COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-71/99
22 Dicembre 1971

T. Letardi and A. Turrin: ROTATING IN EXTREMIS IN A SYNCHROTRON THE VERTICAL POLARIZATION OF THE CIRCULATING ELECTRON BEAM INTO THE HORIZONTAL PLANE BY EXCITING AN IMPERFECTION RESONANCE. -

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(Submitted to Nuclear Instr. and Meth. for publication)

ABSTRACT. -

It is suggested that the vertical polarization of the circulating beam in a synchrotron be turned in the horizontal plane at an energy corresponding to an imperfection resonance of the magnetic moment motion. The evolution in time of the polarization vector is investigated when the radiation loss causes the electrons to approach the resonance energy and to spiral into a suitable forcing perturbation.

Particular attention is given to maintain the spin motion essentially unchanged before the particles leave the phase stable position. It is expected that the spilt beam will have at the resonance radius at each azimuthal location a known direction of the final polarization. The possibility of using such a method on the Frascati 1 GeV Constant Gradient Electron Synchrotron is considered, and it is shown that it should be promising enough to be worth trying.

1. - INTRODUCTION. -

Theoretical considerations[1, 2, 7] suggest the possibility of accelerating, in Electron Synchrotrons, electron beams whose polarization is aligned along the main magnetic field without significant polarization loss. One of the next steps is then to obtain circularly polarized photons by producing longitudinally polarized electrons.

The most straight-forward way to rotate 90^o the direction of the polarization vector consists in deflecting vertically[3] the beam, once extracted. The corresponding deflection angle is given by

$$\delta = \frac{90^o}{1 + \gamma G} , \qquad (1)$$

where γ is the ratio of the total energy E of the electron to its rest energy $m_o c^2$, and $G = \frac{g}{2} - 1 \simeq 1.16 \times 10^{-3}$ is the anomalous part of the magnetic moment (g is the gyromagnetic ratio of the electron).

It follows that δ goes from $\sim 6^o$ for 6 GeV electrons (DESY) to $\sim 28^o$ for 1 GeV electrons (Frascati Synchrotron).

Practical difficulties become therefore stronger and stronger as the final energy decreases.

To overcome any vertical deflection requirement we suggest to bring abruptly at the end of the acceleration cycle the spin vectors of the circulating beam into the horizontal plane by producing artificially a selected resonance between the proper frequency of the magnetic moment of the particle and the periodic perturbation due to a small localized magnetic field, deliberately introduced in a region of small azimuthal width.

In the lab. reference frame the perturbing field is a stationary one. This field is seen by the circulating particle as an impulsive periodic field.

2. - GENERAL CONSIDERATIONS. -

In a reference frame Σ which is attached to the particle and which has one of the coordinate axes pointing in the direction of motion, the electronic spin motion in the field of a Synchrotron is described in first-order approximation by the equation(1, 2, 4, 5, 6, 7)

$$(2) \qquad \dot{\bar{S}} = \omega_p \bar{S} \times \bar{k} + \omega_\perp \frac{B_r}{B_o} \bar{S} \times \bar{n} + \omega_\parallel \frac{B_\theta}{B_o} \bar{S} \times \bar{w} + \frac{\omega_p}{\omega_c} \frac{\dot{z}}{R\Lambda} \bar{S} \times \bar{w},$$

where :

$\bar{S}$ is the polarization vector of the particle ; the dot denotes differentiation with respect the (lab) time ;

$\bar{w}, \bar{n}, \bar{k}$ are the space unit vectors ($\bar{w}$ is the space unit vector pointing in the direction of motion) ;

$\bar{B}_o = \bar{k} B_o$ is the magnetic field at the equilibrium orbit ;

$\bar{B}_r$ and $\bar{B}_\theta$ denote the periodic magnetic fields seen by the particle;

z is the vertical displacement of the particle from the equilibrium orbit ;

$R\Lambda$ is the mean radius (R is the bending radius) ;

ω_c is the angular velocity of revolution ;

$$(3) \quad \begin{cases} \omega_p = \omega_c \gamma G & \text{is the angular velocity of the spin precession about the direction of the guiding field } B_o \text{ ;} \\ \omega_\perp = (1 + \gamma G)\, \omega_c & \\ \omega_\parallel = \dfrac{g}{2}\, \omega_c \,. & \end{cases}$$

The absolute value of S is a constant of motion, say

$$(4) \qquad |\bar{S}| = 1 \,.$$

In equation (2) the first term corresponds to the unperturbed spin motion in a steady field and all the remaining terms are small periodic perturbations.

The analysis(1, 2, 7) of equation (2) shows that in an unperturbed Synchrotron the only resonance of possible concern is the "intrinsic" depolarization resonance (i. e. due to the focusing structure)

(5) $$\gamma G = Q_z ,$$

where Q_z is the number of vertical betatron oscillations per turn. Such a resonance leads to relatively small polarization loss for DESY[(1)], for the Frascati Synchrotron[(2)] and for ARUS[(7)].

