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The Elimination of Second Order Aberrations in Uniform Field Wedge Magnets with Slightly Rotated Edges

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The coefficients of the first and second order optic transformation for the midplane of a uniform field wedge magnet having curved entrance and exit boundaries and rotated input and output faces have been first derived by Brown, Belbeoch, and Bounin [Rev. Sci. Instrum. **35**, 481 (1961)]. Assuming such results and limiting the edge angles to be small (i.e., of the same order of magnitude as the input conditions of the trajectories) simplified analytical expressions can be derived for the above coefficients. Taking advantage of this achievement, it is possible to develop a general discussion concerning the possibility to eliminate the aberration terms. It is shown that for magnets with deflection angles within the interval $(\pi,2\pi)$ a complete elimination of the second order terms in position (x) can be achieved. For magnets with deflection angles in the interval $(0,\pi)$ it is only possible to nullify some pairs of such terms.

INTRODUCTION

IN a magnetic spectrometer it is generally impossible to achieve a complete elimination of all the mathematical aberrations due to second order terms in the input conditions of the trajectories.

When the working conditions of a spectrometer in a specified experiment are well established, it is always found that there are some second order aberration terms that are more effective than the others in reducing the momentum resolution of the instrument. Therefore, the availability of criteria for designing corrections, so as to eliminate the greatest possible number of such undesired terms, is greatly appreciated.

In the past, many successful attempts were made for eliminating at least one of the more serious aberration effects in uniform field spectrometers.[1] This elimination was commonly obtained by substituting the flat edge faces of the magnet with suitably curved ones.[2–4]

Furthermore, it is known that the introduction into a uniform field spectrometer of rotatable edge faces (thus making a wedge magnet) permits one to dispose of two additional parameters (the edge angles) by which it is possible to control the first order horizontal and vertical focusing.

In 1964, Brown, Belbeoch, and Bounin,[5] considering the general case of a uniform field wedge magnet with curved entrance and exit boundaries and rotated input and output faces, developed successful calculations for the coefficients of the second order aberrations terms on the midplane. These calculations were performed assuming a short cutoff distribution for the fringing fields (SCOFF).[6] The expressions of the second order coefficients given by the above authors are rather complicated so that, when these are used for deducing the coefficients of the optic transformation between the object and image planes, very complicated results are obtained. It is, therefore, impossible to develop a general analysis of the conditions required to eliminate the second order terms.

In the present paper, by limiting the magnet edge angles to be of the same order of magnitude as the input conditions of the trajectories, we have derived simpler expressions for all the horizontal aberration coefficients on the image plane. These expressions permit us to discuss systematically the elimination of groups of geometric and chromatic aberration terms. Whenever there are not specific requirements on the vertical focusing, the introduced limitation for the edge angles still permits one to realize valuable displacements of the conjugate points from the positions established by the Barber rule.[7] This is also possible for large deflection angles (see end of Sec. II.A). All the considerations developed in this paper are limited to the midplane of the spectrometer and to a second order approximation of the equations of the trajectories. The SCOFF approximation for the fringing fields is assumed throughout the text. From the knowledge and experience of the authors, the effect on the values of second order coefficients of substituting the SCOFF distributions for the real ones is limited to variations of a few percent. Therefore, the results obtained in this paper can be regarded as valid. Finally, it has been assumed that the effects on the first and second order focusing of additional curvature of isoinduction lines (which may be due to the finite radial width of pole pieces) can be neglected.[6]

I. FIRST AND SECOND ORDER OPTIC COEFFICIENTS FOR SMALL EDGE ANGLES

We will first consider a general spectrometer and a particular trajectory τ_0 of its midplane which we take as central trajectory or optic axis. The input conditions of any trajectory τ of the midplane will be specified giving, at the entrance plane of the magnet, the displacement x_0 of trajectory τ from the central one, the angle θ_0 between the same two trajectories, and the quantity $\gamma=(p-p_0)/p_0$. In this expression p and p_0 indicate the momenta of the particles moving and the trajectories τ and τ_0, respectively. Similarly, we will indicate by x and θ the quantities corresponding to x_0 and θ_0 at the exit plane of the spectrometer. It is well known that the displacement x and the slope $x'=\tan\theta$ of a trajectory at the exit plane can be expressed in general as power series developments of the input quantities x_0, $x_0'=\tan\theta_0$, γ. Using a notation similar to that introduced by Streib,[8] the above developments for the midplane of the magnet are written

$$\eta=\sum_{\lambda,\mu,\nu}(\eta\,|\,x_0^{\lambda}x_0'^{\mu}\gamma^{\nu})x_0^{\lambda}x_0'^{\mu}\gamma^{\nu}, \qquad (1)$$

