\rightarrow\pi^0 p$. One can notice the presence of a dip in this process. On the basis of the absence of a dip in $\eta$ photoproduction at $t\cong-0.5$ (GeV/$c$)$^2$, the authors conclude that photoproduction of $\eta$ mesons at moderate $t$ values does not seem to proceed predominantly via Reggeized $\rho^0$ meson exchange.

In the Regge pole model (2, 164) the differential cross section at high energy depends on the beam energy $E_0$ as $d\sigma/dt\sim E_0^{2\alpha(t)-2}$, in the case in which only one Regge trajectory—here indicated by $\alpha(t)$—can be exchanged in the $t$ channel. On this basis, the disagreement between $\pi^0$ and $\eta$ photoproduction is rather unexpected. In fact, both reactions are expected to be dominated by a single Regge exchange: the $\omega$ trajectory in the case of $\pi^0$ and the $\rho$ trajectory for the $\eta$. Considering that the two trajectories are similar, one would expect the two processes to be similar and to resemble $\pi^-p\rightarrow\pi^0 n$, for which only $\rho$ exchange is permitted. Thus $d\sigma/dt$ versus $t$ should show a sharp dip or local minimum at $t\cong-0.5$ (GeV/$c$)$^2$ and strong shrinkage of the forward peak with increasing energy. This is not the case for the $\eta$, as we have shown in Figure 13.

This striking difference (the $\pi^0-\eta$ puzzle) has been examined by a few authors in recent times. Dar et al (165) propose a peripheral model for high energy exchange reactions, which predicts the different behavior of $\pi^0$ and $\eta$ with unpolarized and polarized photon beams. The model is based on a proper approximation of the partial waves of the scattering amplitude for high energy exchange reactions. By applying it to the experimental results, the authors find that the fits to the data in $\pi^0$ and $\eta$ photoproduction are rather good and may resolve the $\pi^0-\eta$ puzzle. This model can be regarded as an
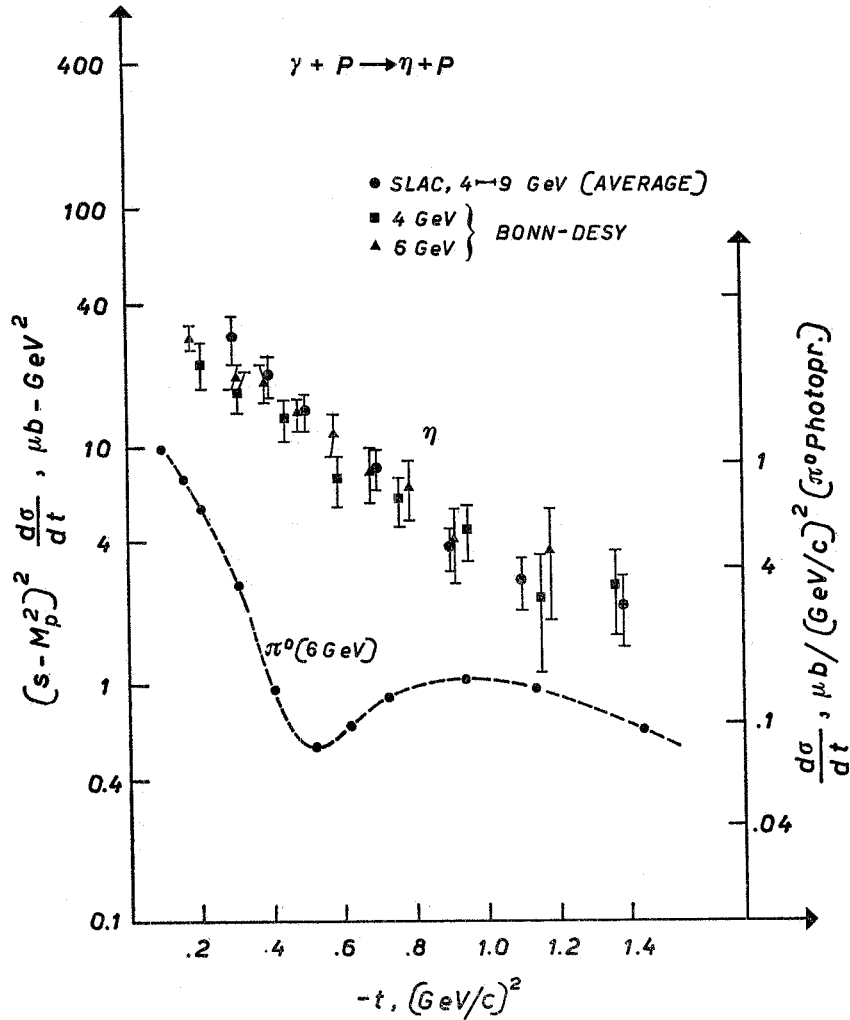
FIGURE 13. Cross sections for $\gamma+p\rightarrow\eta+p$, from SLAC (162) and Bonn-Desy collaboration (163). $d\sigma/dt$ has been multiplied by $(S-M_p{}^2)^2$, with the result that points at different energies fall together. The curve below (ordinates on the right) is $d\sigma/dt$ for reaction $\gamma p\rightarrow\pi^0 p$ at 6 GeV (162), to make evident the $\pi^0-\eta$ puzzle.

approximation to the Regge exchange model with absorption corrections (Regge cuts).

