

LNF-71/89
16 Dicembre 1971

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PARTICLE PRODUCTION CALCULATIONS. -

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This note concerns the calculations of pion nuclear interaction corrections for the 'Bosone' experiment performed on the electron-positron storage ring at Frascati.

The energy dependence of the absorption corrections, as well as the effects of secondary particle production will be discussed. The first two sections will consider the experimental data on absorption and secondary particle production. The third section will describe the calculations required for the 'Bosone' apparatus.

1. - ABSORPTION. -

Wherever possible counter data on absorption has been used, in order to conform as closely as possible to our experimental set up (see Fig. 1).

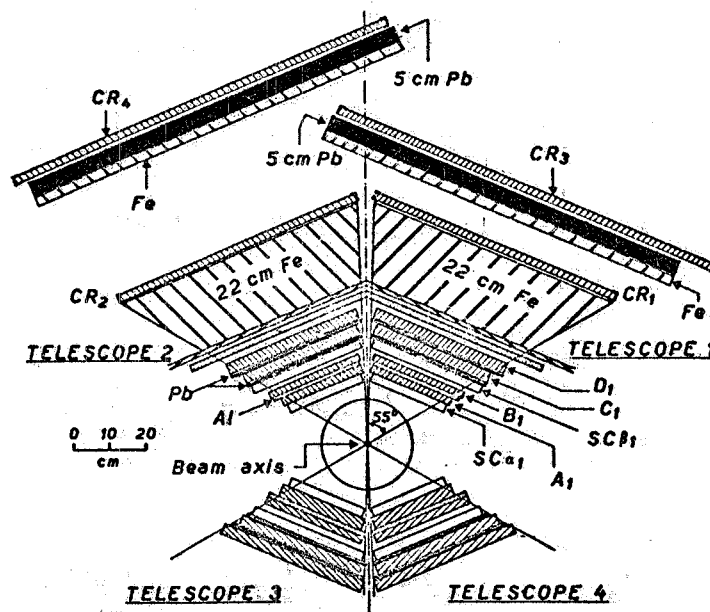


FIG. 1

2.

A typical experimental measurement⁽¹⁾ using good geometry to determine σ_{abs} is shown in Fig. 2. θ is the half angle in radians subtended by the downstream anti-coincidence counter.

σ_{exp} measures the cross-section for pion scattering outside the θ cone defined by the anticounter (a pion absorption is equivalent to a scattering outside the cone). For small angles i. e., with the anti-coincidence counter far downstream, the diffraction peak dominates. For the 'Bosond' experiment diffraction scattering corrections can be ignored since small angle scattering corrections tend to cancel due to edge effects.

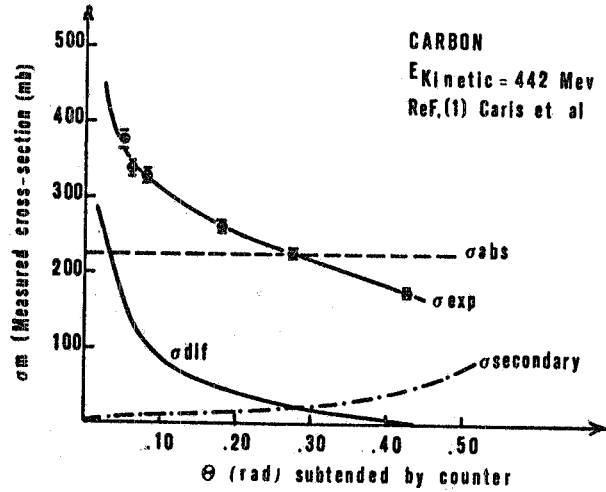


FIG. 2

Two of the absorption experiments^(1, 2) have fit their data with sum of three cross-sections in order to determine σ_{abs} . They assume

$$\sigma_{exp} = \sigma_{diff} + \sigma_{abs} - \sigma_{secondary}$$

An optical model is used to calculate σ_{diff} and σ_{sec} is assumed to have an isotropic distribution (see Fig. 2). σ_{sec} will be discussed later. However, notice that σ_{exp} (the total number of pions scattered outside the angle θ or absorbed) could approach zero if $\sigma_{secondary}$ were large enough. This effect of reducing the "effective absorption" will be discussed later in the section on secondaries.

