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G. Pancheri-Srivastava^(o) and Y. N. Srivastava⁽⁺⁾: A MODEL
DIFFRACTION TERM^(x).

In this letter we wish to present a modification of the Del Giudice-Veneziano ansatz⁽¹⁾ for the Pomeron (diffraction term). This new model, besides enjoying all the good properties of the old one (i. e. crossing symmetry, proper large angle behaviour etc.), now gives only small corrections to the dual resonant model meson-meson scattering lengths and hence does not ruin previous agreement with experiments. We have also constructed off shell Pomeron amplitudes and imposed continuous PCAC in a manner analogous to Arnowitt et al. ⁽²⁾ for the dual Veneziano terms. If we employ t-channel helicity conservation for diffraction dissociation processes (e. g. $\pi N \rightarrow A_1 N$) in agreement with recent experimental results⁽³⁾ and data on A_1 production cross section, we end up with an asymptotic prediction which seems to be borne out qualitatively by experiments.

For elastic $\pi\pi$ scattering Del Giudice and Veneziano suggested the following expression for the Pomeron term:

$$(1) \quad \begin{aligned} P(s, t) &= g \alpha'(t-t_0) H(s, t) \\ H(s, t) &= \sigma^{\alpha(s)} \Gamma(1-\alpha(s)) \Psi(1-\alpha(s), \frac{1}{2}; -g \alpha'(t-4\mu^2)) \end{aligned}$$

where α' is the universal slope of ordinary trajectories, $\alpha(s)$ the non-linear Pomeron trajectory normalized to $\alpha(0) = 1$, g and σ positive

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dimensionless constants and Ψ is the confluent hypergeometric function⁽⁴⁾. The constant t_0 was fixed by the authors to be $2\mu^2$ (μ being the pion mass) in order to eliminate a massless scalar particle. With this choice, however, the scattering lengths, as noted by the authors, turn out to be too large, a fact reflecting the absence of an Adler zero in expression (1). In the spirit of the Freund-Harari conjecture about the dual role of the Pomeron term, any candidate for such a role should possess the proper Adler zeroes if it has to be added as a background builder to the usual Veneziano amplitude which does possess such zeroes. We thus propose that expression (1) be modified as follows

$$(2) \quad \tilde{P}(s, t) = \frac{\alpha' g}{2} (2t+s-\Sigma) \sigma^{\alpha(s)} \Gamma(1-\alpha(s)) \Psi(1-\alpha(s), \frac{1}{2}; -g\alpha'(t-4\mu^2))$$

where Σ is the sum of the squares of the external masses through which the off-shell continuation at the Adler point has to be made. This is, of course, not a smooth continuation but it should be remembered that the continuation in the Lovelace-Veneziano formula was not smooth either. Consider, for instance, a typical term in $\pi\pi$ scattering

$$V(s, t) = \alpha'(2\mu^2 - s - t) B(s, t) = \alpha'(u - \Sigma + 2\mu^2) B(s, t),$$

where B is the Euler beta function. The Adler zero is therefore obtained by continuing Σ off-shell. Our modification (2) is entirely analogous.

Notice that our expression (2) has no scalar massless pole even for one particle off mass shell since, for any elastic amplitude with one particle off shell, $2t+s-\Sigma = t-u = 4p_s q_s z_s$ where z_s is the scattering angle and p_s, q_s denote the initial and final 3-momenta in the s -channel CM system.

With expression (2), the contribution to a scattering amplitude of a Pomeron trajectory exchanged in a given channel, say t , can be written in the very compact form

$$\tilde{P}_{ts} + \tilde{P}_{tu} = g \alpha' \frac{s-u}{2} \left[H(t, s) - H(t, u) \right]$$

Of course the detailed test of the model rests in obtaining phenomenological fits to the diffraction term in high energy scattering processes. Pending such tests we present below two theoretical applications of the model. The first is in calculating the contributions to scattering lengths for $\pi\pi$ and πK where the Veneziano resonance formula makes good predictions. The second is to impose the continuous PCAC constraints which give us rather interesting results.