During the acceleration cycle, other types of resonance may be excited, and these are the so-called "imperfection" resonances. These resonances are found when one of the periodic perturbations in equation (2) has the form

(6) $$A_n \cos (n\omega_c t + \varphi_n) \qquad (n = 1, 2, 3, \ldots)$$

and may occur at energies

(7) $$\gamma_{res} G = n , \quad \text{or} \quad E_{res} = n \frac{m_o c^2}{G} = n \; 440 \text{ MeV} .$$

The corresponding magnetic field perturbations felt by the particles must be periodic in azimuth, and these are the resonances we are interested in.

The problem we have in mind is the following:
Let us suppose that the end of the acceleration cycle is at an energy slightly greater than one of the energies (7), i.e. at an integer multiple of $m_o c^2/G = 440$ MeV. In these conditions the angular velocity value of the spin precession about the main field B_z is slightly greater than $n\omega_c$.

Suppose now that the above mentioned local perturbation (a radial field B_r) is produced (fig. 1) by energizing a single non-linear element placed inside the equilibrium orbit in a straight section. B_r has its maximum value at a radius $r = R - a$ corresponding to $E = E_{res}$. Because of the strong non-linearity introduced, as long as the equilibrium orbit is close to the central orbit, the $(B_r \cdot l)$ field integral is unable to affect the spin motion in a considerable manner.

When the electrons are brought out of synchronism with the R.F. System (by slowly reducing the R.F. peak voltage), their equilibrium orbit contract. After an electron is lost from the phase stable position in the R. F. System, it moves toward the centre of the Machine, as a consequence of the energy losses. Thus, the energy E of the spiralizing electron approaches the resonance value E_{res}, and simultaneously the perturbing action of $(B_r(r) \cdot l)$ becomes increasingly more powerful. In this way the magnetic moment of every particle is eventually bent in the horizontal plane in the course of a few revolutions providing the perturbation field integral sufficiently large to do this.

The behaviour of the spin vector $\bar{S}$ in the reference frame Σ during spiralization is represented schematically in fig. 2.

In the course of one revolution in the machine, the number of spin precessions about $\bar{k}$ is slightly greater than n, and $\Delta\varphi$ is the corresponding advance of angular precession per revolution. As the particle passes through the perturbed field region, the spin vector rotates about $\bar{n}$, and $\Delta\theta$ is the corresponding angular kick received per revolution by the magnetic moment. $\Delta\varphi$ is decreasing and $\Delta\theta$ is increasing at every revolution.

We intend to have the polarization vectors in the horizontal plane at a radius corresponding to $E = E_{res}$ and $r = R - a$. At this radius, the number of precession per revolution is just n in the reference frame Σ and therefore at given azimuthal positions the polarization vector is aligned with the motion direction.

3. - THE PERTURBING MAGNET. -

The radial field B_r may be created by means of a pair of paral lel equal strips carrying a relatively large current (fig. 1).

The strips must be placed symmetrically with respect to the Synchrotron's magnetic median plane.

The expression for the magnetic field B_r for $z = 0$ is :