Other approaches to the $\pi^0-\eta$ puzzle are somewhat different. In particular we remember a paper by M. Colocci (166). The conclusion of this author is again that Regge poles are not capable of providing a reasonable picture of photoproduction processes, so that Regge cuts, or absorption, or

more complicated singularities are definitely needed. The author has developed a successful quantitative model for calculating cut corrections to Regge pole exchange directly in terms of $S$-channel helicity amplitudes.

### 7.3 PRIMAKOFF PRODUCTION OF THE $\eta$ PARTICLE

The Primakoff effect (167) consists in the creation of a meson in the coulomb field of a nucleus. This effect is an important contribution to the cross section of the general photoproduction process

$$\gamma + \text{nucleus} \rightarrow \eta + \text{nucleus} \qquad \qquad 3.1$$
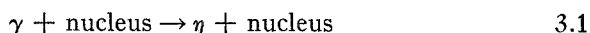
when the energy of the incident photon is high and the momentum transfers involved are sufficiently low. The resulting Primakoff cross section $(d\sigma/d\Omega)_P$ of process 3.1 is directly proportional to the $\Gamma_{\gamma\gamma}$ decay width:

$$\left(\frac{d\sigma}{d\Omega}\right)_P = 8\alpha\Gamma_{\gamma\gamma}z^2 \frac{\beta^3 E^4}{\mu^3} \frac{|F(Q)|^2}{Q^4} \sin^2\theta \qquad \qquad 3.2$$

with $z$ being the charge number of the target nucleus, $\mu$, $\beta$, $\theta$ the eta mass, velocity, and angle; $E$ is the energy of the incident photon. $Q$ is the momentum transfer and $F(Q)$ is the electromagnetic form factor of the nucleus. $F(Q)$ needs corrections to take into account the probability of reabsorption of the $\eta$ in nuclear matter.

The measurement of $\Gamma_{\gamma\gamma}$ of the eta particle through the Primakoff effect has been performed at DESY by Bemporad et al (Bonn-Pisa collaboration) (168). The differential cross section 3.2 of reaction 3.1 has been measured in the region from $0°$ to $4°$, at 4 and at 5.5 GeV incident gamma ray energies, on zinc, silver, and lead nuclei. The experiment was performed with the bremsstrahlung beam on an internal rotating target of the electron synchrotron. The $\eta$s were detected by measuring the angles and energies of their two decay photons in the $\gamma\gamma$ mode.

Extraction of cross section 3.2 from the yield cannot be immediate, due to other possible contributions to the eta production, in particular the coherent nuclear effect. After having extracted $(d\sigma/d\Omega)_P$ from the experimental differential cross section, the authors (168) obtained the following final result (notice that they were detecting the $\eta$ through the $\gamma\gamma$ decay mode):

$$B\Gamma_{\gamma\gamma} = (0.38 \pm 0.083) \text{ keV}$$

Assuming that $B = \Gamma_{\gamma\gamma}/\Gamma_{\text{total}} = 0.375 \pm 0.016$ it follows:

$$\Gamma_{\gamma\gamma} = (1.01 \pm 0.23) \text{ keV} \qquad \qquad 3.3$$

The descending total width of the eta particle is

$$\Gamma_{\text{total}} = (1.01)/(0.375) = (2.7 \pm 0.67) \text{ keV} \qquad \qquad 3.4$$

The knowledge of this absolute width is of fundamental importance in the

theoretical interpretation of the eta decay modes. The nice experiment of Bemporad et al is up to now the only measurement of an absolute width for the eta particle. It is not even necessary to underline the great interest of measuring again through the Primakoff effect the $\Gamma_{\gamma\gamma}$ width at higher energies, as well as to measure the $\Gamma_{\gamma\gamma}$ width of the $X^0$ (Section 4). As far as we know, this program has been started in a few laboratories, but no definite results have been obtained yet. A comment on the possible influence of the coherent nuclear photoproduction when deducing the $\Gamma_{\eta \to \gamma\gamma}$ width from the experiment has been given by Benfatto (169).