$\pi\pi$ and πK scattering lengths :

Using the same notation as in ref. (1), we call S_I^{Pom} the Pome-

ranchuk pole contribution to the $\pi\pi$ amplitudes with pure isospin I (I = 0, 1, 2) in the s channel. It is now

$$\begin{aligned} S_0^{\text{Pom}} &= \frac{cg\alpha'}{2} \left\{ (s-u) \left[H(t, s) - H(t, u) \right] + \right. \\ &\quad \left. + (s-t) \left[H(u, s) - H(u, t) \right] + 3(t-u) \left[H(s, t) - H(s, u) \right] \right\} \\ S_1^{\text{Pom}} &= \frac{cg\alpha'}{2} \left\{ (s-u) \left[H(t, s) - H(t, u) \right] - (s-t) \left[H(u, s) - H(u, t) \right] \right\} \\ S_2^{\text{Pom}} &= \frac{cg\alpha'}{2} \left\{ (s-u) \left[H(t, s) - H(t, u) \right] + (s-t) \left[H(u, s) - H(u, t) \right] \right\} \end{aligned}$$

where the normalization is fixed so that

$$\sigma_{\text{TOT}}^{\pi\pi}(s) \xrightarrow{s \rightarrow \infty} 8\pi^2 cg\alpha' \sigma .$$

At threshold we have

$$(3) \quad S_{0,2}^{\text{Pom}} \xrightarrow[t, u \rightarrow 0]{s \rightarrow 4\pi^2} 2c\sigma z_0 \left\{ \sqrt{\pi z_0} - \sum_{n=1}^{\infty} z_0^n (A_n - B_n \sqrt{\pi z_0}) \right\}$$

$$(4) \quad S_1^{\text{Pom}} \xrightarrow[t, u \rightarrow 0]{s \rightarrow 4\mu^2} c\sigma g\alpha'(t-u) \left\{ \frac{3}{2} \sqrt{\pi z_0} - \sum_{n=1}^{\infty} z_0^n (a_n - b_n \sqrt{\pi z_0}) \right\}$$

with $z_0 = 4\mu^2 g\alpha'$, $A_n = \frac{2^{n-1}}{(2n-1)!!n}$, $B_n = \frac{1}{n!(2n+1)}$ and

$$\begin{aligned} a_n &= A_n \left\{ n+1 - 4\mu^2 \alpha'_P(0) \left[\Psi(n) - \Psi\left(\frac{1}{2}\right) \right] \right\} \\ b_n &= B_n \left\{ n + \frac{3}{2} - 4\mu^2 \alpha'_P(0) \left[\Psi\left(n + \frac{1}{2}\right) - \Psi\left(\frac{1}{2}\right) \right] \right\} \end{aligned}$$

In the above expressions, $\alpha'_P(0)$ is the slope of the Pommeranchuk trajectory which we shall put equal to $\frac{1}{2} \alpha'$ and $\Psi(z) = \left[\frac{d}{dz} \Gamma(z) \right] / \Gamma(z)$.

As in ref. (1), $(\sigma g\alpha')^{-1}$ is the scale factor in a Regge type expansion so that assuming it to be of order 1 (GeV)² we see that $\sigma g \simeq 1$. Since σ has to be larger than 1 to ensure Regge behaviour in s at fixed t of expression (1) or (2) (provided $\alpha(s) \xrightarrow{s \rightarrow \infty} -\infty$), the constant g will be less than 1 and we can neglect in eqs. (3) and (4) terms of order z_0^n with $n > 1$. We then have

4.

$$\mu a_o^{(0)} = \mu a_o^{(2)} \simeq \frac{\mu^2 \sigma_{TOT}^{\pi\pi}(\infty)}{2\pi^2} (\sqrt{\pi z_o} - z_o)$$

$$\mu a_1^{(1)} \simeq - \frac{\mu^2 \sigma_{TOT}^{\pi\pi}(\infty)}{8\pi^2} \left\{ \sqrt{\pi z_o} - \frac{4}{3} z_o \left[1 - 4\mu^2 \alpha_P'(0) \ln 2 \right] \right\}$$

To estimate the order of magnitude we have taken $\sigma_{TOT}^{\pi\pi}(\infty) = 16$ mb and computed the scattering lengths for a few g values : we thus get Table I.

