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COVARIANT OPERATOR-PRODUCT EXPANSION

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## Manifestly Conformal Covariant Operator-Product Expansion.

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### 1. - Introduction.

As is well known, scale invariance <sup>(1)</sup> in conjunction with Wilson's operator product expansion <sup>(2)</sup> appears to play an essential role in the setting-up of a theoretical frame for the interpretation of experiments, such as deep inelastic lepton scattering; which in configuration space depend on the behaviour near the light-cone. It has been suggested that the stronger conformal invariance may indeed apply in such limiting conditions and its implications for equal-time commutators <sup>(3)</sup> and for operator product expansions <sup>(4-6)</sup> have been discussed. In particular, in ref. <sup>(5,6)</sup> the implications of conformal covariance on the operator product expansion on the light-cone <sup>(2,7)</sup> have been fully derived, by a method using the Jacobi identities in ref. <sup>(4,5)</sup>, and by an alternative manifestly covariant method in ref. <sup>(6)</sup>. The latter method takes advantage of the isomorphism between the conformal algebra and the orthogonal algebra  $O_{4,2}$  <sup>(8)</sup> and makes use of a six-dimensional pseudo-Euclidean co-ordinate space. In this note we shall further develop such method to derive a manifestly conformal covariant operator-product expansion valid at all values of  $x^2$ . We must stress that, whereas conformal invariance (as well as scale invariance) of the complete theory may in fact under some form hold near the light-cone, we must exclude the possibility that the complete theory

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<sup>(1)</sup> See, e.g., H. A. KASTRUP: *Phys. Rev.*, **142**, 1060 (1966); G. MACK: *Nucl. Phys.*, **5** B, 499 (1968); M. GELL-MANN: *Lectures at Hawaiian Summer School* (1969).

<sup>(2)</sup> K. WILSON: *Phys. Rev.*, **179**, 1499 (1969).

<sup>(3)</sup> S. CICCARIELLO, R. GATTO, G. SARTORI and M. TONIN: *Ann. of Phys.*, **65**, 265 (1971).

<sup>(4)</sup> S. FERRARA, R. GATTO and A. F. GRILLO: *Nucl. Phys.*, B **34**, 349 (1971).

<sup>(5)</sup> S. FERRARA, R. GATTO and A. F. GRILLO: *Phys. Lett.*, **36** B, 124 (1971).

<sup>(6)</sup> S. FERRARA, R. GATTO and A. F. GRILLO: Frascati preprint LNF, 71/70 (1971) (to be published).

<sup>(7)</sup> R. BRANDT and G. PREPARATA: *Nucl. Phys.*, **27** B, 541 (1971); Y. FRISHMAN: *Phys. Rev. Lett.*, **25**, 966 (1970).

<sup>(8)</sup> For a review of the conformal algebra we refer to G. MACK and A. SALAM: *Ann. of Phys.*, **53**, 174 (1969).

is fully conformal (as well as scale) invariant. Nevertheless, the implications of exact conformal invariance, not restricted to the light-cone alone, might still prove useful in discussing the properties of the so-called skeleton theory <sup>(2)</sup>, and fundamental problems such as that of canonical dimensions, etc. In view of this it appears as a relevant step to obtain a fully conformal covariant operator-product expansion. In addition the causality restriction has to be discussed which, as is well known, is extremely stringent when the entire group of conformal transformations is considered, which contains transformations of four-vectors from inside to outside the light-cone and *viceversa*.

**2. - Manifestly conformal covariant operator-product expansion.**

We limit ourselves for simplicity to the expansion of a product  $A(x)B(x')$ , where  $A(x), B(x)$  are two Lorentz scalars with  $K_\lambda = 0$ , *i.e.* satisfying  $[A(0), K_\lambda] = 0$  and the same for  $B(0)$  (conformal scalars). In the covariant six-dimensional formalism with co-ordinates  $\eta_A$  ( $A = 0, 1, 2, 3, 5, 6$ ) the general form of the expansion is <sup>(9,10)</sup>

$$(1) \quad A(\eta)B(\eta') = \sum_{n=0}^{\infty} E_n(\eta \cdot \eta') D^{(n)A_1 \dots A_n}(\eta, \eta') \psi_{A_1 \dots A_n}(\eta')$$