## 8. OPEN PROBLEMS CONCERNING THE $\eta$, $\eta'$ PARTICLES

We summarize here some open problems and suggestions for future work that an analysis of the present situation seems to indicate. Let us start first with the problems connected with the decay of the $\eta$ and $\eta'$ particles, which still constitute a severe challenge for the experimentalists and are of the highest theoretical and general interest.

One relevant problem is the absolute width of the $\eta$ decay modes, and in particular of the $\eta \to 3\pi$ mode. In fact, the experimental value ($\Gamma_{\eta \to 3\pi^0} \sim 810$ eV) is higher than simple theoretical predictions by a factor 2 to 4. More generally, one can say that the experimental absolute widths of the decays are too high with respect to theory. Some values and some branching ratio may be justified by admitting an appreciable $\eta - \eta'$ mixing (see Section 6), but we must admit that a reduction of the width $\Gamma_{\eta \to 2\gamma}$ would be of some relief to the theory. In this respect we underline the importance of measuring again the width of the $\eta$ through the Primakoff effect at higher energies than those available 5 years ago, for instance, to make sure that the coherent nuclear photoproduction does not alter in some unexpected way the $\eta$ production in the coulomb field (169).

The $\eta$ was chosen as a good tool to verify $C$ invariance in 1965, and we have briefly reviewed the many experiments on this subject, but the problem is still open. The error in the measurement of $A$ (Section 3) has been reduced to within excellent limits, and one might begin to believe that there is a $C$ violation, but the possible interference terms with a $3\pi$ background amplitude (Section 3) may make it difficult to conclude this with certainty. Perhaps one can try to verify $C$ by looking also at the rare decay modes of the $\eta$ (Section 3).

A general problem for many $0^-$ pseudoscalar mesons is the measurement of their widths, which are far narrower than the present resolution of bubble chambers. This is why the Primakoff effect is so important in studying the decays of our particles. It is then natural to hope that the Primakoff effect may soon be applied to the $X^0$ (and perhaps to the $E$) at the maximum available photon energies.

In connection with this problem, we recall that the method of measuring the absolute width of the $X^0 \to \gamma\gamma$ decay mode through the Frascati $e^+e^-$ storage ring (Section 4) may be extended to a number of other cases. In fact,

the bremsstrahlung annihilation process given in Section 4 offers a systematic way to search for all 0 spin particles through $\gamma - \gamma$ annihilation. Notice that this method may be extended in principle to all particles of even spin, independent of parity, and that the $\gamma\gamma$ annihilation cross section increases with energy, so that the future $e^+e^-$ storage rings at the highest energies may be the best instruments to hunt for new and old $0^\pm$ and $2^\pm$ mesons.

We have seen that the $0^-$ pseudoscalar nonet is not yet well established in the $\eta'$ (or $\eta_1$) singlet; in fact, we do not know if the $\eta'$ is the $X^0$, which seems more probable (Section 4), or the $E$ meson. There is, of course, a theoretical possibility, though somewhat annoying, that we have more than one $0^-$ singlet; so we must await new measurements on the $X^0$ and the $E$ particles to know if they are both $I^G$, $J^P = 0^+$, $0^-$ mesons or not, and decide later what to do. This problem may be resolved rather soon if it is, as one can hope, mostly a matter of increasing present statistics on the decay products of $X^0$ and $E$ (see Section 4). Good measurements on the $X^0$ (or the $E$) could eventually allow a verification of the correct mass formula, i.e. linear or quadratic.

As we noticed (Sections 2, 6), the branching ratio of the $\pi^0\gamma\gamma$ decay mode of the $\eta$ is certainly rather small, but it remains critical. A branching ratio $\geq 5\%$ for this mode is still possible from the experiments, but it may still be too high with respect to the present views of vector dominance. It will be difficult, but important, to obtain a more precise experimental result on the $\pi^0\gamma\gamma$ mode. For other conclusions on the $\eta, \eta'$ decays, we refer back to the end of Section 6. Perhaps when summing up the situation, under the hypothesis that all the important experimental results on the $\eta$, $X^0$, $E$ are correct, we are tempted to believe that the electromagnetic interactions of the hadrons are far from being understood and may contain some rather unforeseen features.

The problem connected with $\eta$ and $\eta'$ production are perhaps less specific and not as urgent. They shall be clarified with time, in the course of a concurrent analysis of all the meson channels. It would be very convenient to extend the high energy production (Section 7) to higher $-t$ values so as to have a choice between the different alternatives.

At the low energy end, it is important to improve our knowledge of the $\eta$ production by $K^\pm$ beams (Section 2) in order to reach a final conclusion on the existence of the $\frac{1}{2}^-$ $(S_{\frac{1}{2}})$ baryon octet. The confirmation of this octet, and the clear determination of its properties, would be a nice experimental achievement, providing an interesting test for the proposed classification of baryon resonances.

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