For each of the references (1, 2, 3, 4, 5) the value of σ_{abs} determined from the experiment is shown in Fig. 3. These values of

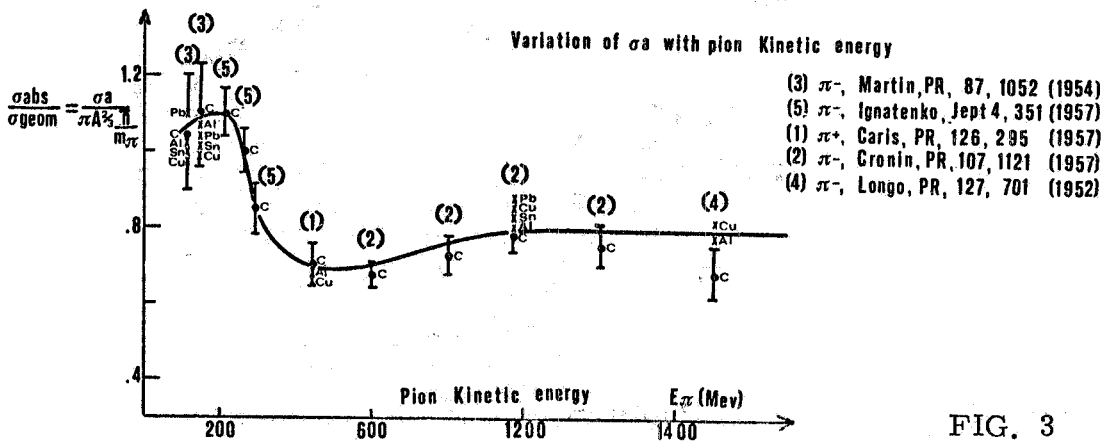


FIG. 3

σ_{abs} have been divided by $\sigma_{\text{geom}} = \pi(\hbar/m_{\pi}c)^2 \times A^{2/3}$, the "natural" geometrical cross-section for each element. The graph indicates that within errors at each energy the ratio of absorption to geometric cross-section is the same for all elements. This allows one to write a single energy dependence for $\sigma_a/\sigma_{\text{geom}}$ for all elements.

The energy dependence of the absorption cross-sections in Fig. 3 shows the effect of the 3-3 resonance near 200 MeV, but is almost constant ($0.7 = \sigma_a/\sigma_{\text{geom}}$) at higher energies, consistent with the usual convention of using $2/3 \sigma_{\text{geom}} = \sigma_{\text{abs}}$.

2. - SECONDARIES. -

In Ref.(2, 3) simultaneously with the absorption measurement an attempt was also made to measure secondary particle production by pions. In Fig. 2, the experimental raw data of Caris et al. (1) can be seen. A down stream counter, subtending a half angle θ , is put in anticoincidence with a pion trigger before the target. The experimental counting rate then is a measure of the number of particles absorbed or scattered outside the angle θ . Notice that at the largest angles measured (25°) the experimental cross-section is 176 ± 3 mb for Carbon, however, the absorption was determined to be 225 mb from this measurement. That is, the measured particle "loss" at 25° was 20% less than that calculated from purely absorption. This is due, of course, to the production of secondary particles into the solid angle defined by θ . These secondaries, produced when a pion is absorbed, anticoincidence in the down stream counter and reduce the effective pion absorption. Both Caris(1) and Cronin(2) have fit their data (457 \rightarrow 970 MeV) using a secondary particle cross-section of the form $2\pi\eta(1 - \cos\theta) = \sigma_{\text{sec}}$, that is, assuming a isotropic production of secondaries. The experimental best fit values of η (mb/ster) are shown for Carbon target in Fig. 4. The low energy points were determined from estimates of maximum secondary production in Ref. (5). Two features are apparent; first, there is a very small contribution below

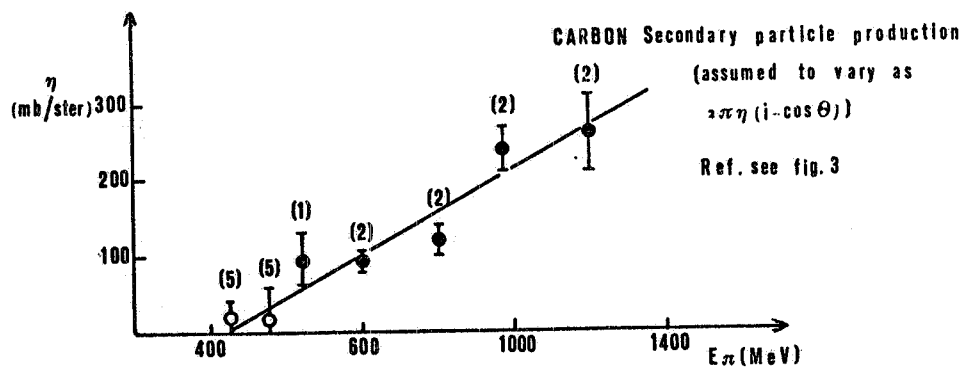


FIG. 4

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250 MeV and second, there is a rather steep linear dependence on incident pion kinetic energy.