where  $D^{(n)A_1 \dots A_n}(\eta, \eta')$  is an « orbital » tensor operator defined over  $\eta^2 = 0 = \eta'^2$  and regular at  $\eta \cdot \eta' = 0$ ,  $E_n(\eta \cdot \eta')$  is a *c*-number of the form

$$(2) \quad E_n(\eta \cdot \eta') = \gamma_n(\eta \cdot \eta')^{\frac{1}{2}(\lambda_A + \lambda_B - \lambda_n - n)},$$

and  $\psi_{A_1 \dots A_n}(\eta)$  are irreducible tensor representations of  $SU_{2,2}$  (the spinor group associated to  $O_{4,2}$ ) homogeneous of degree  $\lambda_n = -l_n$ , containing Lorentz tensors of maximum order  $n$ , and satisfying supplementary conditions  $\eta^{A_1} \psi_{A_1 \dots A_n}(\eta) = 0$  and  $\partial^{A_1} \psi_{A_1 \dots A_n}(\eta) = 0$ . The expansion in eq. (1) is manifestly conformal covariant on the hypercones  $\eta^2 = 0$  and  $\eta'^2 = 0$ . In ref. <sup>(6)</sup> it was noted that the most general form of the operator  $D^{(n)A_1 \dots A_n}(\eta, \eta')$  is (see eq. (10) of ref. <sup>(6)</sup>)

$$(3) \quad D^{(n)A_1 \dots A_n}(\eta, \eta') = \sum_{m=0}^n \eta^{A_1} \dots \eta^{A_{n-m}} \eta'^{A_{n-m+1}} \dots \eta'^{A_n} D^{(n,m)}(\eta, \eta') c_{nm},$$

where  $c_{nm}$  are constants,  $D^{(n,m)}(\eta, \eta')$  is a differential operator defined on  $\eta^2 = 0 = \eta'^2$  and homogeneous of degree  $h = \frac{1}{2}(\lambda_A - \lambda_B + \lambda_n - n) + m$  in  $k/k'$  ( $\lambda_A, \lambda_B, \lambda_n$  are the degrees of homogeneity of  $A, B$  and  $\psi_{A_1 \dots A_n}$ , and we recall that  $x_\mu = k^{-1} \eta_\mu$ ,  $k = \eta_5 + \eta_6$ ). The operator  $D^{(n,m)}$  is uniquely given as formal power

$$(4) \quad D^{(n,m)}(\eta, \eta') = D^h(\eta, \eta'),$$

where

$$(5) \quad D(\eta, \eta') = \eta \cdot \eta' \square'_6 - 2\eta \cdot \partial'(1 + \eta' \cdot \partial')$$

<sup>(9)</sup> The covariant notations in six dimensions are the same used in our previous paper (ref. <sup>(9)</sup>), to which we refer for all definitions and for a summary of the formalism.

<sup>(10)</sup> We observe that we have taken, for simplicity, each irreducible representation  $\psi_{A_1 \dots A_n}(\eta)$  without multiplicity. Obviously, in general a sum over the same tensor representation (with possible different homogeneity degrees) is understood.

Indeed,  $D(\eta, \eta')$  is the only operator defined on the hypercones and homogeneous of degree one in  $k/k'$ . We note that the two terms in  $D(\eta, \eta')$ , eq. (5), both contain a term proportional to  $\partial/\partial\eta'^2$ , and their sum is indeed the only combination which remains well defined on  $\eta'^2 = 0$ . It can be rewritten as

$$(6) \quad D(\eta, \eta') = (\eta \cdot \eta')^{-1} \eta^A \eta'^C g^{BD} L'_{AB} L'_{CD},$$

where  $L'_{AB} = i(\eta'_A \partial'_B - \eta'_B \partial'_A)$  are the orbital generators of  $O_{4,2}$ . For  $\eta \cdot \eta' = 0$  (corresponding to the light-cone,  $(x - x')^2 = 0$ )  $D(\eta, \eta') \propto \eta \cdot \partial'$  and one recovers the result of ref. (6). We shall now state two useful lemmata:

1) The  $n + 1$  covariants defined in eq. (3) are all proportional. From the supplementary conditions

$$(7) \quad \eta^{A_1} \psi_{A_1 A_2 \dots A_n}(\eta) = 0, \quad \partial^{A_1} \psi_{A_1 A_2 \dots A_n}(\eta) = 0,$$

one has

$$(8) \quad \eta'^{A_1} D^h(\eta, \eta') \psi_{A_1 \dots A_n}(\eta') = \eta'^{A_1} 2h(2 - h + \lambda_n) D^{h-1}(\eta, \eta') \psi_{A_1 \dots A_n}(\eta').$$