η being one of the two quantities x or $x'=\tan\theta$, and λ, μ, ν being positive integers or zero so that $\lambda+\mu+\nu\geqslant 1$. By suitably limiting the intervals of the values taken by x_0, x_0', γ, it is possible to make accurate calculations of the values of x and x' merely retaining the first and second order terms in the developments (1) and dropping all the higher order ones. This procedure often is sufficient for all the practical needs and when it is valid we can also write $x_0'=\theta_0$, $x'=\theta$, such equalities being true to the second order.

Let us now consider, in particular, a uniform field wedge magnet having entrance and exit curved boundaries and rotated input and output faces (see Fig. 1). For a spectrometer with such a geometry, the first and second order coefficients between the entrance and exit planes $(\eta\,|\,x_0^{\lambda}x_0'^{\mu}\gamma^{\nu})$ have been calculated by the authors of Ref. 5 and listed at the end of Sec. VI of that paper. Since we shall derive all the subsequent considerations from the analytical expressions of such coefficients, we shall standardize the mathematical symbols and the convections for the signs of the coordinates and angles to those used by the above authors. The meaning of a few other symbols is clarified by Fig. 1.

We start noting that all the coefficients of Ref. 5 can be regarded as sums whose terms are products of powers (with exponents positive integers or zero) of the quantities $\tan\beta_1$, $\tan\beta_2$, $\sec\beta_1$, $\sec\beta_2$, $1/R_1$, and $1/R_2$. The coefficients of such terms are trigonometric functions of angle α whose absolute values have an upper bound of the order of unit. Furthermore, in the above mentioned calculations

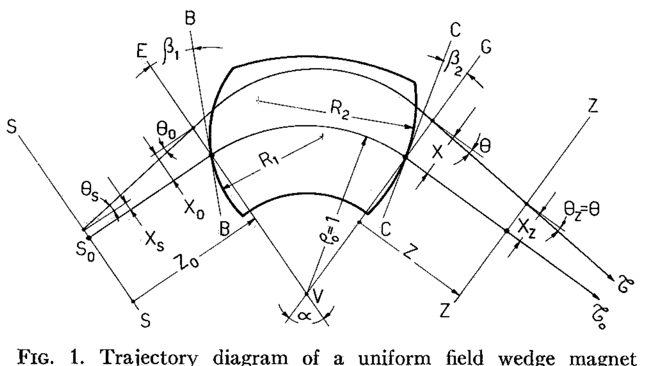

FIG. 1. Trajectory diagram of a uniform field wedge magnet having curved entrance and exit boundaries and rotated input and output faces.

of first and second order optic coefficients, the angles β_1 and β_2 are allowed to vary throughout quite a large interval, even if in practice $|\beta_1|$ and $|\beta_2|$ are kept rather less than $\pi/2$.

With the aim of obtaining more simplified analytical expressions for the above coefficients, we introduce in such expressions the following series developments (valid for $\beta_k^2<\pi^2/4$):

$$\begin{aligned}\tan\beta_k&=1+\tfrac{1}{3}\beta_k^3+(2/15)\beta_k^5+\cdots\\ \sec\beta_k&=1+\tfrac{1}{2}\beta_k^2+(5/24)\beta_k^4+\cdots.\end{aligned}\qquad (k=1,2)$$