(8) $$B_r(\Delta r) = \frac{\mu_o I}{2\pi h} \Sigma \text{ artg}$$

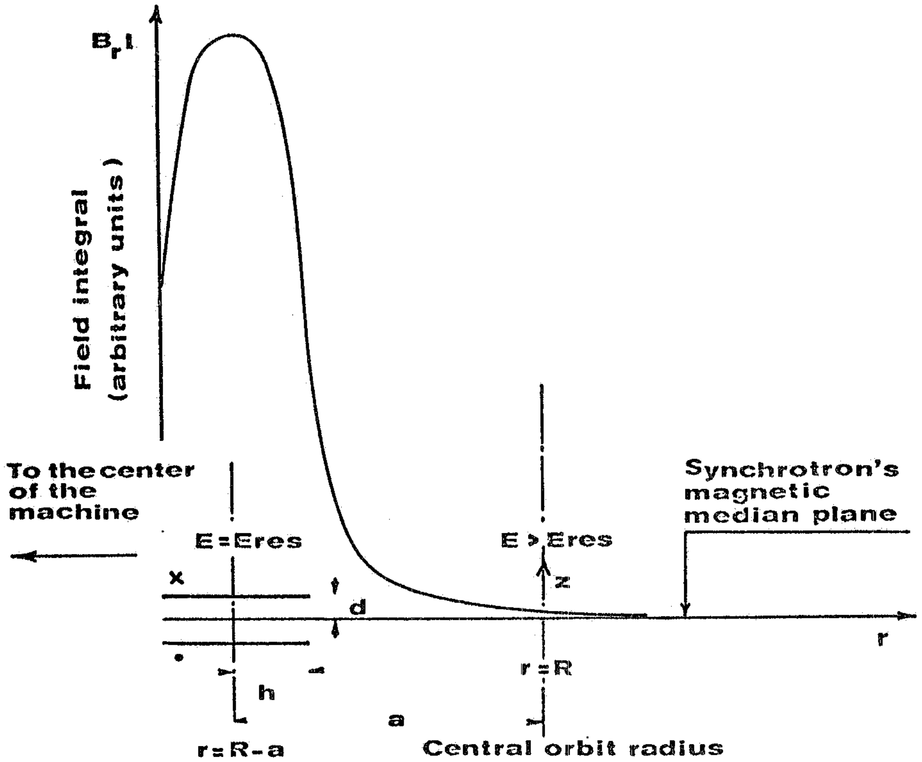

FIG. 1 - Shape of the perturbing radial magnetic field (on the median plane). r denotes the radial position. The cross section of the perturbing magnet consisting of a pair of parallel equal current strips (as described in sect. 3) is represented. The signs x and • represent current flows of opposite directions perpendicular to the plane of the figure. The length l of each strip is much greater than its width 2h.

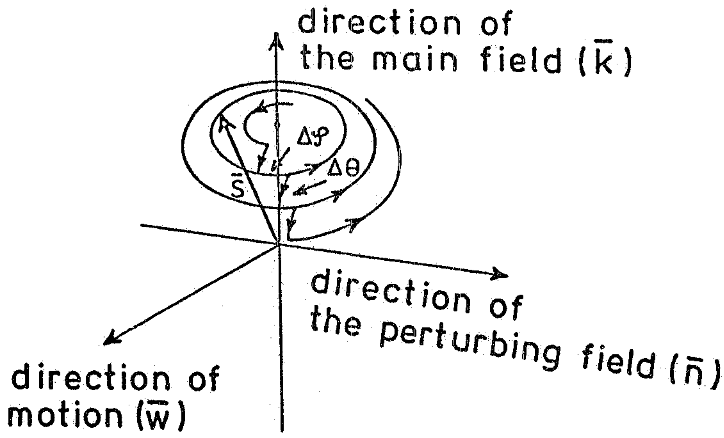

FIG. 2 - Sketch of the spin motion in the reference frame Σ, when the particle falls under the influence of the perturbing field B_r. The magnitudes of $\Delta\varphi$ and $\Delta\theta$ are exaggerated for the sake of clearness.

where

(8') $$\Sigma \text{ artg} = \text{artang}\,\frac{\Delta r + a + h}{d} - \text{artang}\,\frac{\Delta r + a - h}{d} \;;$$

and I is the total current in each strip;

2h is the width of each strip;

a is the distance of the axis of the strip pair from the central orbit;

2d is the spacing between the strips;

$\Delta r = r - R$.

The shape of $B_r(\Delta r)$ is outlined in fig. 1.

4. - ANALYSIS OF THE POLARIZATION VECTOR MOTION IN THE PERTURBED MAGNETIC FIELD. -

To investigate the polarization vector motion under the action of the perturbing field B_r near the $\gamma G = n$ resonance, it is sufficient to leave in eq. (2) only the first perturbing term.

Equation (2) becomes

(9) $$\dot{\bar{S}} = \omega_c \gamma G \bar{S} \times \bar{k} + \omega_r \bar{S} \times \bar{n}$$

where

(10) $$\omega_r = (1 + \gamma G)\,\omega_c\,\frac{B_r(\Delta r, t)}{B_o} \;.$$

Because a lumped perturbation has been considered, $\omega_r \neq 0$ only when a particle passes through the non-linear perturbing field region.

From here downwards the evolution in time of $\bar{S}$ will be studied in a reference frame(6) Σ' which is attached to the particle and which rotates about the main field direction with angular velocity

(11) $$\Omega = n\omega_c = \gamma_{res} G\, \omega_c$$

(see equation (7)).

Equation (9), when trasformed to this new rotating frame, becomes

$$\dot{\overline{S}} = \Delta\omega \overline{S} \times \overline{k} + \omega_r \overline{S} \times \overline{j} . \tag{12}$$

Here,

$$\Delta\omega = (\gamma G - n)\,\omega_c \tag{12'}$$

and $\overline{j}$ is a unit space vector.