In Fig. 5 the production cross-section (η) at 970 MeV is indicated for several elements.

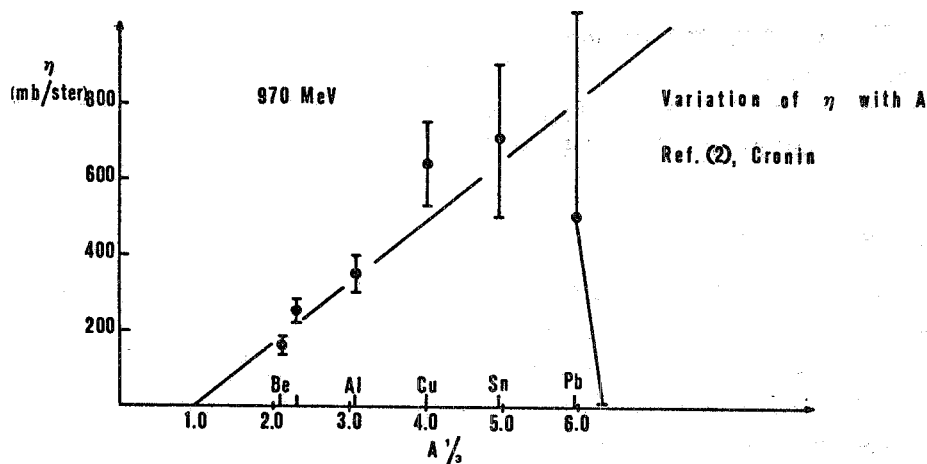


FIG. 5

By using the A dependence at 970 MeV shown in Fig. 5 and assuming $\eta = 0$ at 250 MeV the secondary production cross-section can be calculated for any element at all incident pion energies.

Several of the assumptions made in Ref. (1, 2) about secondary particle production have been examined experimentally. In particular, Fig. 6 shows the angular dependence of inelastic pion production⁽⁵⁾. Although 300 MeV pairs do not contribute significantly to secondary particle production it is clear that there is not strong forward peaking. At most, we would over estimate the secondary particle production by 15-20% by assuming an isotropic distribution and using the best fit value of η at 25° as in Ref. (1, 2).

Also measurements of the energy distributions of the secondaries have been carried out in several experiments⁽⁵⁾. This information is very important since it allows us to calculate range losses of secondaries in our apparatus. The energy distributions in general show that the secondary energy is not distributed uniformly from the incident energy down to zero but tends bunch at energies lower than 1/2 the incident pion energy. The experimental

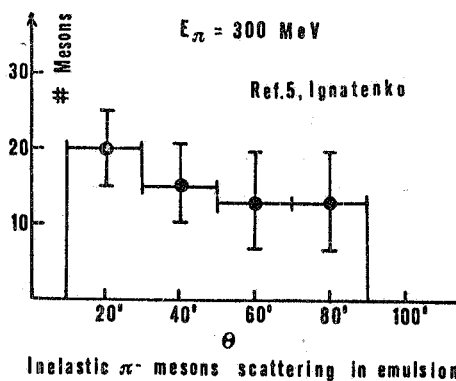


FIG. 6

results have been plotted in Fig. 7 in a manner useful for calculation. For each secondary percentage energy loss (F_E) the fraction of secondaries with an energy loss less than F_E have been plotted. That is, for any cut of energy (expressed as a fraction of the incident pion energy) the graph indicates directly the % of secondaries that remain. The solid line in Fig. 7 is an average used in the calculation.