2) One has

$$(9) \quad D^h(\eta, \eta') = (L - 1)^h \sum_{J=0}^h \binom{h}{J} \frac{(\eta \cdot \eta')^J (2\eta \cdot \partial')^{h-J} \square_6'^J}{(L - 1)^J},$$

$$L = -k' \frac{\partial}{\partial k'} \text{ and } (L - 1)^J = (l_n - 1)(l_n - 1 + 1) \dots (l_n - 1 + J - 1).$$

In terms of the variables  $(x, k)$  one has the identities (11)

$$(10) \quad \square_6' - 4(1 - L) \frac{\partial}{\partial \eta'^2} = \frac{1}{k'^2} \square_4',$$

$$(11) \quad \eta \cdot \partial' - 2(\eta \cdot \eta') \frac{\partial}{\partial \eta'^2} = \frac{k}{k'} [(x - x') \partial' - L],$$

$$(12) \quad D(\eta, \eta') = 2 \frac{k}{k'} \left[ (L - (x - x') \partial')(1 - L) - \left( \frac{x - x'}{2} \right)^2 \square_4' \right],$$

which together with eq. (14) of ref. (6) allow us to write eq. (9) as

$$(13) \quad D^h(\eta, \eta') = \left( \frac{k}{k'} \right)^h (-2)^h \frac{\Gamma(l_n - 1 + h)}{\Gamma(l_n - 1)} \sum_{J=0}^{\infty} \frac{1}{J!} \frac{\Gamma(-h + J)}{\Gamma(-h)} \frac{\Gamma(l_n - 1)}{\Gamma(l_n - 1 + J)} \cdot \frac{\Gamma(l_n + h + J)}{\Gamma(l_n + 2J)} \left( - \left( \frac{x - x'}{2} \right)^2 \right)^J {}_1F_1(J - h, l_n + 2J; (x - x') \cdot \partial') \square'^{J/h}.$$

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(11) It is instructive to notice from eq. (10) that only for  $l = 1$  is  $\square_6'$  defined on the hypercone, corresponding to the fact that only for the canonical value  $l = 1$  is the Klein-Gordon equation conformally covariant.

The lemmata 1) and 2) can be proved by induction for integer  $h$  and then extended through analytical continuation to all values of  $h$  <sup>(12)</sup>; we also used the identity

$$(-\eta \cdot \partial')^\beta = \left(\frac{k}{k'}\right)^\beta \frac{\Gamma(-\lambda + \beta)}{\Gamma(-\lambda)} {}_1F_1(-\beta, -\lambda; (x-x') \cdot \partial'),$$

where  $\lambda$  is the homogeneity degree of the operator on which  $(\eta \cdot \partial')^\beta$  is acting (see ref. (6)). We note that, for  $h$  positive integer, the terms with  $J > h$  in eq. (13) all vanish. Inserting into eq. (13) the well-known integral representation for  ${}_1F_1$  (see ref. (5,6)), one obtains

$$(14) \quad D^h(\eta, \eta') = \left(\frac{k}{k'}\right)^h (-2)^h \frac{\Gamma(l_n - 1 + h)}{\Gamma(-h) \Gamma(l_n - 1)} \sum_{J=0}^{\infty} \frac{1}{J!} \frac{\Gamma(l_n - 1)}{\Gamma(l_n - 1 + J)} \left[-\left(\frac{x-x'}{2}\right)^2\right]^J \cdot \int_0^1 du u^{-h+J-1} (1-u)^{l_n+h+J-1} \exp[u(x-x') \partial'] \square'^J,$$

and after performing the summation over  $J$

$$(15) \quad D^h(\eta, \eta') = \left(\frac{k}{k'}\right)^h (-2)^h \frac{\Gamma(l_n - 1 + h)}{\Gamma(-h) \Gamma(l_n - 1)} \cdot \int_0^1 du u^{-h-1} (1-u)^{l_n+h-1} \exp[u(x-x') \partial'] {}_0F_1\left(l_n - 1; -\left(\frac{x-x'}{2}\right)^2 \square' u(1-u)\right).$$