When such substitutions are carried out and a suitable rearrangement is accomplished, the expressions of the coefficients result in sums of terms formed by products of powers of the quantities β_1, β_2, $1/R_1$, and $1/R_2$. The coefficients of these terms are again trigonometric functions of angle α with upper bounds as above specified. Therefore, the developments [Eq. (1)] will assume, for the case considered, the following analytical form:

$$\eta=\sum_{\lambda,\mu,\nu}\left[\sum_{\rho,\sigma\,0}^{\infty}\left(F_{\eta}^{\lambda\mu\nu\rho\sigma}(\alpha)\frac{1}{R_1^{a}}\frac{1}{R_2^{b}}\beta_1^{\rho}\beta_2^{\sigma}\right)\right]x_0^{\lambda}\theta_0^{\mu}\gamma^{\nu}, \qquad (2)$$

with $\eta=x$, θ and λ, μ, ν, ρ, σ positive integers or zero so that $\lambda+\mu+\nu=1$, 2. In addition, the exponents a and b can assume the values zero or one only.

If we now limit the angles β_k to be of the same order of magnitude as the entrance conditions x_0, θ_0, γ, and if $R_1\approx 1$, $R_2\approx 1$, we then could neglect in Eq. (2) all the terms for which $\lambda+\mu+\nu+\rho+\sigma>2$, without reducing the required accuracy in the calculation of the η value. When the above procedure is carried out, we obtain for the first and second order optic coefficients the expressions listed in Table I. Note that in the expressions of the first order coefficients of this table (for which $\lambda+\mu+\nu=1$) only one of the two angles β_k can appear, owing to the equation $\rho+\sigma=0$ or 1. For the second order coefficients (for which $\lambda+\mu+\nu=2$) none of the angles β_k will appear in the corresponding expressions, since $\rho+\sigma=0$.

We are now interested in the coefficients of the optic transformation between the object plane (SS) and any plane (ZZ) behind the magnet (see Fig. 1). Introducing in the developments (2) the drift region transformation

$$\begin{aligned}x_0&=x_s+z_0\theta\\ \theta_0&=\theta_s\end{aligned}\quad\text{and}\quad\begin{aligned}x_z&=x+z\theta\\ \theta_z&=\theta,\end{aligned}$$

we obtain for the displacement x_z the expression

$$\begin{aligned}x_z=(x_z|x_s)x_s&+(x_z|\theta_s)\theta_s+(x_z|\gamma)\gamma+(x_z|x_s^2)x_s^2\\ &+(x_z|\theta_s^2)\theta_s^2+(x_z|\gamma^2)\gamma^2+(x_z|x_s\theta_s)x_s\theta_s\\ &+(x_z|x_s\gamma)x_s\gamma+(x_z|\theta_s\gamma)\theta_s\gamma,\end{aligned}\qquad (3)$$

whose coefficients assume (after some rearrangement) the expressions listed in Table II.

TABLE I. Analytical expressions for the coefficients of the second order expansions of $x=x(x_0,\theta_0,\gamma)$ and $\theta=\theta(x_0,\theta_0,\gamma)$. The reported expressions are valid for the spectrometer shown in Fig. 1 and for edge angles satisfying the hypothesis specified in the text.

Coefficients	Analytical expressions
$(x\|x_0)$	$\cos\alpha+\beta_1\sin\alpha$
$(x\|\theta_0)$	$\sin\alpha$
$(x\|\gamma)$	$1-\cos\alpha$
$(x\|x_0^2)$	$-\frac{1}{2}\sin^2\alpha+(1/2R_1)\sin\alpha$
$(x\|\theta_0^2)$	$\frac{1}{2}\cos\alpha(1-\cos\alpha)$
$(x\|\gamma^2)$	$-\frac{1}{2}\sin^2\alpha$
$(x\|x_0\theta_0)$	$\frac{1}{2}\sin(2\alpha)$
$(x\|x_0\gamma)$	$\sin^2\alpha$
$(x\|\theta_0\gamma)$	$\sin\alpha(1-\cos\alpha)$
$(\theta\|x_0)$	$-\sin\alpha+(\beta_1+\beta_2)\cos\alpha$
$(\theta\|\theta_0)$	$\cos\alpha+\beta_2\sin\alpha$
$(\theta\|\gamma)$	$\sin\alpha+\beta_2(1-\cos\alpha)$
$(\theta\|x_0^2)$	$(1/2R_1)\cos\alpha+(1/2R_2)\cos^2\alpha$
$(\theta\|\theta_0^2)$	$-\frac{1}{2}\sin\alpha+(1/2R_2)\sin^2\alpha$
$(\theta\|\gamma^2)$	$-\sin\alpha+(1/2R_2)(1-\cos\alpha)^2$
$(\theta\|x_0\theta_0)$	$(1/R_2)\sin\alpha\cos\alpha$
$(\theta\|x_0\gamma)$	$\sin\alpha+(1/R_2)(1-\cos\alpha)\cos\alpha$
$(\theta\|\theta_0\gamma)$	$(1/R_2)\sin\alpha(1-\cos\alpha)$