TABLE I

g	1	1/2	1/4
$\mu a_o^{(0)}$	0.017	0.013	0.09
$-\mu a_1^{(1)}$	0.004	0.003	0.002

These values for, say, $g = 1/2$ represent a 10%, 15% and 25% correction to the current algebra $l=0, 1, 2$ scattering lengths and are smaller if $\sigma_{TOT}^{\pi\pi}(\infty)$ and/or g are smaller. They seem to be of the correct magnitude thus encouraging us to use expression (2) for the calculation of the Pommeranchuk contribution to πK scattering amplitude. The Pommeranchuk trajectory being exchanged only in the $l_t = 0$ channel, we can write

$$S_{1/2}^{Pom} = S_{3/2}^{Pom} = c' g \alpha' \frac{s-u}{2} \left[H'(t, s) - H'(t, u) \right]$$

where $H'(t, s) = \sigma^{\alpha(t)} \Gamma(1-\alpha(t)) \Psi(1-\alpha(t), \frac{1}{2}; -g\alpha'(s - (m+\mu)^2))$, m being the k meson mass. Again the above amplitude, when extrapolated off the mass shell in the manner already specified, has zeroes when any of the external momenta is sent to zero. Proceeding as before we get

$$(m+\mu) a_o^{(1/2)} = \frac{m\mu}{2\pi} \sigma_{TOT}^{\pi K}(\infty) \left[\sqrt{\pi x} - \sum_{n=1}^{\infty} x^n (A_n - B_n \sqrt{\pi x}) \right]$$

with $x = 4 m \mu \alpha' g$. Taking $\sigma_{TOT}^{\pi K}(\infty) = 12$ mb we again estimate the scattering lengths for different g values as shown in Table II.

We see that this contribution to πK scattering length is, for $\frac{1}{16} \leq g \leq 1$, of the same order of magnitude as the one provided by the

TABLE II

g	1	1/2	1/4	1/16
$\mu a_o^{(1/2)}$	0.017	0.013	0.01	0.005

usual Veneziano term⁽²⁾ (the current algebra value is similar). It is interesting to calculate the threshold cross section for πK scattering using our Pommeranchuk term summed with the dual Veneziano term. For $K^+\pi^-$ we obtain

$$\frac{\sigma_{K^+\pi^-}^{\text{Pom+Ven}}(\text{threshold})}{\sigma_{K^+\pi^-}^{\text{Ven}}(\text{threshold})} \approx (1 + 0.14 \sqrt{g})^2$$

which for $g \approx 1$ can considerably reduce the discrepancy between the measured value by Trippe et al. and the Veneziano model value (see footnote 16 of ref. (2)).

Continuous PCAC :

Consider for example $K\pi \rightarrow K\pi$ and $K\pi \rightarrow KA_1$ amplitudes with last π and A_1 off their mass shells. We try to relate the Pomeron parts of these amplitudes via PCAC. Let the Pomeron term for the process

$$K(p_1) + \pi(p_2) \rightarrow K(p_3) + \pi(q)$$

be

$$T_\pi = c_o(t) (s-u) [H(t, s) - H(t, u)]$$

where $c_o(t)$ has no off shell momentum dependence. For the process

$$K(p_1) + \pi(p_2) \rightarrow K(p_3) + A_1(q)$$

let us write

$$T_A = \epsilon_\mu T_A^\mu$$

where

$$T_A^\mu = (p_1 - p_3)^\mu D_1 + (p_1 + p_3)^\mu D_2 + q^\mu D_3$$

PCAC implies that⁽²⁾

$$(5) \quad \frac{2 F_\pi m_A^2}{g_A} T_\pi = (t - \mu^2 + q^2) D_1 + (s-u) D_2 + 2 q^2 D_3$$

6.

In eq. (5) m_A is the A_1 meson mass and the PCAC constants appearing at the left hand side can be fixed from KSFR and Weinberg sum rules⁽²⁾ to be

$$F_\pi m_A = g_A .$$

Asymptotic t-channel helicity conservation for the A_1 production process requires that the leading term in $T_{\lambda=1}^t$ vanish, $T_{\lambda=1}^t$ being the helicity + 1 amplitude for the A_1 meson in the t-channel CM system. Since $T_{\lambda=1}^t \sim D_2 \sim \beta_2(t) s^{\alpha(t)-1}$, t-channel helicity conservation implies $\beta_2(t) = 0$.