The stability conditions of the spin motion for the particles circulating close to the central orbit (before they slip out of phase stability) are investigated first. We will assume $\Delta\omega \simeq$ constant. Besides, one sees readily that the spin flip-producing resonance is driven essentially by the zero-th harmonic Fourier component of ω_r, namely

$$\langle \omega_r \rangle = \frac{1}{T}\int_0^{\tau} \omega_r \, d\tau = \omega_r \frac{\tau}{T} , \tag{13}$$

where T is the revolution period of the particle and τ denotes his transit time through the perturbed region ($\tau << T$). Neglecting periodic terms of ω_r (which cannot be responsible for the resonant spin flip) equation (12) expressed in terms of the three orthogonal components of the polarization vector in the reference frame Σ' becomes

$$\begin{cases} \dot{S}_u = \Delta\omega S_v - \langle \omega_r \rangle S_z \\ \dot{S}_v = - \Delta\omega S_u \\ \dot{S}_z = \langle \omega_r \rangle S_u \end{cases} \tag{14}$$

where S_u and S_v are the horizontal components of $\overline{S}$.

The system of equations (14) must be solved with the initial conditions $S_u(0) = 0$; $S_v(0) = 0$; $S_z(0) = 1$. One is primarily interested in the variation in time of the vertical component S_z of the polarization vector.

Obviously, both the quantum-mechanical[6] and the classical[2] formulation of the problem contained in system (14) lead to the same expression for S_z: The expectation value of the spin in the z-direction is expressed by

(15) $$S_z = 1 - 2\left|g\right|^2 ,$$

where g is the solution of the second order differential equation

(15') $$\ddot{g} - i\,\Delta\omega\,\dot{g} + \frac{\langle\omega_r\rangle^2}{4}\,g = 0$$

with the initial conditions $g(0) = 0$; $\dot{g}(0) = \dfrac{\langle\omega_r\rangle}{2}$.

Assuming $\langle\omega_r\rangle$ = constant, one obtains for S_z

(16) $$S_z = 1 - \frac{\langle\omega_r\rangle^2}{(\Delta\omega)^2 + \langle\omega_r\rangle^2}\left[1 - \cos\left(\sqrt{(\Delta\omega)^2 + \langle\omega_r\rangle^2}\; t\right)\right]$$

so that

(16') $$\frac{(\Delta\omega)^2 - \langle\omega_r\rangle^2}{(\Delta\omega)^2 + \langle\omega_r\rangle^2} \leq S_z \leq 1 .$$

From (16') one can conclude that the vertical polarization remains unaffected when

(16") $$\langle\omega_r\rangle^2 \ll (\Delta\omega)^2 .$$

We will now investigate the polarization vector behaviour when the radiation loss causes an electron to spiral into the above-mentioned perturbed magnetic field, after being lost from the phase stable position.

Let us assume that at every passage of the particle through the perturbing field region the coordinate axes of Σ and Σ' coincide. Let us image, for concreteness, that at every passage the u-axis is directed along the direction of motion (and the v-axis is directed along the direction of the perturbing field). From equation (12) it follows that in the part of the orbit that lie within the unperturbed azimuthal region (where $\omega_r = 0$) the polarization vector motion is described by the system

$$(17) \qquad \dot{S}_u = \Delta\omega S_v \; ; \qquad \dot{S}_v = - \Delta\omega S_u \; ; \qquad \dot{S}_z = 0 \; .$$

Likewise, in the part of the orbit that lie within the perturbed azimuthal region (where $\omega_r \neq 0$) the polarization vector evolution is governed by the system

$$(18) \qquad \dot{S}_u = - \omega_r S_z \; ; \qquad \dot{S}_v = 0 \; ; \qquad \dot{S}_z = \omega_r S_u \; .$$

The solutions of systems (17) and (18) are the following: If $\bar{S}_{in}$ denotes the value of $\bar{S}$ at the entrance position of the $\omega_r = 0$ region, one sees that at the end of such a region

$$(17') \qquad \begin{cases} S_{u_{out}} = S_{u_{in}} \cos(\Delta\omega \cdot T) + S_{v_{in}} \sin(\Delta\omega \cdot T) \\ S_{v_{out}} = - S_{u_{in}} \sin(\Delta\omega \cdot T) + S_{v_{in}} \cos(\Delta\omega \cdot T) \\ S_{z_{out}} = S_{z_{in}} \; . \end{cases}$$

In the same manner, at the exit position of the $\omega_r \neq 0$ region $\bar{S}_{out}$ is given in terms of $\bar{S}_{in}$ by the transformation

$$(18') \qquad \begin{cases} S_{u_{out}} = S_{u_{in}} \cos(\omega_r \tau) - S_{z_{in}} \sin(\omega_r \tau) \\ S_{v_{out}} = S_{v_{in}} \\ S_{z_{out}} = S_{u_{in}} \sin(\omega_r \tau) + S_{z_{in}} \cos(\omega_r \tau) \; . \end{cases}$$

