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1. - INTRODUCTION -

As well known, scale invariance[(1)] in conjunction with Wilson's operator product expansion[(2)] appears to play an essential role in the setting up of a theoretical frame for the interpretation of experiments, such as deep inelastic lepton scattering, which in configuration space depend on the behaviour near the light-cone. It has been suggested that the stronger conformal invariance may indeed apply in such limiting conditions and its implications for equal-time commutators[(3)] and for operator product expansions[(4,5,6)] have been discussed. In particular in refs. (5) and (6) the implications of conformal invariance on the operator product expansion on the light-cone[(2,7)] have been fully derived, by a method using the Jacobi identities in refs. (4) and (5), and by an alternative manifestly covariant method in ref. (6). The latter method takes advantage of the isomorphism between the conformal algebra and the orthogonal algebra $0(4,2)$[(8)] and makes use of a six-dimensional pseudoeuclidean coordinate space. In this note we shall further develop such method to derive a manifestly conformal covariant operator product expansion valid at all values of x^2. We must stress that whereas conformal invariance (as well as scale invariance) of the complete theory may in fact under some form hold near the light-cone, we must ex-

(x) - Istituto di Fisica dell'Università di Roma e Istituto Nazionale di Fisica Nucleare, Sezione di Roma.

clude the possibility that the complete theory is fully conformal (as well as scale) invariant. Nevertheless the implications of exact conformal invariance, not restricted to the light-cone alone, might still prove useful in discussing the properties of the so-called skeleton theory[2], and fundamental problems such as that of canonical dimensions, etc. In view of this, it appears as a relevant step to obtain a fully conformal covariant operator product expansion. In addition the causality restriction has to be discussed which, as well known, is extremely stringent when the entire group of conformal trasformations is considered, which contains transformations of fourvectors from inside to outside the light-cone and viceversa.

2. - MANIFESTLY CONFORMAL COVARIANT OPERATOR PRODUCT EXPANSION. -

We limit ourselves for simplicity to the expansion of a product $A(x)B(x')$ where $A(x)$, $B(x)$ are two Lorentz scalars with $K_\lambda = 0$, i.e. satisfying $[A(0), K_\lambda] = 0$ and the same for $B(0)$ (conformal scalars). In the covariant six-dimensional formalism with coordinates η_A($A = 0, 1, 2, 3, 5, 6$) the general form of the expansion is[9,17]

$$A(\eta)B(\eta') = \sum_{n=0}^{\infty} E_n(\eta\cdot\eta')D^{(n)A_1\ldots A_n}(\eta, \eta')\,\psi_{A_1\ldots A_n}(\eta') \tag{1}$$

where: $D^{(n)A_1\ldots A_n}(\eta, \eta')$ is an "orbital" tensor operator defined over $\eta^2 = 0 = \eta'^2$ and regular at $\eta\cdot\eta' = 0$; $E_n(\eta\cdot\eta')$ is a c-number of the form

$$E_n(\eta\cdot\eta') = \gamma_n(\eta\cdot\eta')^{1/2(\lambda_A + \lambda_B - \lambda_n - n)} \tag{2}$$

and $\psi_{A_1\ldots A_n}(\eta)$ are irreducible tensor representations of SU (2,2), (the spinor group associated to 0(4,2)) homogeneous of degree $\lambda_n = -l_n$, containing Lorentz tensors of maximum order n, and satisfying supplementary conditions $\eta^{A_1}\psi_{A_1\ldots A_n}(\eta) = 0$ and $\partial^{A_1}\psi_{A_1\ldots A_n}(\eta) = 0$. The expansion in Eq. (1) is manifestly (conformal covariant on the hypercones $\eta^2 = 0$ and $\eta'^2 = 0$. In ref. (6) it was noted that the most general form of the operator $D^{(n)A_1\ldots A_n}(\eta, \eta')$ is (see Eq. 10) of ref. (6))

$$\begin{aligned} D^{(n)A_1\ldots A_n}(\eta, \eta') = \sum_{m=0}^{n} \eta^{A_1}\ldots\eta^{A_{n-m}}\eta'^{A_{n-m+1}}\ldots\ldots \\ \ldots\eta'^{A_n} D^{(n_1 m)}(\eta, \eta')\, C_{nm} \end{aligned} \tag{3}$$