Equation (15) is meant as a formal operator expression with all derivatives located at the right and acting to the right. Using eq. (15) one has for the expansion in eq. (1)

$$(16) \quad A(x) B(x') = \sum_{n=0}^{\infty} \left[ \frac{1}{(x-x')^2} \right]^{\frac{1}{2}(l_A+l_B+n-l_n)} e_n^{AB} \int_0^1 du u^{\frac{1}{2}(l_A-l_B+l_n+n)-1} \cdot (1-u)^{\frac{1}{2}(l_B-l_A+l_n-n)-1} \exp[u(x-x') \partial'] {}_0F_1\left(l_n - 1; \left(\frac{x-x'}{2}\right)^2 \square' u(1-u)\right) x^{A_1} \dots x^{A_n} \psi_{A_1 \dots A_n}(x'),$$

where  $\eta^A = kx^A$  and  $\psi_{A_1 \dots A_n}(x) = k^{l_n} \psi_{A_1 \dots A_n}(\eta)$ . The tensors  $O_{A_1 \dots A_n}(x)$  which transform according to the conformal algebra in space-time are given (see ref. (6)) by

$$(17) \quad O_{A_1 \dots A_n}(x) = (\exp[-ix \cdot \pi] \psi)_{A_1 \dots A_n}(x),$$

where the « internal generator »  $\pi_\mu = S_{6\mu} + S_{5\mu}$  acts as

$$(18) \quad (\pi_\mu O)_{A_1 \dots A_n}(0) = i \sum_{i=1}^n [(g_{6A_i} + g_{5A_i}) O_{A_1 \dots \hat{A}_i \dots A_n \mu}(0) - g_{\mu A_i} (O_{A_1 \dots \hat{A}_i \dots A_n 6}(0) + O_{A_1 \dots \hat{A}_i \dots A_n 5}(0))]$$

<sup>(12)</sup> This is a standard method in representation theory.

(the notation  $O_{A_1 \dots \widehat{A}_i \dots A_n}$  means that  $A_i$  is omitted), and the components  $O_{\alpha_1 \dots \alpha_j x x \dots}$  (where  $x$  are 5 or 6) can be obtained through the supplementary conditions (eq. (7)) <sup>(13)</sup>

$$(19) \quad O_{\alpha_1 \dots \alpha_{n-k} 66 \dots}(x) = 2^{-k} \frac{\Gamma(l_n - 2 - n)}{\Gamma(l_n - 2 - n + k)} \partial_{\mu_1 \dots} \partial_{\mu_k} O^{\mu_1 \dots \mu_k}_{\alpha_1 \dots \alpha_{n-k}}(x)$$

(the substitution  $5 \leftrightarrow 6$  does not change the value on account of the first supplementary condition). The covariant product in eq. (16) can be written as

$$(20) \quad x^{A_1} \dots x^{A_n} \psi_{A_1 \dots A_n}(x') = \sum_{J=0}^n \binom{n}{J} \frac{\Gamma(l_n - 2 - n)}{\Gamma(l_n - 2 - n + J)} \left[ -\frac{1}{2}(x - x')^2 \right]^J \cdot (x - x')^{\alpha_1} \dots (x - x')^{\alpha_J} \partial'_{\mu_1 \dots} \partial'_{\mu_J \dots} O^{\mu_1 \dots \mu_J}_{\alpha_1 \dots \alpha_{n-J}}(x').$$

Finally, taking  $x' = 0$ , from eq. (16) one obtains <sup>(14)</sup>

$$(21) \quad A(x)B(0) = \sum_{n=0}^{\infty} \left( \frac{1}{x^2} \right)^{\frac{1}{2}(l_A + l_B + n - l_n)} c_n^{AB} \int_0^1 du u^{\frac{1}{2}(l_A - l_B + i_n + n) - 1} (1 - u)^{\frac{1}{2}(l_B - l_A + l_n - n) - 1} \cdot x_{u_A}^{A_1} \dots x_{u_A}^{A_n} \left[ {}_0F_1 \left( l_n - 1; \frac{-x^2}{4} u(1-u)(u^{-2} \square + 2iu^{-1}\pi \cdot \partial - \pi^2) \right) O \right]_{A_1 \dots A_n}(ux),$$

where  $x_{u_A} = \{(1-u)x_\mu, \frac{1}{2}[1 + (1-u)^2 x^2], \frac{1}{2}[1 - (1-u)^2 x^2]\}$ .