After passage through the two successive field regions, i. e. after one complete revolution of the particle, $\bar{S}_{out}$ is connected with $\bar{S}_{in}$ by the transformation

$$(19) \qquad \begin{vmatrix} S_{u_{out}} \\ S_{v_{out}} \\ S_{z_{out}} \end{vmatrix} = \begin{vmatrix} \cos(\Delta\theta) & 0 & -\sin(\Delta\theta) \\ 0 & 1 & 0 \\ \sin(\Delta\theta) & 0 & \cos(\Delta\theta) \end{vmatrix} \begin{vmatrix} \cos(\Delta\varphi) & \sin(\Delta\varphi) & 0 \\ -\sin(\Delta\varphi) & \cos(\Delta\varphi) & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} S_{u_{in}} \\ S_{v_{in}} \\ S_{z_{in}} \end{vmatrix} ,$$

where (see fig. 2)

(19')
$$\Delta\varphi = \Delta\omega \cdot T \, ; \qquad \Delta\theta = \omega_r \tau \; .$$

Starting with proper initial conditions, the motion of the spin vector for successive revolutions may be followed by applying at each revolution the transformation (19). At each step of the calculation, the $\Delta\varphi$ and $\Delta\theta$ values must be changed because :

- for the $\Delta\varphi$ value : the energy value of the particle decreases by a small amount corresponding to the radiation loss per revolution and, consequently,
- for the $\Delta\theta$ value : the equilibrium orbit moves a small distance inward, corresponding to the orbit contraction per revolution.

The dependence of $\Delta\varphi$ on the number N of revolution after spiralization is started is

(20)
$$\Delta\varphi = 2\pi n \left[\frac{L}{E_{ris}} \left(\frac{a}{\sigma} - N \right) + \frac{\Delta E}{E} \right] ,$$

where

L is the radiation loss per revolution at $E \simeq E_{ris}$;

σ is the corresponding closed orbit contraction per revolution (the center of the resonance is assumed to lie at $\Delta r = -a = -N_{max}\sigma$);

$\frac{\Delta E}{E}$ is the fractional displacement in energy of an off-momentum particle.

$\Delta\theta$ is expressed by

(21)
$$\Delta\theta \cong (1 + \gamma_{res} G) \frac{B_r(\Delta r) \cdot 1}{B_o R} ,$$

where

(21') $\Delta r = - N\sigma$ in the absence of radial betatron oscillations ;

$B_r(\Delta r)$ is given by equations (8) and (8') ;

B_oR is the magnetic rigidity;
l is the length of each current strip.

5. - NUMERICAL RESULTS FOR THE FRASCATI ELECTRON SYNCHROTRON. -

The analysis outlined above has been applied for computing the whole process of turning $\bar{S}$ into the horizontal plane for the Frascati Electron Synchrotron. It is felt that the promising numerical results obtained here could be achieved for other Electron Accelerators, such as Alternating Gradient Synchrotrons, too.

The maximum value of γG that can be reached in the 1 GeV Frascati Synchrotron is γG = 2.25, so that the $\gamma G = n = 2$ resonance must be used as the most advantageous imperfection resonance for our purpouses. The corresponding resonance energy is E_{res} = 880 MeV. The following parameters have been used in the calculations :

Beam parameters at E = 880 MeV	Strip pair parameters
$\pm(\frac{\Delta E}{E})_{max} = \pm 1. \times 10^{-3}$	a = 6.5 cm
L = 15 keV	h = 1.5 cm
σ = 0.17 mm	d = 0.5 cm .

Thus, the beam must be accelerated up to the energy $E = E_{res} + \frac{L\,a}{\sigma} =$ = 885.75 MeV, and the corresponding number of revolutions available to deliver then the spin vector in the horizontal plane is N_{max} = 383.

The optimum I· l value required for a particle leaving the stable position with $\Delta E/E = 0$ ("best" particle) is

$$I \cdot l = 600 \text{ Amp} \cdot \text{m} .$$

In fig. 3 the evolution of the spin vector of the "best" particle during spiralization is shown.

The ratio $\langle \omega_r \rangle / \Delta\omega$ has been computed (at $\Delta r = -1$ cm), and is $< 0.6 \times 10^{-2}$, then condition (16") is largerly satisfied for particles that are not jet lost from the phase stable position during the spill-out time.

Particles having different momenta that are lost from the phase stable position have different spin vector evolutions, as shown in figs. 4 and 5, that refer to the two limiting cases $\pm(\Delta E/E)_{max}$. At $\Delta r =$ $= -a$, the polarization vector of the "best" particle will have a known direction at each azimuthal position (the direction at the location of the perturbing magnet can be easily obtained on inspection of fig. 3).

At $\Delta r = -a$, spin vectors of off-momentum particles having different $\Delta E/E$ values fan out into the small aperture of a cone having its axis coincident with the spin direction of the "best" particle. The polarization of these particles, referred to this direction, has been computed and the results are shown in fig. 6.