where C_{nm} are constants, $D^{(n,m)}(\eta, \eta')$ is a differential operator defined on $\eta^2 = 0 = \eta'^2$ and homogeneous of degree $h = 1/2(\lambda_A - \lambda_B + \lambda_n - n) + m$ in k/k' ($\lambda_A, \lambda_B, \lambda_n$ are the degrees of homogeneity of A, B and $\psi_{A_1..A_n}$, and we recall that $x_\mu = k^{-1}\eta_\mu$ $k = \eta_5 + \eta_6$). The operator $D^{(n,m)}$ is uniquely given as formal power

(4) $$D^{(n,m)}(\eta, \eta') = D^h(\eta, \eta')$$

where

(5) $$D(\eta, \eta') = \eta \cdot \eta' \,\Box'_6 - 2\eta \cdot \partial'(1 + \eta' \cdot \partial')$$

Indeed, $D(\eta, \eta')$ is the only operator defined on the hypercones and homogeneous of degree one in k/k'. We note that the two terms in $D(\eta, \eta')$, Eq. (5), both contain a term proportional to $\partial/\partial \eta'^2$, and their sum is indeed the only combination which remains well-defined on $\eta'^2 = 0$. It can be rewritten as

(6) $$D(\eta, \eta') = (\eta \cdot \eta')^{-1} \eta^A \eta^C g^{BD} L'_{AB} L'_{CD}$$

where $L'_{AB} = i(\eta'_A \partial'_B - \eta'_B \partial'_A)$ are the orbital generators of 0(4,2). For $\eta \cdot \eta' = 0$ (corresponding to the light-cone, $(x-x')^2=0$) $D(\eta, \eta') \propto \eta \cdot \partial'$ and one recovers the result of ref. (6).

We shall now state some useful lemmas:

I) The n+1 covariants defined in Eq. (3) are all proportional. From the supplementary conditions

(7) $$\eta^{A_1} \psi_{A_1 A_2 .. A_n}(\eta) = 0 \qquad \partial^{A_1} \psi_{A_1 A_2 .. A_n}(\eta) = 0$$

one has

(8) $$\eta'^{A_1} D^h(\eta, \eta') \psi_{A_1 .. A_n}(\eta') = \eta^{A_1} 2h(2-h+\lambda_n) D^{h-1}(\eta, \eta') \psi_{A_1 .. A_n}(\eta')$$

II) One has:

(9) $$D^h(\eta, \eta') = (L-1)^h \sum_{J=0}^{h} \binom{h}{J} \frac{(\eta \cdot \eta')^J (2\eta \cdot \partial')^{h-J} \Box'^J_6}{(L-1)^J}$$

$$L = -k' \frac{\partial}{\partial k'} \text{ and } (L-1)^J = (1_n-1)(1_n-1+1)\dots(1_n-1+J-1)$$

In terms of the variables (x, k) one has the identities[10]

(10) $$\Box'_6 - 4(1-L)\frac{\partial}{\partial \eta'^2} = \frac{1}{k'^2}\Box'_4$$

(11) $$\eta\cdot\partial' - 2(\eta\cdot\eta')\frac{\partial}{\partial \eta'^2} = \frac{k}{k'}\left[(x-x')\partial' - L\right]$$

(12) $$D(\eta,\eta') = 2\frac{k}{k'}\left[(L-(x-x')\partial')(1-L) - \left(\frac{x-x'}{2}\right)^2\Box'_4\right]$$

which together with Eq. (14) of ref. (6) allow us to write Eq. (9) as

(13) $$D^h(\eta,\eta') = \left(\frac{k}{k'}\right)^h(-2)^h\frac{\Gamma(1_n-1+h)}{\Gamma(1_n-1)}\sum_{J=0}^{\infty}\frac{1}{J!}\frac{\Gamma(-h+J)}{\Gamma(-h)}\cdot$$
$$\cdot\frac{\Gamma(1_n-1)}{\Gamma(1_n-1+J)}\frac{\Gamma(1_n+h+J)}{\Gamma(1_n+2J)}\left(-\left(\frac{x-x'}{2}\right)^2\right)^J {}_1F_1(J-h, 1_n+2J; (x-x')\partial')\Box'^J$$

