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G. Parisi and M. Testa : THE Σ-TERM AND THE SCALE BREAKING

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The Σ-Term and the Scale Breaking (*).

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The purpose of the present paper is to investigate some consequences of a breaking pattern of scale invariance proposed in ref. (1).

The general scheme (2) is based on the following decomposition of the hadronic Lagrangian L:

$$L = L_s + \varepsilon u + \varepsilon_0 \sigma_0 + \varepsilon_8 \sigma_8 , \tag{1}$$

where u causes a breaking of scale invariance but not of $SU_3 \times SU_3$ and σ_0 and σ_8 are the usual breaking terms belonging to a $(3, \overline{3}) + (\overline{3}, 3)$ representation of $SU_3 \times SU_3$ (3). In ref. (1) we further proposed the invariance of L_s under $U_3 \times U_3$; we also assumed that this invariance is broken by u, which transforms as a $(1, \overline{1}) + (\overline{1}, 1)$ representation of $U_3 \times U_3$. From an analysis of the spectrum of scalar and pseudoscalar mesons we concluded that the operator dimension of u is 3. We are thus led to suggest for the meson part of the hadron Lagrangian the usual expression of the σ-model:

$$\left\{\begin{aligned} L &= L_{\text{kinetic}} + \lambda[\operatorname{Tr}(\mathscr{M}\mathscr{M}^+)]^2 + \mu \operatorname{Tr}[(\mathscr{M}\mathscr{M}^+)^2] + \\ &\qquad + \varepsilon(\det\mathscr{M} + \det\mathscr{M}^+) + L_{\text{baryon-meson}} + \varepsilon_0\sigma_0 + \varepsilon_8\sigma_8 , \\ \mathscr{M} &\equiv \sum_{k=0}^{8} (\sigma_k + i\pi_k)\lambda_k , \qquad \lambda_0 \equiv \sqrt{\tfrac{2}{3}}\, I , \end{aligned}\right. \tag{2}$$

where the unknown baryonic term is scale and $U_3 \times U_3$ invariant and the u-term is identified with the expression $\det\mathscr{M} + \det\mathscr{M}^+$ (4). If an analysis of the mass spectrum of

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(2) L. BROWN and M. GELL-MANN: *Lectures at Hawai Summer School, 1969*, CALT 68-244 (1970).

(3) S. L. GLASHOW and S. WEINBERG: *Phys. Rev. Lett.*, **20**, 224 (1968); M. GELL-MANN, R. J. OAKES and B. RENNER: *Phys. Rev.*, **175**, 2195 (1968).

(4) We are not considering the existence of fundamental spin-one meson fields.

spin-zero mesons is carried out on the ground of a phenomenological Lagrangian constructed with the four not more than quadrilinear $SU_3 \times SU_3$ invariants [5], one gets a fit of 8 masses (π, K, η, X^0, δ, $\varkappa$, τ_{0+} (1070), α peak [6]) in terms of 4 parameters. By this procedure it is possible to fix the coefficient of the trilinear term. If this is identified with the same coefficient in the true Lagrangian (2), we are led to the value

$$\varepsilon = 4.2\, m_\pi \, . \tag{3}$$

We can now write down the fundamental identity of dilation symmetry breaking for baryons in the limit of exact SU_3:

$$M_B = \langle B|\theta^\mu_\mu|B\rangle = -\varepsilon\langle B|u|B\rangle - 3\varepsilon_0\langle B|\sigma_0|B\rangle \, , \tag{4}$$

here we used canonical dimensions for the σ and π fields. Under the hypothesis that the dominant contribution to the first term of (4) comes from tree-like diagrams, we obtain

$$M_B = -3\varepsilon\eta_0^2 \tfrac{2}{3}\sqrt{\tfrac{2}{3}}\, 2\langle B|\sigma_0|B\rangle - 3\varepsilon_0\langle B|\sigma_0|B\rangle \, , \qquad \eta_0 \equiv \langle 0|\sigma_0|0\rangle \, . \tag{5}$$

The values of the parameters η_0 and ε_0 have been estimated by using Glashow-Weinberg equations as applied to pseudoscalar mesons [7] and their value turns out to be

$$\varepsilon_0 = 9.9\, m_\pi^3 \qquad \eta_0 = m_\pi \, . \tag{6}$$

Using the observed mean baryon mass $M_B = 1150\,\text{MeV}$ we can evaluate from (5) the value of $-\varepsilon_0\langle B|\sigma_0|B\rangle$. It turns out

$$-\varepsilon_0\langle B|\sigma_0|B\rangle \approx 260\,\text{MeV} \, . \tag{7}$$

From (7) it is possible to evaluate the so-called Σ-term:

$$\Sigma \equiv -\tfrac{1}{3}(\sqrt{2}\,\varepsilon_0 + \varepsilon_8)\langle B|\sqrt{2}\,\sigma_0 + \sigma_8|B\rangle \, , \tag{8}$$

which has been recently the object of much theorethical work [8-12]. One gets

$$\Sigma = 28\,\text{MeV} \, , \tag{9}$$

using the values $\varepsilon_8/\varepsilon_0 \simeq -1.25$ [3,13] and $\varepsilon_8\langle B|\sigma_8|B\rangle \simeq 210\,\text{MeV}$.

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(13) If the values of ref. (7) are used, $-\varepsilon_8/\varepsilon_0 \approx 1.29$ and the value $\Sigma \approx 20\,\text{MeV}$ is obtained.

The first quantitative estimate of (8) from experimental data on meson-nucleon scattering is due to KIM and VON HIPPEL [8] who got

$$\Sigma_{\text{K.v.H.}} = 26\ \text{MeV}\ . \tag{10}$$

The correctness of the calculations of ref. [8] was questioned by CHENG and DASHEN [9], who, by means of a dispersion-theoretical method, obtain

$$\Sigma_{\text{C.D.}} = 110\ \text{MeV}\ . \tag{11}$$

The value of the Σ-term was shown to be critically dependent on the s- and p-wave scattering lengths by ALTARELLI, CABIBBO and MAIANI [10], who on the basis of the smooth behaviour of the π-$\mathcal{N}$ amplitude find $\Sigma_{\text{A.C.M.}} = (80 \pm 30)\ \text{MeV}$.

HÖHLER, JAKOB and STRAUSS [11] criticize the work by CHENG and DASHEN both from theoretical and numerical point of views and propose

$$\Sigma_{\text{H.J.S.}} = 40\ \text{MeV}\ . \tag{12}$$

More recently ERICSON and RHO [12], performing a calculation similar to that of ref. [8] but starting from meson-nucleus scattering, get

$$\Sigma_{\text{E.R.}} = 34\ \text{MeV}\ . \tag{13}$$

Both $\Sigma_{\text{H.J.S.}}$ and $\Sigma_{\text{E.R.}}$ are in qualitative agreement with (9).

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