A single perturbing magnet having the calculated strength value $(I \cdot l) = 600$ Amp · m gives a strong distortion of the closed orbit for the vertical betatron motion in the Frascati Synchrotron. This distortion can be greatly reduced by using - instead of a single perturbing magnet - two perturbing magnets, having strength $\frac{1}{2}(I \cdot l) = 300$ Amp · m each, placed in two opposite straight sections of the Synchrotron and excited with the same polarity.

The corresponding maxima of the distorted closed orbit occur at the azimuthal positions midway between the perturbing magnets locations and are expressed by

$$(22) \qquad z_{max} \cong \frac{R}{Q_z} \frac{\frac{1}{2}(B_r l)}{B_o R} \frac{1}{2 \sin(Q_z \frac{\pi}{2})},$$

where $R = 360$ cm is the equilibrium orbit radius, and $Q_z \cong 0.9$ is the number of vertical betatron oscillations per revolution. At $\Delta r =$ $= -a$, $z_{max} \simeq +7$ mm.

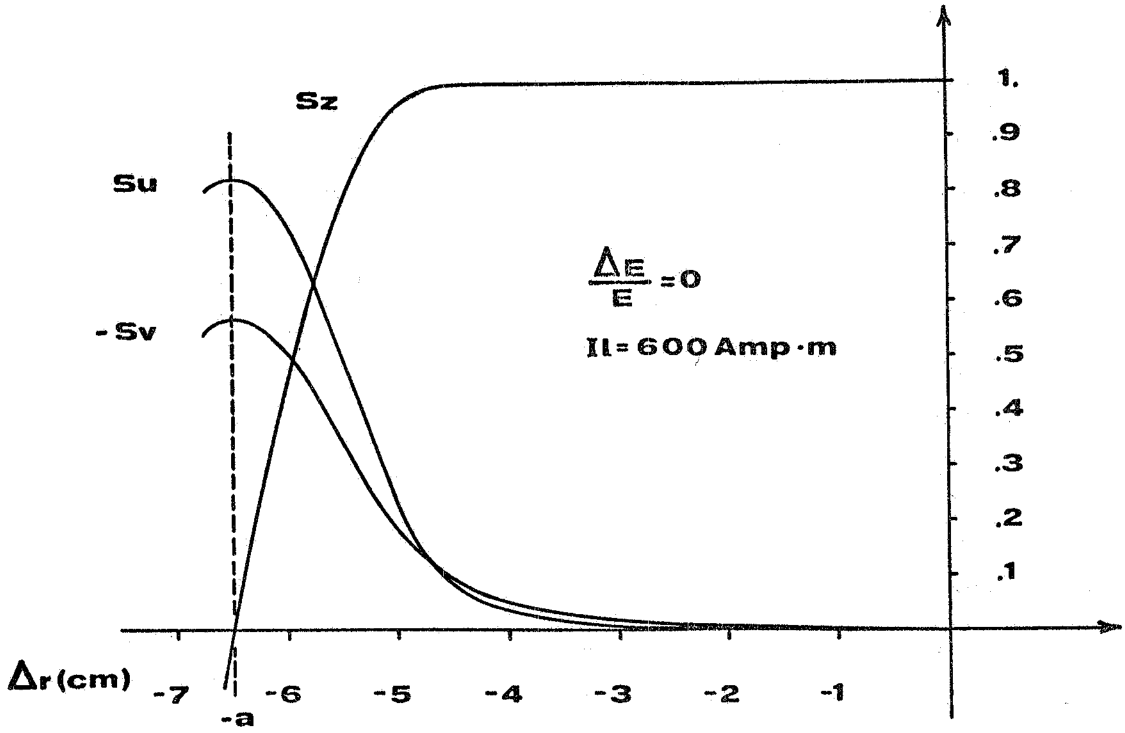

FIG. 3 - Spin vector evolution of the "best" particle at the azimuthal location of the perturbing field.