The lemmas (I) and (II) can be proved by induction for integer h and then extended through analytical continuation to all values of h[11]; we also used the identity

$$(-\eta\cdot\partial')^\beta = \left(\frac{k}{k'}\right)^\beta\frac{\Gamma(-\lambda+\beta)}{\Gamma(-\lambda)}\;{}_1F_1(-\beta_1-\lambda;(x-x')\partial')$$

where λ is the homogeneity degree of the operator on which $(\eta\,\partial')^\beta$ is acting (see ref. (6)). We note that, for h positive integer, the terms with $J > h$ in Eq. (13) all vanish. Inserting into Eq. (13) the well-known integral representation for ${}_1F_1$ (see refs. (5) and (6)) one obtains

(14) $$D^h(\eta,\eta') = \left(\frac{k}{k'}\right)^h(-2)^h\frac{\Gamma(1_n-1+h)}{\Gamma(-h)\Gamma(1_n-1)}\sum_{J=0}^{\infty}\frac{1}{J!}\frac{\Gamma(1_n-1)}{\Gamma(1_n-1+J)}\cdot$$
$$\cdot\left[-\left(\frac{x-x'}{2}\right)^2\right]^J\cdot\int_0^1 du\, u^{-h+J-1}(1-u)^{1_n+h+J-1}e^{u(x-x')\partial'}\Box'^J$$

and after performing the summation over J

$$D^h(\eta,\eta') = (\frac{k}{k'})^h(-2)^h \frac{\Gamma(l_n-1+h)}{\Gamma(-h)\Gamma(l_n-1)} \int_o^1 du\, u^{-h-1}(1-u)^{l_n+h-1} \cdot \tag{15}$$
$$\cdot\, e^{u(x-x')\partial'}\; {}_oF_1(l_n-1;\, -(\frac{x-x'}{2})^2 \Box' u(1-u))$$

Eq. (15) is meant as a formal operator expression with all derivatives located at the right and acting to the right. Using Eq. (15) one has for the expansion in Eq. (1)

$$A(x)B(x') = \sum_{n=0}^{\infty} \left[\frac{1}{(x-x')^2}\right]^{1/2(l_A+l_B+n-l_n)} C_n^{AB} \cdot$$
$$\cdot \int_o^1 du\, u^{1/2(l_A-l_B+l_n+n)-1} (1-u)^{1/2(l_B-l_A+l_n-n)-1} e^{u(x-x')\partial'} \cdot \tag{16}$$
$$\cdot\; {}_oF_1(l_n-1;\, -(\frac{x-x'}{2})^2 \Box' u(1-u)) x^{A_1} .. x^{A_n} \psi_{A_1 .. A_n}(x')$$

where $\eta^A = k x^A$ and $\psi_{A_1 .. A_n}(x) = k^{l_n} \psi_{A_1 .. A_n}(\eta)$. The tensors $O_{A_1 .. A_n}(x)$ which transform according to the conformal algebra of space-time are given (see ref. (6) by

$$O_{A_1 .. A_n}(x) = (e^{-ix\cdot\pi} \psi)_{A_1 .. A_n}(x) \tag{17}$$

where the "internal generator" $\pi_\mu = S_{6\mu} + S_{5\mu}$ acts as

$$(\pi_\mu O)_{A_1 .. A_n}(0) = i \sum_{i=1}^{n} (g_{6A_i} + g_{5A_i}) O_{A_1 .. \hat{A}_i .. A_n \mu}(O) - \tag{18}$$
$$- g_{\mu A_i}(O_{A_1 .. \hat{A}_i .. A_n 6}(0) + O_{A_1 .. \hat{A}_i .. A_n 5}(0))$$