Let the plane (ZZ) now become the image of the object plane (SS). Then, we can write to second order for the radial displacement x_i of τ from τ_0,

$$x_i=(x_i|x_s)x_s+(x_i|\gamma)\gamma+(x_i|x_s^2)x_s^2+(x_i|\theta_s^2)\theta_s^2+(x_i|\gamma^2)\gamma^2 +(x_i|x_s\theta_s)x_s\theta_s+(x_i|x_s\gamma)x_s\gamma+(x_i|\theta_s\gamma)\theta_s\gamma. \quad (4)$$

The coefficients of all the terms in Eq. (4) are obtained from the corresponding ones of Table II putting $z=z_i$, where z_i is the distance of the image plane from the exit face of the magnet measured along τ_0. In Eq. (4) we have put

$$(x_i|\theta_s)=0. \quad (5)$$

TABLE II. Analytical expressions for the coefficients of the second order expansion of $x_z=x_z(z|x_s,\theta_s,\gamma)$, z being the distance of the plane (ZZ) from the exit plane (VG) of the magnet (see Fig. 1).

Coefficients	Analytical expressions
$(x_z\|x_s)$	$(x\|x_0)+z(\theta\|x_0)=\cos\alpha+\beta_1\sin\alpha-z[\sin\alpha-(\beta_1+\beta_2)\cos\alpha]$
$(x_z\|\theta_s)$	$z_0(x\|x_0)+(x\|\theta_0)+z[z_0(\theta\|x_0)+(\theta\|\theta_0)]$ $=z_0(\cos\alpha+\beta_1\sin\alpha)+\sin\alpha-z\{z_0[\sin\alpha-(\beta_1+\beta_2)\cos\alpha]$ $-\cos\alpha-\beta_2\sin\alpha\}$
$(x_z\|\gamma)$	$(x\|\gamma)+z(\theta\|\gamma)=1-\cos\alpha+z[\sin\alpha+\beta_2(1-\cos\alpha)]$
$(x_z\|x_s^2)$	$(x\|x_0^2)+z(\theta\|x_0^2)$ $=-\frac{1}{2}\sin^2\alpha+(1/2R_1)\sin\alpha+z[(1/2R_1)\cos\alpha+(1/2R_2)\cos^2\alpha]$
$(x_z\|\theta_s^2)$	$z_0^2(x\|x_0^2)+z_0(x\|x_0\theta_0)+(x\|\theta_0^2)+z[z_0^2(\theta\|x_0^2)+z_0(\theta\|x_0\theta_0)+(\theta\|\theta_0^2)$ $=z_0^2\{-\frac{1}{2}\sin^2\alpha+(1/2R_1)\sin\alpha+z[(1/2R_1)\cos\alpha$ $+(1/2R_2)\cos^2\alpha]\}+z_0[1+(1/R_2)z]\sin\alpha\cos\alpha-z[\frac{1}{2}\sin\alpha$ $-(1/2R_2)\sin^2\alpha]+\frac{1}{2}\cos\alpha(1-\cos\alpha)$
$(x_z\|\gamma^2)$	$(x\|\gamma^2)+z(\theta\|\gamma^2)=-\frac{1}{2}\sin^2\alpha-z[\sin\alpha-(1/2R_2)(1-\cos\alpha)^2]$
$(x_2\|x_s\theta_s)$	$2z_0(x\|x_0^2)+(x\|x_0\theta_0)+z[2z_0(\theta\|x_0^2)+(\theta\|x_0\theta_0)]$ $=2z_0\{-\frac{1}{2}\sin^2\alpha+(1/2R_1)\sin\alpha+z[(1/2R_1)\cos\alpha$ $+(1/2TR_2)\cos^2\alpha]\}+[1+(1/R_2)z]\sin\alpha\cos\alpha$
$(x_z\|x_s\gamma)$	$(x\|x_0\gamma)+z(\theta\|x_0\gamma)=\sin^2\alpha+z[\sin\alpha+(1/R_2)\cos\alpha(1-\cos\alpha)]$
$(x_z\|\theta_s\gamma)$	$z_0(x\|x_0\gamma)+(x\|\theta_0\gamma)+z[z_0(\theta\|x_0\gamma)+(\theta\|\theta_0\gamma)]$ $=z_0\{\sin^2\alpha+z[\sin\alpha+(1/R_2)\cos\alpha(1-\cos\alpha)]\}$ $+[1+(1/R_2)z]\sin\alpha(1-\cos\alpha)$