### 3. - Properties of the expansion.

The expansion obtained, eqs. (16) and (21), is conformally covariant, as it is evident from its derivation from the form in eq. (1). The form obtained here is particularly interesting since it reduces directly to the light-cone expansion of ref. <sup>(4,5)</sup> at  $x^2 = 0$ , where the  ${}_0F_1$  function becomes  ${}_0F_1(l_n - 1; 0) = 1$  and one can write

$$x^{A_1} \dots x^{A_n} \psi_{A_1 \dots A_n}(x') = (x - x')^{\alpha_1} \dots (x - x')^{\alpha_n} O_{\alpha_1 \dots \alpha_n}(x');$$

nonleading terms in  $x^2$  can simply be obtained by expanding  ${}_0F_1$  in a power series in  $x^2$ . We now come to an interesting selection rule, which one obtains by noting that eq. (20) is clearly unacceptable for  $l_n = 2 + n$ , *i.e.* for «canonical dimensions» of nonscalar representations. The reason is that eq. (7) cannot be imposed for  $l_n = 2 + n$  unless  $O_{\alpha_1 \dots \alpha_n}(x)$  is conserved (compare with the necessary and sufficient condition for canonical dimensions given in ref. <sup>(4)</sup>), but then all components in eq. (19) vanish. One can show <sup>(15)</sup> that the pathology of the components in eq. (19) for  $l_n = 2 + n$  is reflected in the circumstance that the homogeneous set of equations giving the coefficients of the contribution of a representation of spin  $n$  to the expansion in eq. (21) has, for

<sup>(13)</sup> A detailed presentation will be given in a forthcoming paper.

<sup>(14)</sup> A simpler form of this expansion will be given elsewhere.

<sup>(15)</sup> The pathology is associated to the following degeneracy of the representations of the stability algebra at  $x = 0$ : for  $l_n = 2 + n$  both  $O_{\alpha_1 \dots \alpha_n}(0)$  and  $\partial^{\alpha_1} O_{\alpha_1 \dots \alpha_n}(0)$  commute with  $K_\lambda$  (see the theorem of ref. <sup>(4)</sup> referred to in the text).

$l_n = 2 + n$ , eigensolutions only for  $l_A = l_B$  <sup>(16)</sup>. A complete discussion of the properties of representations with canonical dimension will be given in a forthcoming paper. We also observe that this case includes the free-field theory, *i.e.* conserved four-tensors (see ref. <sup>(4)</sup>). Therefore the conformal covariance of eq. (21) implies that the contribution of spin  $n$  occurs only for  $l_A = l_B$  <sup>(17)</sup>. For  $l_A = l_B$  eq. (16) (we actually take  $A = B$ ) becomes

$$(22) \quad A(x)A(0) = \sum_{n=0}^{\infty} \left(\frac{1}{x^2}\right)^{l_A-1} c_n \int_0^1 du [u(1-u)]^n \exp [ux \cdot \partial] \cdot {}_0F_1 \left( n + 1; \frac{-x^2}{4} \square u(1-u) \right) x^{\alpha_1} \dots x^{\alpha_n} O_{\alpha_1 \dots \alpha_n}(0).$$

Finally we show how the covariant expansion can be derived from the three-point function. We limit ourselves to the scalar contribution to the expansion (for simplicity) and write

$$(23) \quad A(x)B(0) = \left(\frac{1}{x^2}\right)^{\frac{1}{2}(l_A+l_B-l)} \int_0^1 du u^{\frac{1}{2}(l_A-l_B+l)-1} (1-u)^{\frac{1}{2}(l_B-l_A+l)-1} \cdot \exp [ux \cdot \partial] {}_0F_1 \left( l_n - 1; \frac{-x^2}{4} \square u(1-u) \right) O(0).$$

On the other hand (see for instance ref. <sup>(6)</sup>),

$$(24) \quad \langle 0|C(y)A(x)B(0)|0\rangle = C_{ABC} \left[ \frac{1}{(y-x)^2} \right]^{\frac{1}{2}(l_C+l_A-l_B)} \left[ \frac{1}{x^2} \right]^{\frac{1}{2}(l_A+l_B-l_C)} \left[ \frac{1}{y^2} \right]^{\frac{1}{2}(l_C+l_B-l_A)}.$$