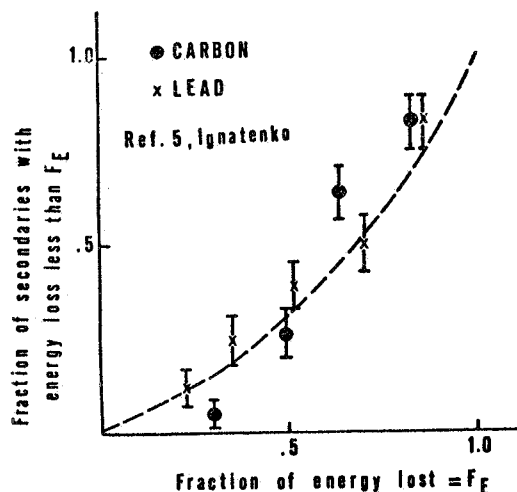


FIG. 7

3. - GENERAL CALCULATION PROCEDURES. -

3.1. - Absorption.

We wish to calculate the fraction of pions that remain in the beam after passing through a series of consecutive absorbers, consisting of differing materials.

If N_0 is the initial number of pions, then

$$N_0 e^{-x_i/\lambda_{abs}}$$

is the number remaining after the first absorber. x_i is the path length and λ_{abs} is the absorption length. Clearly then, for a series of N absorbers the expression for the number remaining is

$$N_0 e^{-\sum_{i=1}^N x_i/\lambda_{iabs}}$$

Since

$$\lambda_{geom} = \frac{A}{\sigma_{geom} N_0} \text{ (gm/cm}^2\text{)}$$

(A is the atomic #, N_0 Avogadro's number)

Then using

$$\sigma_{geom} R(E) = \sigma_{abs}$$

($R(E)$ is given in Fig. 3)

we can write the number of pions remaining after N slabs as

6.

$$(1) \quad \# = N_0 e^{-\sum_{i=1}^N x_i/\lambda_{i\text{geom}} R(E)}$$

Our apparatus (Fig. 1) has three counter planes of interest; trigger (A, B, C), RC mark (CR₁, CR₂) and cosmic ray anticoincidence (CR₃, CR₄). For each plane equation (1) has been calculated:

$$\begin{aligned} \text{Trigger} &= N_0 e^{-0.269 R(E)} \\ \text{RC mark} &= N_0 e^{-2.09 R(E)} \\ \text{Final anti} &= N_0 e^{-2.67 R(E)} \end{aligned}$$

For a 400 MeV pion, the fraction of remaining pions at each counter plane is

$$\begin{aligned} \text{Trigger} &= 0.80 \\ \text{RC} &= 0.17 \\ \text{Anti} &= 0.11 \end{aligned}$$

These are calculations for a single pion entering one telescope.

3.2. - Secondary particle production.

We can write the probability to produce a secondary at a point x in an absorber of thickness T as

$$e^{-x/\lambda_{\text{abs}}} \frac{\Delta x}{\lambda_{\text{abs}}} p(\theta, E)$$

where $e^{-x/\lambda_{\text{abs}}}$ is the fraction of pions at the point x ; $\Delta x/\lambda_{\text{abs}}$ is the probability of absorption; and $p(\theta, E_m)$ is the probability to produce a secondary (once a pion has been absorbed) within a specified angle, θ , and with a certain minimum energy. However, the probability that this secondary remains (is not absorbed) after it passes through the remainder of the absorber, is

$$e^{-(T-x)/\lambda_{\text{abs}}}$$

so that we can write

$$\# \text{ secondaries} = N_0 e^{-T/\lambda_{\text{abs}}} \frac{T}{\lambda_{\text{abs}}} p(\theta, E)$$

and the expression for the total $\#$ of particles (pions + secondaries) is

$$N_0 e^{-T/\lambda_{\text{abs}}} \left(1 + \frac{T}{\lambda_{\text{abs}}} p(\theta, E) \right).$$

Unfortunately, the experimental measurement of secondaries does not give us $p(\theta, E)$ directly, since Ref. (1, 2) assume an expression for the total number of pions remaining as

$$e^{-T/\lambda_{\text{abs}}} e^{T\sigma_s N_0/A} \quad (\sigma_s = 2\pi\eta(1-\cos\theta))$$

However, we can make the identification

$$\frac{T}{\lambda_{\text{abs}}} \bar{p}(\theta, E) \Rightarrow \frac{T\sigma_s N}{A}$$

and write

$$\# \text{ secondaries} = N_0 e^{-x/\lambda_{\text{abs}}} \frac{x\sigma_s(\theta, E)N_0}{A}$$

We can proceed with the calculation for our apparatus by noticing that for the second absorber (for instance) the number of secondaries that are produced and appear after the N^{th} absorber is

$$N_0 e^{-x_1/\lambda_{1\text{abs}}} e^{-x_2/\lambda_{2\text{abs}}} \frac{x_2\sigma_{\text{sec}}N_0}{A} e^{-\sum_{i=3}^N x_i/\lambda_{i\text{abs}}}$$

so, summing the secondaries from all N absorbers gives

$$(2) \quad \# \text{ secondaries} = \sum_{i=1}^N \frac{x_i\sigma_i(\theta, E)N_0}{A} \left[e^{-\sum_{i=1}^N x_i/\lambda_{i\text{abs}}} \right]$$

Notice that $\sigma_i(\theta, E)$ is the cross-section for production of secondaries into the counter solid angle seen by each absorber and with an energy above the range cut for each absorber.