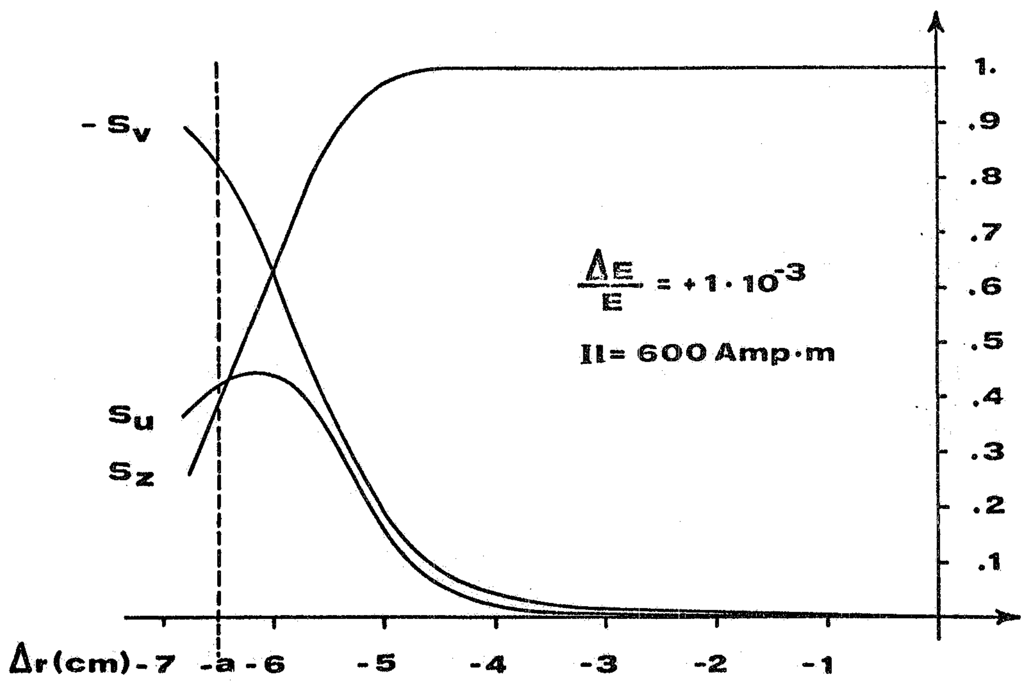

FIG. 4 - Spin vector evolution of an off momentum particle ($\Delta E/E = +1. \times 10^{-3}$).

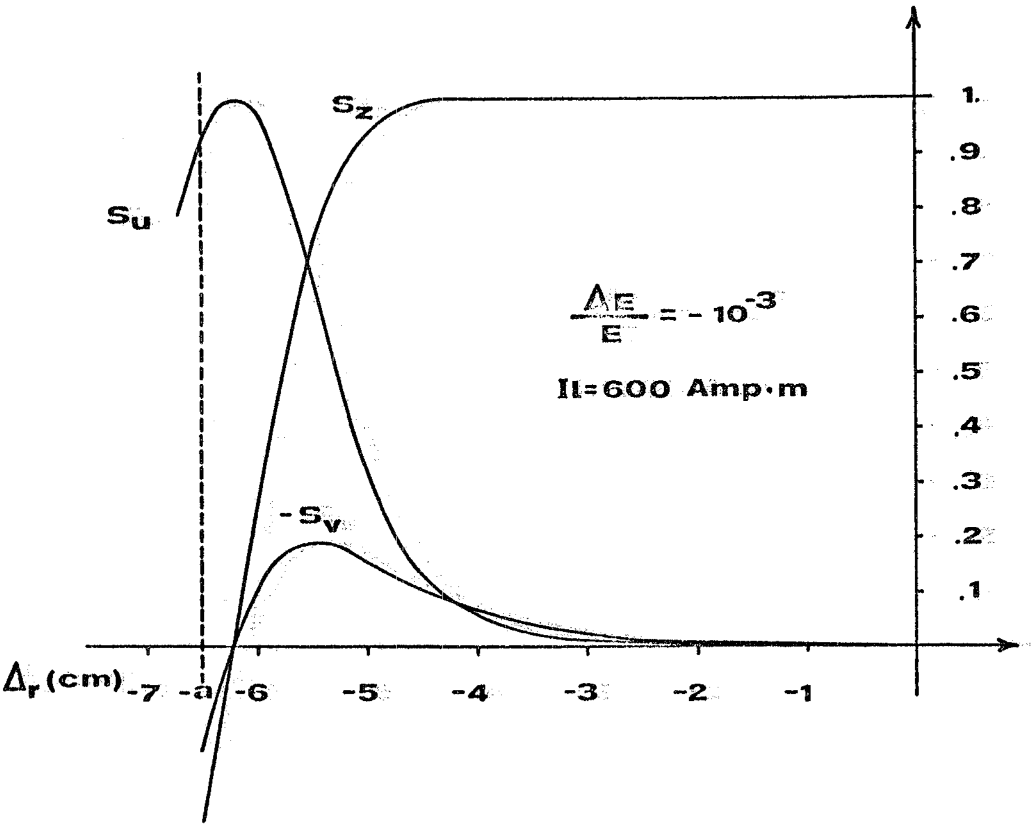

FIG. 5 - Spin vector evolution for $\Delta E/E = -1. \times 10^{-3}$.