As is well known, this is the condition for the conjugate points. For given values of the magnet parameters α, β_1, β_2, Eq. (5) gives a relation between the distance z_0 and z_i of the conjugate planes from the magnet edge faces. From Eq. (5) we can deduce

$$z_0=\frac{z_i(\cos\alpha+\beta_2\sin\alpha)+\sin\alpha}{z_i[\sin\alpha-(\beta_1+\beta_2)\cos\alpha]-(\cos\alpha+\beta_1\sin\alpha)}. \quad (6)$$

II. CONDITIONS REQUIRED TO ELIMINATE THE SECOND ORDER TERMS

A. Elimination of the Aberration Terms for $\alpha>\pi$

It will be useful to discuss first the case $\pi<\alpha<2\pi$. We will consider that α may range throughout the open interval $(\pi,2\pi)$ even if in the current practice of spectrometer designs, deflection angles greatly exceeding π are rarely taken into account.

Now, assuming Eq. (5) to be satisfied, we will attempt to find the conditions that will render the coefficients $(x_i|x_s^2)$, $(x_i|x_s\theta_s)$, and $(x_i|\theta_s^2)$ zero simultaneously. With the help of Table II, we can write the following set of equations:

$$\sin^2\alpha+\frac{1}{R_1}\sin\alpha+z_i\left(\frac{1}{R_1}\cos\alpha+\frac{1}{R_2}\cos^2\alpha\right)=0$$

$$\left(1+\frac{1}{R_2}z_i\right)\sin\alpha\cos\alpha=0$$

$$\cos\alpha(1-\cos\alpha)-z_i\left(\sin\alpha-\frac{1}{R_2}\sin^2\alpha\right)=0.$$

By excluding values of the type $\alpha=k\pi/2$ with $k=1, 2, 3, 4$ (for which $\sin\alpha\cos\alpha=0$) we can obtain the following roots:

$$z_i=(\cos\alpha-1)/\sin\alpha=-\tan(\alpha/2) \quad (7a)$$

$$R_1=R_2=(1-\cos\alpha)/\sin\alpha=\tan(\alpha/2). \quad (7b)$$

Equation (7a) has a simple geometrical interpretation; it states merely that the image point must lie on the bisectrix of the angle $(2\pi-\alpha)$.

We then consider the coefficients $(x_i|x_s\gamma)$, $(x_i|\theta_s\gamma)$, and $(x_i|\gamma^2)$ of the remaining second order terms in Eq. (4). The conditions that must be imposed for such coefficients to be zero result in the following set of equations (see Table II):

$$\sin^2\alpha+z_i[\sin\alpha+(1/R_2)\cos\alpha(1-\cos\alpha)]=0$$

$$[1+(1/R_2)z_i]\sin\alpha(1-\cos\alpha)=0$$

$$-\tfrac{1}{2}\sin^2\alpha-z_i[\sin\alpha-(1/2R_2)(1-\cos\alpha)^2]=0.$$

It is not difficult to see that these equations can be satisfied with the values of z_i and R_2 given by Eqs. (7a) and (7b). Therefore we can conclude that, when the deflection angle is given, values of z_i, R_1, and R_2 can be calculated by Eqs. (7a) and (7b) so to nullify all the second order terms in Eq. (4).