(the notation $O_{A_1 .. \hat{A}_i .. A_n}$ means that A; is omitted) and the components $O_{\alpha_1 .. \alpha_J xx ..}$ (where x are 5 or 6) can be obtained through the supplemen tary conditions (Eq. (7))[12]

$$(19)\qquad O_{\alpha_1 \cdot\cdot \alpha_{n-k}\, 66 \cdot\cdot}(x) = 2^{-K} \frac{\Gamma(l_n-2-n)}{\Gamma(l_n-2-n+k)}\, \partial_{\mu_1} \cdot\cdot\, \partial_{\mu}\, O^{\mu_1 \cdot\cdot \mu_K}_{\alpha_1 \cdot\cdot \alpha_{n-k}}(x)$$

(the substitution 5 or 6 does not change the value on account of the first supplementary condition). The covariant product in Eq. (16) can be written as

$$(20)\qquad \begin{aligned} x^{A_1} \ldots x^{A_n}\, \psi_{A_1 \cdot\cdot A_n}(x') &= \sum_{J=0}^{n} \binom{n}{J} \frac{\Gamma(l_n-2-n)}{\Gamma(l_n-2-n+J)} \left[-\frac{1}{2}(x-x')^2 \right]^J \\ &\cdot (x-x')^{\alpha_1} \cdot\cdot (x-x')^{\alpha_{n-J}}\, \partial'_{\mu_1} \cdot\cdot\, \partial'_{\mu_J}\, O^{\mu_1 \cdot\cdot \mu_J}_{\alpha_1 \cdot\cdot \alpha_{n-J}}(x') \end{aligned}$$

Finally, taking $x' = 0$, from Eq. (16) one obtains[13]

$$(21)\qquad \begin{aligned} A(x)B(0) &= \sum_{n=0}^{\infty} \left(\frac{1}{x^2}\right)^{1/2(l_A+l_B+n-l_n)} C_n^{AB} \int_o^1 du\, u^{1/2(l_A-l_B+l_n+n)-1} \\ &\cdot (1-u)^{1/2(l_B-l_A+l_n-n)-1}\, x_u^{A_1} \cdot\cdot x_u^{A_n} \Big[{}_oF_1\Big(l_n-1; \frac{-x^2}{4}\, u(1-u) \cdot \\ &\cdot (u^{-2}\Box + 2iu^{-1}\pi\cdot\partial) - \pi^2\Big)\Big) O \Big]_{A_1 \cdot\cdot A_n}(ux) \end{aligned}$$

where

$$x_{u_A} \equiv \left\{ (1-u)x_\mu, \frac{1}{2}\left[1+(1-u)^2 x^2\right], \frac{1}{2}\left[1-(1-u)^2 x^2\right] \right\}$$

3. - PROPERTIES OF THE EXPANSION -

The expansion obtained, Eqs. (16) and (21) is conformally covariant, as it is evident from its derivation the form in Eq. (1). The form obtained here is particularly interesting since it reduces directly to the light-cone expansion of ref. (4) and (5) at $x^2 = o$, where the ${}_0F_1$ function beco mes ${}_0F_1(l_n-1;\,0) = 1$ and one can write $x^{A_1}..x^{A_n}\psi_{A_1..A_n}(x') = (x-x')^{\alpha_1}..(x-x')^{\alpha_n}O_{\alpha_1...\alpha_n}(x')$; non leading terms in x^2 can simply be obtained by expanding ${}_0F_1$, in power series in x^2. We now come to an interesting selection rule, which one obtains by noting that Eq. (20) is clearly unacceptable for $l_n = 2 + n$, i.e. for "canonical dimensions" of non-scalar representations. The reason is that Eq. (7) cannot be imposed for $l_n = 2 + n$ unless $O_{\alpha_1..\alpha_n}(x)$ is conserved (compare with the necessary and sufficient condition for canonical dimensions given in ref. (4)), but then all components in Eq. (19) are undefined. One can show[14] that the pathology of the components in Eq. (19) for $l_n = 2 + n$ is reflected in the circumstance that the homogeneous set of equations, giving the coefficients of the contribution of a representation of spin n to the expansion in eq. (21), has for $l_n = 2+n$, eigensolutions only for $l_A = l_B$[19]. A complete discussion of the properties of representations with canonical-dimension will be given in a forthcoming paper. We also observe that this case includes the free-field theory i.e. conserved four-tensors (see ref. (4)). Therefore the conformal covariance of Eq. (21) implies that the contribution of spin n occurs only for $l_A = l_B$[15, 19]. For $l_A = l_B$ Eq. (16) (we actually take A = B) becomes

$$A(x)A(0) = \sum_{n=0}^{\infty} \left(\frac{1}{x^2}\right)^{l_A - 1} C_n \int_0^1 du \left[u(1-u)\right]^n e^{ux\cdot\partial} \,\cdot$$