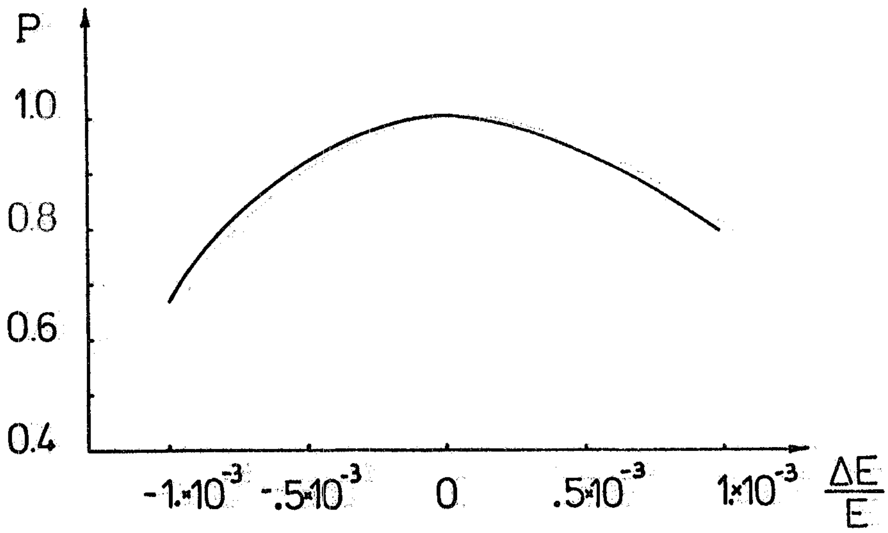

FIG. 6 - Off momentum particles polarization (at $\Delta r = -a$) computed with respect to the polarization direction of the "best" particle.

The minima of the distorted closed orbit occur at the longitudinal mid-points of the perturbing lenses and are expressed by

(22')
$$z_{min} = z_{max} \cos (Q_z \frac{\pi}{2}) .$$

At $\Delta r = -a$, $z_{min} \simeq +1$ mm.

Finally, it must be pointed out that the problem of the polarization vector motion has been approached by considering the action of the perturbing magnetic filed/(s) $B_r(\Delta r)$ during spiralization in the absence of radial betatron oscillations. In the actuality the radial position Δr of a particle at the location of either magnetic perturbation is given by

(23)
$$\Delta r \cong -M \frac{\sigma}{2} + x_o \cos (Q_x M \pi + \psi)$$

where

- M is the number of half revolutions after spiralization is started;
- x_o is the amplitude of the radial betatron oscillation;
- $Q_x \cong 2/3$ is the number of radial betatron oscillations per revolution;
- ψ is the initial phase.

It will be necessary to examine - wery briefly - the situation, to see if our previous conclusions need any revision.

We will do so in the resonant conditions ($\Delta\varphi \rightarrow 0$) and in the assumption that $Q_x = 2/3$. The angular kick recevied by the magnetic moment from the two perturbing magnets in the course of $1 + \frac{1}{2}$ revolutions is

(24)
$$\sum_{0}^{2} {}_m \Delta\theta_m \cong (1+\gamma G) \frac{\frac{1}{2} 1}{B_o R} \sum_{0}^{2} {}_m B_r (-M \frac{\sigma}{2} + x_o \cos (\frac{2}{3} m\pi + \psi)).$$

We expand $B_r(\Delta r)$ in the neighbourhood of $-M\frac{\sigma}{2}$, as follows,

(25)
$$B_r(-M \frac{\sigma}{2} + x_o \cos (\frac{2}{3} m\pi + \psi)) =$$

$$= B_r(-M\frac{\sigma}{2}) + \left(\frac{dB_r}{dr}\right)_{\Delta r=-M(\sigma/2)} x_o \cos\left(\frac{2}{3}m\pi + \psi\right) + \text{higher order terms.}$$

Substituting (25) in (24) one finds

$$\sum_0^2 {}_m \Delta\theta_m \cong (1+\gamma G)\frac{\frac{1}{2}\,1}{B_o R}\left[3B_r(-M\frac{\sigma}{2}) + \right. \tag{24'}$$

$$\left. + \left(\frac{dB_r}{dr}\right)_{\Delta r=-M(\sigma/2)} x_o \sum_0^2 {}_m \cos\left(\frac{2}{3}m\pi + \psi\right)\right].$$

$\sum_0^2 {}_m \cos\left(\frac{2}{3}m\pi + \psi\right) \equiv 0$ whatever the initial phase value, so that we may conclude that in first order approximation the spin motion is unaffected by the radial betatron motion.

6. - CONCLUSION. -

It has been shown that an imperfection resonance can be forced in an Electron Synchrotron in such a way that the initial vertical polarization of the beam results without significant loss a longitudinal one at the resonance radius, and at given symmetrical azimuthal locations.

ACKNOWLEDGEMENTS. -

Sincere thanks are due to Prof. G. Bologna and to Prof. A. Reale for their interest in this effort.

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