Before proceeding, it will be useful to make some remarks. For an assigned value of angle α in the interval $(\pi, 2\pi)$ (excluding $\alpha=\frac{3}{2}\pi$), Eq. (7a) gives a positive value for z_i as it must in order to give a real image of object plane (SS). Knowing z_i and fixing the values of angles β_1 and β_2, it will be possible to compute from Eq. (6) the distance z_0 of the object plane (SS) from the magnet entrance boundary. It is interesting to remark that the choice $\beta_1=\beta_2=0$ yields the result $z_0=z_i$. This result, in the case $x_s=0$, leads to an overlap of the conjugate points [see Fig. 2(a)]. This overlap disappears for the case $x_s\neq 0$ since the conjugate points belong to the distinct planes (SS) and (II). If one is not interested in a mirror-like behavior of the magnet, it is necessary to choose at least one of the two edge angles different from zero [see Fig. 2(b)]. With regard to this fact, we wish to emphasize

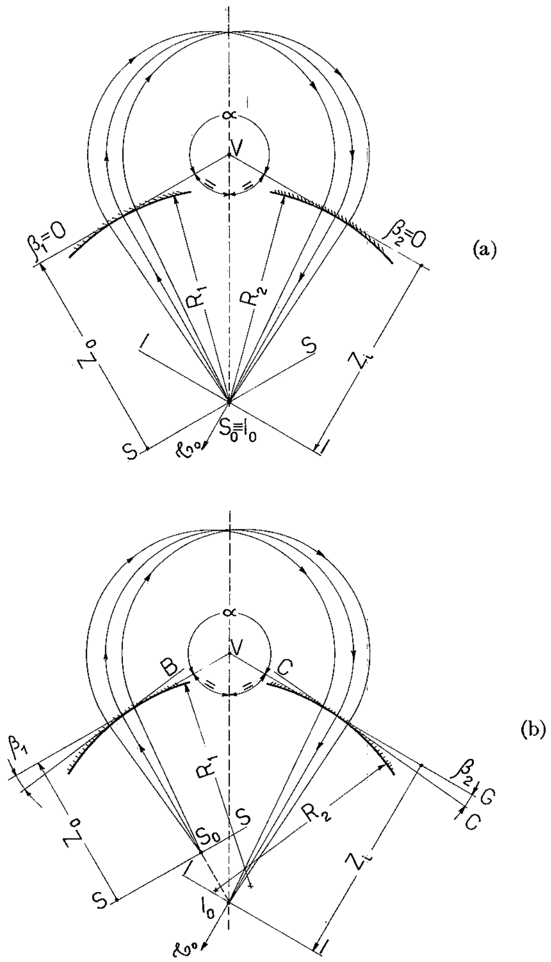

FIG. 2. Trajectory diagrams for a magnet with $\pi<\alpha<2\pi$. (a) In the case $\beta_1=\beta_2=0$ the conjugate points S_0 and I_0 are overlapped. (b) For β_1 and β_2 different from zero, the conjugate points S_0 and I_0 are separated. In order to eliminate the second order aberrations in x_s, we have assumed for z_i, R_1, and R_2 the values given by Eqs. (7a) and (7b) in the text.

that, even with edge angles within the limits of our basic hypothesis, it is possible to obtain valuable displacements of the object point from the overlap position. For instance, assuming z_i given by Eq. (7a) and putting $\beta_1=\beta_2=-0.05$, Eq. (6) yields $z_0/z_i=0.72$ for $\alpha=(7/6)\pi$, and $z_0/z_i=0.85$ for $\alpha=\frac{4}{3}\pi$.

B. Elimination of the Aberration Terms for $0<\alpha<\pi$

Let us now examine if there are in this case sets of terms in Eq. (4) that can be made zero simultaneously. We start by considering the set of three coefficients,

$$(x_i|x_s\gamma),\quad (x_i|\theta_s\gamma),\quad (x_i|\gamma^2). \tag{8}$$

It is not difficult to see that none of the three pairs of coefficients derived from set (8) can be zero simultaneously. In fact, all three corresponding sets of two equations admit as the sole solution Eqs. (7a) and (7b). For $0<\alpha<\pi$, this solution has no practical significance.