(22)

$$\cdot\; {}_0F_1\left(n+1;\, -\frac{x^2}{4}\Box u(1-u)\right) x^{\alpha_1}..x^{\alpha_n} O_{\alpha_1..\alpha_n}(0)$$

Finally we show how the covariant expansion can be derived from the three-point function. We limit ourselves to the scalar contribution to the expansion (for simplicity) and write

$$A(x)B(0) = \left(\frac{1}{x^2}\right)^{1/2(l_A+l_B-l)} \int_0^1 du\, u^{1/2(l_A-l_B+l)-1} \,\cdot$$

(23)

$$\cdot\; (1-u)^{1/2(l_B-l_A+l)-1}\, e^{ux\cdot\partial}\; {}_0F_1\left(l_n-1;\, -\frac{x^2}{4}\Box u(1-u)\right) O(0)$$

On the other hand (see for instance ref. (6))

$$\langle 0|C(y)A(x)B(0)|0\rangle = C_{ABC}\left[\frac{1}{(y-x)^2}\right]^{1/2(l_C+l_A-l_B)} \cdot \tag{24}$$

$$\cdot \left[\frac{1}{x^2}\right]^{1/2(l_A+l_B-l_C)}\left[\frac{1}{y^2}\right]^{1/2(l_C+l_B-l_A)}$$

We use the identity[18]

$$\left[\frac{1}{y^2}\right]^{1/2(l_C+l_B-l_A)}\left[\frac{1}{(y-x)^2}\right]^{1/2(l_C+l_A-l_B)} \propto \int_0^1 du\, u^{1/2(l_C+l_A+l_B)-1} \cdot$$

$$\cdot (1-u)^{1/2(l_C+l_B-l_A)-1}\left[(y-ux)^2\right]^{-l_C}\left[1+x^2\frac{u(1-u)}{(y-ux)^2}\right]^{-l_C} = \tag{25}$$

$$= \int_0^1 du\, u^{1/2(l_C+l_A-l_B)-1}(1-u)^{1/2(l_C+l_B-l_A)-1}\sum_{h=0}^{\infty}\frac{1}{h!}\frac{\Gamma(l_C+h)}{\Gamma(l_C)} \cdot$$

$$\cdot (-x^2)^h\left[u(1-u)\right]^h\left[\frac{1}{(y-ux)^2}\right]^{l_C+h}$$

We next note that[16]

$$\langle 0|\Box^h C(x)C(y)|0\rangle \propto \Box^h\left[\frac{1}{(x-y)^2}\right]^{l_C} = \tag{26}$$

$$= 4^h\frac{\Gamma(l_C+h)\,\Gamma(l_C-1+h)}{\Gamma(l_C)\,\Gamma(l_C-1)}\left[\frac{1}{(y-x)^2}\right]^{l_C+h}$$

$$\left[\frac{1}{(y-ux)^2}\right]^{l_c+h} = e^{ux\partial}\left(\frac{1}{4}\right)^h\frac{\Gamma(l_c)\,\Gamma(l_c-1)}{\Gamma(l_c+h)\,\Gamma(l_c-1+h)}\langle 0|C(y)\Box^h C(0)|0\rangle \tag{27}$$

The contribution in Eq. (24) thus comes from the expansion

$$\int_0^1 du\, u^{1/2(l_C+l_A-l_B)-1} (1-u)^{1/2(l_C+l_B-l_A)-1}\, 1^{ux\cdot\partial} \sum_{h=0}^{\infty} \frac{1}{h!} \cdot$$

$$(28) \qquad \cdot \frac{\Gamma(l_C-1)}{\Gamma(l_C-1+h)} \left(\frac{-x^2}{4}\right)^h (u(1-u)\Box)^h\, G(0) = \int_0^1 du\, u^{1/2(l_C+l_A-l_B)-1} \cdot$$

$$\cdot (1-u)^{1/2(l_C-l_B-l_A)-1}\, e^{ux\cdot\partial}\, {}_0F_1(l_C-1; \frac{-x^2}{4}\, u(1-u)\Box) C(0)$$

which coincides with that in Eq. (23).