In the spirit of ref. (2), we assume that, for not too large values of the off shell momentum q, we need to keep only up to quadratic dependence in q in the invariant amplitudes. This leads us to write for the D_i 's :

$$D_{1,3} = d_{1,3} (s-u) [H(t,s) - H(t,u)]$$

$$D_2 = d_2 (q^2 - m_A^2) [H(t,s) - H(t,u)]$$

where the d_i 's are constants. Notice that the form chosen for D_2 ensures on shell (for the A_1 -meson) t-channel helicity conservation. Using the above equations we obtain

$$(6) \quad d_3 = -\frac{1}{2} (d_1 + d_2)$$

$$c_0(t) = \frac{1}{2m_A} \left[(t - \mu^2) d_1 - m_A^2 d_2 \right]$$

From (6) the reader will notice why we did not simply set $d_2=0$, since this choice would imply a zero in the pion residue function at $t = \mu^2$, which in itself is unreasonable besides the embarrassing consequence that it leads to the ratio of A_1 cross section to π cross section much larger than unity instead of viceversa.

Now if we use the factorization property, we can obtain for the ratio of πA_1 coupling vs. $\pi\pi$ coupling to the Pomeron

$$(7) \quad \frac{\beta_{\pi A_1}^{\lambda=0}(t)}{\beta_{\pi\pi}(t)} = \frac{d_1 \sqrt{[t - (m_A - \mu)^2][t - (m_A + \mu)^2]}}{m_A^2 d_2 + (\mu^2 - t) d_1}$$

Assuming the same t-dependence of the Pomeron term in $\pi N \rightarrow \pi N$ and $\pi N \rightarrow A_1 N$, eq. (7) allows us to compute the ratio of the integrated

diffraction cross sections, i. e.

$$\frac{\sigma(\pi N \rightarrow \pi N)}{\sigma(\pi N \rightarrow A_1 N)} \approx \frac{\beta_{\pi\pi}^2(0)}{\beta_{\pi A_1}^2(0)} \approx \left(\frac{d_2}{d_1}\right)^2$$

having assumed $|d_2| \approx |d_1|$. To compute the ratio d_1/d_2 , we use the experimental value of $\sigma_{\pi N}^{el}(\infty) \approx 4$ mb and the fact that A_1 production cross section at 16 GeV/c is about $250 \pm 50 \mu\text{b}$ ⁽⁵⁾. This tells us that $|d_1/d_2| \approx 1/4$. We now have a prediction: if d_1 and d_2 have opposite signs, then from eq. (6) or (7) we notice that the $\pi\pi$ coupling to the P_0 meron, $c_0(t)$, has a zero at the momentum transfer value t_0 such that

$$-t_0 = -\mu^2 + m_A^2 \left(-\frac{d_2}{d_1}\right) \approx 4 m_A^2 = 4.4 (\text{GeV}/c)^2.$$

Thus we expect a dip in the diffraction part of $d\sigma/dt$ for πN scattering at $t \approx -4.4 (\text{GeV}/c)^2$. In fitting πN data at large t , Barger and Phillips⁽⁶⁾ find that their Pomeron term has a minimum around $t \approx -3 (\text{GeV}/c)^2$. Also, Beretvas and Booth⁽⁷⁾ have obtained from experiment the t dependence of the Pomeron coupling in πN system. Figure 1 of ref. (7) shows that the Pomeron coupling at $t = -3 (\text{GeV}/c)^2$ is $\approx 1.5 \times 10^{-3}$ times its value at $t=0$ and that at higher (i. e. more negative) t values it rises again. In a recent letter Akerlof et al.⁽⁸⁾ have done a careful experimental analysis of the reactions $\pi^{\pm} p \rightarrow \pi^{\pm} p$ and $\pi^{\pm} p \rightarrow K^{\pm} \Sigma^{\pm}$ in the region $2.2 < -t < 3.5 (\text{GeV}/c)^2$. They find a dip in the elastic reactions at $-t = (2.8 \text{ GeV}/c)^2$ but not in the inelastic one. Due to this reason they reach the conclusion (in agreement with us) that the dip structure is diffractive in origin. Our calculation in fact shows a qualitative agreement with the above experimental results.

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