Now, with regard to the coefficients

$$(x_i|x_s^2),\quad (x_i|x_s\theta_s),\quad (x_i|\theta_s^2)$$

of the geometric aberration terms in Eq. (4), we note that the simultaneous cancellation of the pair

$$(x_i|x_s^2),\quad (x_i|x_s\theta_s) \tag{9}$$

leads to the relation $z_i=-R_2$ which, when introduced into the set of equations

$$(x_i|x_s\gamma)=0$$
$$(x_i|\theta_s\gamma)=0$$
$$(x_i|\gamma^2)=0,$$

reproduces Eqs. (7a) and (7b). Therefore, we cannot hope to nullify simultaneously any set of three coefficients formed by pair (9) and any one of the three coefficients (8). From the above considerations it follows that it is impossible to nullify simultaneously any set of four coefficients. Thus, the next step in our analysis will be to consider the sets of three coefficients not yet examined. These coefficients can be grouped as follows:

$$(x_i|x_s^2)\quad (x_i|\theta_s^2)\quad (x_i|x_s\gamma)$$
$$(x_i|x_s^2)\quad (x_i|\theta_s^2)\quad (x_i|\theta_s\gamma)$$
$$(x_i|x_s^2)\quad (x_i|\theta_s^2)\quad (x_i|\gamma^2)$$
$$(x_i|x_s\theta_s)\quad (x_i|\theta_s^2)\quad (x_i|x_s\gamma)$$
$$(x_i|x_s\theta_s)\quad (x_i|\theta_s^2)\quad (x_i|\theta_s\gamma)$$
$$(x_i|x_s\theta_s)\quad (x_i|\theta_s^2)\quad (x_i|\gamma^2).$$

It can be seen that each set of three equations obtained by equating to zero the analytical expressions (given in Table II) for the coefficients of each row of the above table admits Eqs. (7a) and (7b) as the sole solution. Neverthe-less, for $0<\alpha<\pi$ these solutions are not significant.

TABLE III. This table has been deduced from Table II for $\alpha<\pi$ and for symmetrical focusing $[\beta_1=\beta_2=0,\ z_0=z_i=\cot(\alpha/2)]$.

Coefficients	Analytical expressions
$(x_i\|x_s)$	-1
$(x_i\|\theta_s)$	0
$(x_i\|\gamma)$	2
$(x_i\|x_s^2)$	$-\frac{1}{2}\sin^2\alpha+\frac{1}{2}[1/R_1+(1/R_2)\cos^2\alpha]\cot(\alpha/2)$
$(x_i\|x_s\theta_s)$	$-\sin\alpha+[1/R_1+(1/R_2)\cos\alpha]\cot^2(\alpha/2)$
$(x_i\|x_s\gamma)$	$\sin^2\alpha+\cos\alpha+1+(1/R_2)\sin\alpha\cos\alpha$
$(x_i\|\theta_s^2)$	$\frac{1}{2}(1/R_1+1/R_2)\cot^3(\alpha/2)-1$
$(x_i\|\theta_s\gamma)$	$\sin\alpha+2\cot(\alpha/2)+(1/R_2)(1+\cos\alpha)$
$(x_i\|\gamma^2)$	$-\frac{1}{2}\sin^2\alpha-\cos\alpha-1+(1/2R_2)(1-\cos\alpha)\sin\alpha$