We would like to thank S. Bonora, G. Sartori, and M. Tonin for discussions on the subject.

REFERENCES AND FOOTNOTES -

(1) - See e.g. H.A. Kastrup, Phys. Rev. 142, 1060 (1966); G. Mack, Nuclear Physics B5, 499 (1968); M. Gell-Mann, Lectures at Hawaian Summer School (1969).
(2) - K. Wilson, Phys. Rev. 179, 1499 (1969).
(3) - S. Ciccariello, R. Gatto, G. Sartori and M. Tonin, Annals of Physics, 65, 265 (1971).
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(5) - S. Ferrara, R. Gatto, A.F. Grillo, Phys. Letters 36 B, 124 (1971); and ibidem Erratum.
(6) - S. Ferrara, R. Gatto, A.F. Grillo, Frascati preprint LNF, 71/70 (1971) (to be published).
(7) - R. Brandt and G. Preparata, Nuclear Physics, B 27, 541 (1971); Y. Frishman, Phys. Rev. Letters 25, 966 (1970).
(8) - For a review of the conformal algebra we refer to G. Mack and A. Salam, Annals of Physics 53, 174 (1969).
(9) - The covariant notations in six-dimension are the same used in our previous paper (ref. (6)), to which we refer for all definitions and for a summary of the formalism.
(10) - It is instructive to notice from Eq. (10) that only for $l = 1$ is $\Box_6'$ defined on the hypercone, corresponding to the fact that only for the canonical value $l = 1$ is the Klein-Gordon equation conformally covariant.

(11) - This is a standard method in representation theory.
(12) - A detailed presentation will be given in a forthcoming paper.
(13) - A simpler form of this expansion will be given elsewhere.
(14) - The pathology is associated to the following degeneracy of the representations of the stability algebra at $x = 0$: for $l_n = 2 + n$ both $O_{\alpha_1 \ldots \alpha_n}(0)$ and $\partial^{\alpha_1} O_{\alpha_1 \ldots \alpha_n}(0)$ commute with K_λ (see the theorem of ref. (4) referred to in the text).
(15) - This result can also be understood from translation invariance on hermitean basis. The expansions for $A(x)B(0)$ and $B(x)A(0)$ can be related (see ref. (5)) by a sequence of translation, $x \rightarrow -x$ hermitean conjugation. Then $l_A \leftrightarrow l_B$. However the non-leading terms proportional to divergences have under such sequence of operations behaviour opposite to that of the terms proportional to $\Box$ (dalambertian): they must therefore have a factor $l_A - l_B$.
(16) - One has $l = l_C$ from a selection rule of the conformal algebra on two-point function.
(17) - We observe that we have taken, for simplicity, each irreducible representation $\psi_{A_1 \ldots A_n}(\eta)$ without multiplicity. Obviously in general a sum over the same tensor representation (with possible different homogeneity degrees) is understood.
(18) - The integral representation in Eq. (25) is nothing but a Riemann--Liouville Fractional integral (see Bateman, Tables of Integral Transforms, Vol. 2 pag. 186). The proportionality constant turns out to be

$$\Gamma(l_C) / \Gamma\left(\frac{l_C - l_A - l_B}{2}\right) \Gamma\left(\frac{l_C + l_B - l_A}{2}\right).$$

(19) - It can be shown that for $l_n = 2+n$ the divergences of the tensors $O_{\alpha_1 \ldots \alpha_n}(x)$ can be added as an independent irreducible representation starting by from $\partial^\alpha O_{\alpha \alpha_1 \ldots \alpha_{n-1}}(x)$. This is a consequence of footnote (14).
In the covariant formalism this means that the n-1 order tensor

$$\partial^A \psi_{AA_1 \ldots A_{n-1}}(\eta)$$

is a genuine tensor (it satisfies the supplementary conditions (7)) and its components can be derived as in Eq. (19)).