3.3. - Calculation procedure.

The following list contains the calculational steps required to evaluate the secondary production as a function of incident pion energy.

- a) For a set of incoming pion energies calculate the pion energy at each absorber in the apparatus, as its energy is degraded by range losses.
- b) From the energy dependence of the secondary production cross-section (η) given in Fig. 4 calculate η (mb/ster) at each absorber for each incident energy.
- c) Using the solid angle of the counter (trigger, RC, final Anti) seen by the secondaries from each absorber, and assuming isotropic pro-

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duction calculate $\sigma_{\text{sec}} = (\Delta\Omega/4\pi)\eta$ (between the two wire chambers, α_i, β_i the solid angles were restricted so that the projected secondary track would be within ± 30 mm of the source as we require in our analysis)

d) Use the σ_{sec} (calculated in step c) to calculate the probability of production and then use Fig. 7 to calculate the fraction of secondaries that remain after the energy cut due to range losses (this cut is different for each absorber and for each of the 3 counters).

e) Sum the number of secondaries from each absorber (as in equation (2)) and multiply by the absorption factor .

The total number of pions that remain at the trigger (or the other two counter planes of interest, depending on the calculation) is given by (1) and (2)

$$\# \text{ pions} = N_0 e^{-\sum_{i=1}^N x_i/\lambda_{i\text{abs}}} \underbrace{\left[1 + \sum_{i=1}^N \frac{x_i \sigma_{i\text{sec}} N_0}{A} \right]}_C$$

As can be seen, when the factor C is equal to

$$e^{\sum_{i=1}^N x_i/\lambda_{i\text{abs}}}$$

all of the absorbed pions have produced a secondary that reached the final counter. Clearly for energies higher than this "balanced absorption energy" there no longer is a correction for pion absorption.

For our apparatus this energy point is

Trigger	800 MeV
RC mark	1000 MeV
Final Anti	1500 MeV.

Table I contains C calculated for a series of energies for each counter of interest (trigger, RC mark, Final Anti).

All of the calculations in Table I refer to absorption corrections for a single track. Of interest, however, is the effect of absorption and secondary particle production on the Montecarlo calculation of the efficiency of the experimental apparatus.

Referring to Fig. 1 it can be seen that events will be "lost" when a pion is absorbed before the trigger ($A \cdot B(C+D)$ required), but at the

same time high energy pions which would trigger the anticoincidence counter CR₁ (and therefore result in a "lost" event) are absorbed before the anticounter. The extent of this cancellation depends on the energy distribution of the final state particles and on the relative absorption lengths before the two counters. It turns out, that at 1800 MeV center of mass energy for a four charged final state there is a cancellation of absorption effects to within 1 %.

TABLE I

Trigger				
Pion K energy	400 MeV	500 MeV	1000 MeV	1500 MeV
C =	1.047	1.099	1.413	1.573
RC mark				
Pion K energy	600 MeV	1000 MeV	1500 MeV	
C =	1.588	5.159	10.61	
Final Anti				
Pion K energy	600 MeV	1000 MeV	1500 MeV	
C =	1.133	3.24	8.389	

Since the addition of secondaries should correspond roughly to a reduction of the absorption cross-section, it is expected that this cancellation of "absorption effects" will also be found when we consider both effects.

For example, at 2000 MeV C. M. energy for the $2C + 1N$ final state we have examined the cancellation in detail. This example was chosen to maximize the effects of secondaries by using a high energy and low multiplicity. It was found that although secondary production reduced the trigger event losses by 42 %. There was a corresponding increase in secondary produced anti coincidences that resulted in a net increase of the efficiency by only 6 %. This represents a near maximum effect since for lower energy and high multiplicity the secondary production cross-section is reduced. Notice however that the cancellation of absorption effects is a result of the particular absorber-counter configuration of our apparatus and secondary production effects can be quite dramatic on a single counter.

10.

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