We use the identity <sup>(18)</sup>

$$(25) \quad \left[ \frac{1}{y^2} \right]^{\frac{1}{2}(l_C+l_B-l_A)} \left[ \frac{1}{(y-x)^2} \right]^{\frac{1}{2}(l_C+l_A-l_B)} \propto \int_0^1 du u^{\frac{1}{2}(l_C+l_A-l_B)-1} \cdot (1-u)^{\frac{1}{2}(l_C+l_B-l_A)-1} [(y-ux)^2]^{-l_C} \left[ 1 + x^2 \frac{u(1-u)}{(y-ux)^2} \right]^{-l_C} = \int_0^1 du u^{\frac{1}{2}(l_C+l_A-l_B)-1} (1-u)^{\frac{1}{2}(l_C+l_B-l_A)-1} \sum_{h=0}^{\infty} \frac{1}{h!} \frac{\Gamma(l_C+h)}{\Gamma(l_C)} (-x^2)^h [u(1-u)]^h \left[ \frac{1}{(y-ux)^2} \right]^{l_C+h}.$$

<sup>(16)</sup> It can be shown that for  $l_n = 2 + n$  the divergences of the tensors  $O_{\alpha_1 \dots \alpha_n}(x)$  can be added as an independent irreducible representation starting from  $\partial^\alpha O_{\alpha \alpha_1 \dots \alpha_{n-1}}(x)$ . This is a consequence of the footnote <sup>(15)</sup>. In the covariant formalism this means that the  $(n-1)$ -th tensor  $\partial^A \psi_{AA_1 \dots A_{n-1}}(\eta)$  is a genuine tensor, *i.e.* it satisfies the two supplementary conditions and its components can be evaluated as in eq. (19).

<sup>(17)</sup> This result can also be understood from translation invariance on Hermitean basis. The expansions for  $A(x)B(0)$  and  $B(x)A(0)$  can be related (see ref. <sup>(6)</sup>) by a sequence of translation,  $x \rightarrow -x$  Hermitean conjugation. Then  $l_A \leftrightarrow l_B$ . However, the nonleading terms proportional to divergences have under this sequence of operations behaviour opposite to that of the terms proportional to  $\square$  (d'Alembertian): they must therefore have a factor  $l_A - l_B$ .

<sup>(18)</sup> The integral representation in eq. (25) is nothing but a Riemann-Liouville fractional integral (see *Bateman Manuscript Project, Tables of Integral Transforms*, Vol. 2, p. 186). The proportionality constant turns out to be  $\Gamma(l_C)/\Gamma((l_C+l_A-l_B)/2) \Gamma((l_C+l_A-l_B)/2)$ .

We next note that <sup>(19)</sup>

$$(26) \quad \langle 0 | \square^h C(x) C(y) | 0 \rangle \propto \square^h \left[ \frac{1}{(x-y)^2} \right]^{l_C} = 4^h \frac{\Gamma(l_C + h) \Gamma(l_C - 1 + h)}{\Gamma(l_C) \Gamma(l_C - 1)} \left[ \frac{1}{(y-x)^2} \right]^{l_C+h},$$

$$(27) \quad \left[ \frac{1}{(y-ux)^2} \right]^{l_C+h} = \exp [ux \cdot \partial] \left( \frac{1}{4} \right)^h \frac{\Gamma(l_C) \Gamma(l_C - 1)}{\Gamma(l_C + h) \Gamma(l_C - 1 + h)} \langle 0 | C(y) \square^h O(0) | 0 \rangle.$$

The contribution in eq. (24) thus comes from the expansion

$$(28) \quad \int_0^1 du u^{\frac{1}{2}(l_C+l_A-l_B)-1} (1-u)^{\frac{1}{2}(l_C+l_B-l_A)-1} \exp [ux \cdot \partial] \sum_{h=0}^{\infty} \frac{1}{h!} \frac{\Gamma(l_C - 1)}{\Gamma(l_C - 1 + h)} \cdot \left( \frac{-x^2}{4} \right)^h (u(1-u) \square)^h C(0) = \int_0^1 du u^{\frac{1}{2}(l_C+l_A-l_B)-1} (1-u)^{\frac{1}{2}(l_C+l_B-l_A)-1} \cdot \exp [ux \cdot \partial] {}_0F_1 \left( l_C - 1; \frac{-x^2}{4} u(1-u) \square \right) C(0),$$

which coincides with that in eq. (23).

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<sup>(19)</sup> One has  $l = l_C$  from a selection rule of the conformal algebra on two-point function.