Let us turn now to the pairs of coefficients. If we neglect the following pairs:

$$(x_i|x_s\gamma)\quad (x_i|\theta_s\gamma)$$
$$(x_i|x_s\gamma)\quad (x_i|\gamma^2)$$
$$(x_i|\theta_s\gamma)\quad (x_i|\gamma^2),$$

which we have seen cannot be made zero simultaneously, 12 pairs remain to be considered.[9] We can now remark that for a given value of α (less than π), we dispose of six parameters (R_1, R_2, β_1, β_2, z_0, and z_i) and of only three equations. These are given by Eq. (5) and by those two equations obtained by equating to zero the expressions of the coefficients of the selected pair. The number of variables being redundant with respect to number of equations, we can therefore assign three additional conditions. With the aim to obtain simplified expressions, we will put $\beta_1=0$, $\beta_2=0$, and $z_0=z_i$ (even if this choice is obviously not the only possible one). For the above choice, Eq. (5) becomes

$$z_i^2\sin\alpha-2z_i\cos\alpha-\sin\alpha=0.$$

This admits only the positive root

$$z_i=(1+\cos\alpha)/\sin\alpha=\cot(\alpha/2).$$

When this value of z_i is inserted into the expressions of the coefficients listed in Table II, these expressions become those of Table III. It is only a matter of simple algebra to show that it is possible to nullify, in the case of symmetrical focusing, each of the 12 pairs listed in Table IV assuming for R_1 and R_2 the expressions reported in the same row of such a table.

TABLE IV. Analytical expressions for the curvature of the entrance and exit pole edges that nullify the pairs of second order coefficients specified on the same row of the table. The results are valid for $\alpha<\pi$ and for symmetrical focusing $[\beta_1=\beta_2=0,\ z_0=z_i=\cot(\alpha/2)]$.

Pairs of coefficients	$1/R_1$	$1/R_2$
$(x_i\|x_s^2)$ $(x_i\|x_s\theta_s)$	$\tan(\alpha/2)$	$-\tan(\alpha/2)$
$(x_i\|x_s^2)$ $(x_i\|\theta_s^2)$	$[1-2\tan^2(\alpha/2)\cot^2\alpha]\tan(\alpha/2)$	$[1-2\csc^2\alpha\tan(\alpha/2)]\tan(\alpha/2)$
$(x_i\|x_s\theta_s)$ $(x_i\|\theta_s^2)$	$\tan^3(\alpha/2)$	$\tan^3(\alpha/2)$
$(x_i\|x_s^2)$ $(x_i\|x_s\gamma)$	$\cot(\alpha/2)$	
$(x_i\|x_s\theta_s)$ $(x_i\|x_s\gamma)$	$\dfrac{3-\cos\alpha}{\sin\alpha}$	$-\dfrac{\sin\alpha+\cot(\alpha/2)}{\cos\alpha}$
$(x_i\|\theta_s^2)$ $(x_i\|x_s\gamma)$	$2\tan^3(\alpha/2)+\dfrac{\sin\alpha+\cot(\alpha/2)}{\cos\alpha}$	
$(x_i\|x_s^2)$ $(x_i\|\theta_s\gamma)$	$\sin^2\alpha\tan(\alpha/2)+(3-\cos\alpha)\cos\alpha\cot\alpha$	
$(x_i\|x_s\theta_s)$ $(x_i\|\theta_s\gamma)$	$\cot(\alpha/2)$	$\dfrac{\cos\alpha-3}{\sin\alpha}$
$(x_i\|\theta_s^2)$ $(x_i\|\theta_s\gamma)$	$2\tan^3(\alpha/2)+\dfrac{3-\cos\alpha}{\sin\alpha}$	
$(x_i\|x_s^2)$ $(x_i\|\gamma^2)$	$[1-2\cot\alpha\cot(\alpha/2)]\cot(\alpha/2)$	
$(x_i\|x_s\theta_s)$ $(x_i\|\gamma^2)$	$\tan(\alpha/2)-2\csc^2(\alpha/2)\cot\alpha$	$[1+\csc^2(\alpha/2)]\cot(\alpha/2)$
$(x_i\|\theta_s^2)$ $(x_i\|\gamma^2)$	$2\tan^3(\alpha/2)-[1+\csc^2(\alpha/2)]\cot(\alpha/2)$	

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[1] A list of papers concerning the partial elimination of aberrations in this kind of spectrometers can be found in the bibliography of the book: J. J. Livingood, *The Optics of Dipole Magnets* (Academic, New York, 1969), p. 238; see also Chap. 18.

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[9] These pairs are those listed in the first column of Table IV.